GOVERNMENT OF TAMIL NADU

PHYSICS

HIGHER SECONDARY FIRST YEAR

VOLUME - 2

Untouchability is Inhuman and a Crime

A publication under Free Textbook Programme of Government of Tamil Nadu

Department of School Education

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Government of Tamil Nadu

First Edition - 2018

NOT FOR SALE

Content Creation

State Council of Educational
Research and Training

© SCERT 2018

Printing & Publishing

Tamil NaduTextbook and Educational
Services Corporation

www.textbooksonline.tn.nic.in

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CONTENTS

PHYSICS

BKTBL2
E-book

BKHY9Y
Assessment

B163_11_Phy_EM
DIGI links

Let's use the QR code in the text books ! How ?
- Download the QR code scanner from the Google PlayStore/ Apple App Store into your smartphone
- Open the QR code scanner application
- Once the scanner button in the application is clicked, camera opens and then bring it closer to the QR code in the text book.
- Once the camera detects the QR code, a url appears in the screen.Click the url and goto the content page.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

HOW TO USE THE BOOK

Scope of Physics	• Awareness on higher learning - courses, institutions and required competitive exams • Financial assistance possible to help students to climb academic ladder
Learning Objectives:	• Overview of the unit • Gives clarity on the goals and intention of the topics
DO YOU KNOW?	• Additional facts related to the topics covered to facilitate curiosity driven learning
Example problems	• To ensure understanding, problems/illustrations are given at every stage before advancing to next level
	• Visual representation of concepts with illustrations • Videos, animations, and tutorials
ICT	• To harness the digital skills to class room learning and experimenting
Summary	• Recap of salient points of the lesson
Concept Map	• Schematic outline of salient learning of the unit
Evaluation	• Evaluate students' understanding and get them acquainted with the application of physical concepts to numerical and conceptual questions
Books for Reference	• List of relevant books for further reading
Solved examples	• Solutions to exercise problems are accessible here. In addition, a few solved examples are given to facilitate students to apply the concepts learnt.
Competitive Exam corner	• Model Questions - To motivate students aspiring to take up competitive examinations such as NEET, JEE, Physics Olympiad, JIPMER etc
Practical	• List of practical and the description of each is appended for easy access.
Glossary	• Scientific terms frequently used with their Tamil equivalents

Back wrapper: Solvay Conference 1927, Belgium

Photograph of the attendees of the most famous fifth solvay international conference held in october 1972. 17 of the 29 scientists found in this photograph are Nobel laureates (shown in bold).

Front row (L to R) : **I. Langmuir**, **M. Planck**, **Marie Curie**, **H.A. Lorentz**, **A. Einstein**, P. Langevin, Ch.E. Guye, **C.T.R. Wilson**, **O.W. Richardson**

Middle row (L to R) : **P. Debye**, M. Knudsen, **W.L. Bragg**, H.A. Kramers, **P.A.M. Dirac**, **A.H. Compton**, **L.de Broglie**, **M. Born**, **N. Bohr**

Back row (L to R) : A. Piccard, E. Henriot, P. Ehrenfest, Ed. Herzen, Th.De Donder, **E. Schrödinger**, E.Verschaffelt, **W. Pauli**, **W. Heisenberg**, R.H. Fowler, L. Brillouin

—Photographie by Benjamin Couprie

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

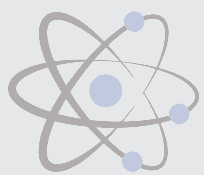
Scope of Physics - Higher Education

Exams

- JEE-Joint Entrance Examination
- Physics Olympiad Exam
- NEET- National Eligibility and Entrance Test
- NEST- National Entrance Screening Test
- AIEEE- All India Engineering Entrance Exam
- AIIMS- All India Institute of Medical Science (Entrance Examination)
- JIPMER- Jawaharlal institute of Postgraduate Medical Education and Research (Entrance Examination)
- KVPY- Kishore Vaigyanik Protsahan Yojana
- JAM- Joint Admission Test
- TIFR GS - Tata Institute of Fundamental Research Graduate School Admissions Examination
- JEST- Joint Entrance Screening Test
- NET- National Eligibility Test (CSIR and UGC)
- GATE-Graduate Aptitude Test in Engineering
- ICAR -AIEEA-Indian Council of Agricultural Research All India Entrance Examination

After completing +2

- B.Sc (Physics)
- Integrated M.Sc (Physics) (Central Universities)
- Integrated M.Sc (in Central Research Institutes through NEST and KVPY with stipend)
- B.Sc./B.S./B.Stat./B.Math./M.S. in Mathematics, Chemistry and Biology. (KVPY)
- B.E/B.Tech/ B.Arch (JEE, AIEEE in IITs and NITs) MBBS/ B.D.S/B.Pharm (NEET, JIPMER, AIIMS)
- B.Sc. (Agriculture) (ICAR -AIEEA)
- Dual Degree Program BS & MS (JEE, JEST in IITs and IISERs)
- B.Sc (Hospitality administration)
- B.Sc (Optoectronics)
- B.Sc (Optometry)
- B.Tech (Optics and Optoelectronics)



After completing undergraduate course in Physics (B.Sc Physics)

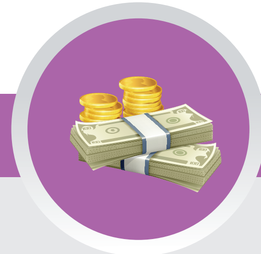
- M.Sc. (Physics) (In Central and State Universities and Colleges)
- M.Sc. Physics (JAM in IISc ,IITs and NITs)
- M.Sc. (In State and Central Universities)
- Medical Physics
- Materials Science
- Energy
- Earth Sciences
- Space science
- Oceanography
- Remote sensing
- Electronics
- Photonics
- Optoelectronics
- Acoustics
- Applied electronics
- Astronomy and Astrophysics
- Nanoscience and Nanotechnology
- Biostatistics
- Bio informatics
- Vacuum sciences

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Opportunities after B.Sc. Physics

Jobs in Government Sector

- Indian Forest Services
- Scientist Job in ISRO, DRDO, CSIR labs
- Union Public Service Commission
- Staff Selection Commission
- Indian Defence Services etc.
- Public Sector Bank
- State PCS
- Grade III & Compiler Post
- Tax Assistant
- Statistical Investigator
- Combined Higher Secondary
- Combined Graduate Level Exam etc.



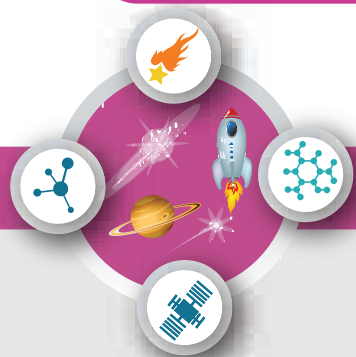
Financial assistance to pursue higher education

Scholarships for graduate and post graduate courses

- International Olympiad: for getting stipend for Higher Education in Science and Mathematics
- DST – INSPIRE Scholarships (for UG and PG)
- DST – INSPIRE Fellowships (for Ph.D)
- UGC National Fellowship (for Ph.D)
- Indira Gandhi Fellowship for Single Girl Child (for UG and PG)
- Moulana Azad Fellowship for Minorities (for Ph.D)
- In addition various fellowships for SC/ST/PWD, OBC etc. are available.
- Visit website of University Grants Commission (UGC) and Department of Science and Technology (DST)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Institutes in india to pursue research in physics



After completing Post Graduate course in Physics (M.Sc Physics)

Topics of Research

- Quantum Physics and Quantum Optics
- Astrophysics, Astronomy
- String theory, Quantum Gravity
- Mathematical Physics, Statistical Mechanics
- Quantum Field Theory
- Particle Physics and Quantum Thermodynamics
- Quantum Information Theory
- Condensed Matter Physics, Materials Science
- Electro Magnetic Theory
- Black Holes, Cosmology
- Crystal Growth, Crystallography
- Spectroscopy, Atomic, Molecular and Optical Physics
- Nano Science and Nanotechnology
- Energy and Environment Studies
- Biophysics, Medical Physics
- Cryptography, Spintronics
- Optics and Photonics
- Meteorology and Atmospheric Science

Research Institutions in various areas of science

Name of the Institution	Website
Indian Institute of Science (IISc) Bangalore	www.iisc.ac.in
Raman Research Institute (RRI) Bangalore	www.rri.res.in
Institute of Mathematical Sciences (IMSc) Chennai	www.imsc.res.in
Indian Association for Cultivation of Science (IACS) Calcutta	www.iacs.res.in
Chennai Mathematical Institute (CMI) Chennai	www.cmi.ac.in
Tata Institute of Fundamental Research (TIFR) Mumbai	www.tifr.res.in
Bhaba Atomic Research Centre (BARC) Mumbai	www.barc.gov.in
SN Bose centre Basic Natural science Calcutta	www.bose.res.in
Indian Institute of Space Science and Technology (IIST) Trivandrum	www.iist.ac.in
Physics Research Laboratory (PRL) Ahmedabad	www.prl.res.in
Indian Institute of Astrophysics (IIA) Bangalore	www.iiap.res.in
Institute of Physics (IOP) Bhubaneswar	www.iopb.res.in
Institute for Plasma Research (IPR) Gujarat	www.ipr.res.in
Inter University Centre for Astronomy and Astrophysics (IUCAA) Pune	www.iucaa.in
Indira Gandhi Centre for Atomic Research (IGCAR), Kalpakkam	www.igcar.gov.in
Hyderabad Central University, Hyderabad	www.uohyd.ac.in
Delhi University, Delhi	www.du.ac.in
Mumbai University, Mumbai	www.mu.ac.in
SavithiribaiPhule Pune university, Pune	www.unipune.ac.in
National Institute of Science Education and Research (NISER), Bhubaneshwar	www.niser.ac.in
IISER Educational Institutions	www.iiseradmission.in
Indian Institute of Technology in various places (IIT's)	www.iitm.ac.in
National Institute of Technology (NITs)	www.nitt.edu
Jawaharlal Nehru University (JNU)	www.jnu.ac.in
Central Universities	www.ugc.ac.in
State Universities	https://www.ugc.ac.in
CSIR – Academy (National laboratories, Delhi, Hyderabad, Trivandrum, Chennai, Calcutta etc)	

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

UNIT 6

GRAVITATION

"The most remarkable discovery in all of astronomy is that the stars are made up of atoms of same kind as those in the Earth" – Richard Feynman

Learning Objectives

In this unit, the student is exposed to

- Kepler's laws for planetary motion
- Newton's law of gravitation
- connection between Kepler's laws and law of gravitation
- calculation of gravitational field and potential
- calculation of variation of acceleration due to gravity
- calculation of escape speed and energy of satellites
- concept of weightlessness
- advantage of heliocentric system over geocentric system
- measurement of the radius of Earth using Eratosthenes method
- recent developments in gravitation and astrophysics

BL37MP

6.1 INTRODUCTION

We are amazed looking at the glittering sky; we wonder how the Sun rises in the East and sets in the West, why there are comets or why stars twinkle. The sky has been an object of curiosity for human beings from time immemorial. We have always wondered about the motion of stars, the Moon, and the planets. From Aristotle to Stephen Hawking, great minds have tried to understand the movement of celestial objects in space and what causes their motion.

The 'Theory of Gravitation' was developed by Newton in the late 17th century to explain the motion of celestial objects and terrestrial objects and answer most of the queries raised. In spite of the study of gravitation and its effect on celestial objects, spanning last three centuries, "gravitation" is still one of the active areas of research in physics today. In 2017, the Nobel Prize in Physics was given for the detection of 'Gravitational waves' which was theoretically predicted by Albert Einstein in the year 1915. Understanding planetary motion, the formation of stars and galaxies, and recently massive objects like black holes and their life cycle have remained the focus of study for the past few centuries in physics.

Geocentric Model of Solar System

In the second century, Claudius Ptolemy, a famous Greco-Roman astronomer,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

developed a theory to explain the motion of celestial objects like the Sun, the Moon, Mars, Jupiter etc. This theory was called the geocentric model. According to the geocentric model, the Earth is at the center of the universe and all celestial objects including the Sun, the Moon, and other planets orbit the Earth. Ptolemy's model closely matched with the observations of the sky with our naked eye. But later, astronomers found that even though Ptolemy's model successfully explained the motion of the Sun and the Moon up to a certain level, the motion of Mars and Jupiter could not be explained effectively.

Heliocentric Model of Nicholas Copernicus

In the 15th century, a Polish astronomer, Nicholas Copernicus (1473-1543) proposed a new model called the 'Heliocentric model' in which the Sun was considered to be at the center of the solar system and all planets including the Earth orbited the Sun in circular orbits. This model successfully explained the motion of all celestial objects.

Around the same time, Galileo, a famous Italian physicist discovered that all objects close to Earth were accelerated towards the Earth at the same rate. Meanwhile, a noble man called Tycho Brahe (1546-1601) spent his entire lifetime in recording the observations of the stellar and planetary positions with his naked eye. The data that he compiled were analyzed later by his assistant Johannes Kepler (1571–1630) and eventually the analysis led to the deduction of the laws of the planetary motion. These laws are termed as 'Kepler's laws of planetary motion'.

6.1.1 Kepler's Laws of Planetary Motion

Kepler's laws are stated as follows:

1. **Law of orbits:**
   Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci.

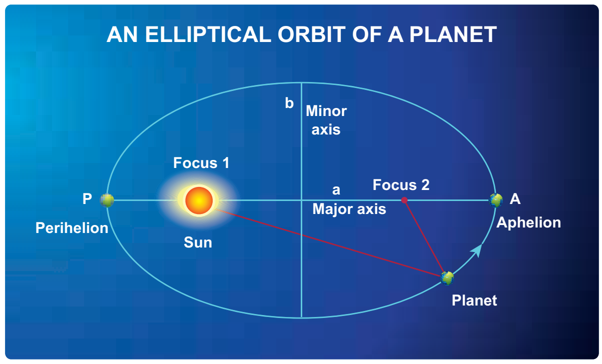


Figure 6.1 An ellipse traced out by a planet around the Sun.

The closest point of approach of the planet to the Sun 'P' is called perihelion and the farthest point 'A' is called aphelion (Figure 6.1). The semi-major axis is 'a' and semi-minor axis is 'b'. In fact, both Copernicus and Ptolemy considered planetary orbits to be circular, but Kepler discovered that the actual orbits of the planets are elliptical.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

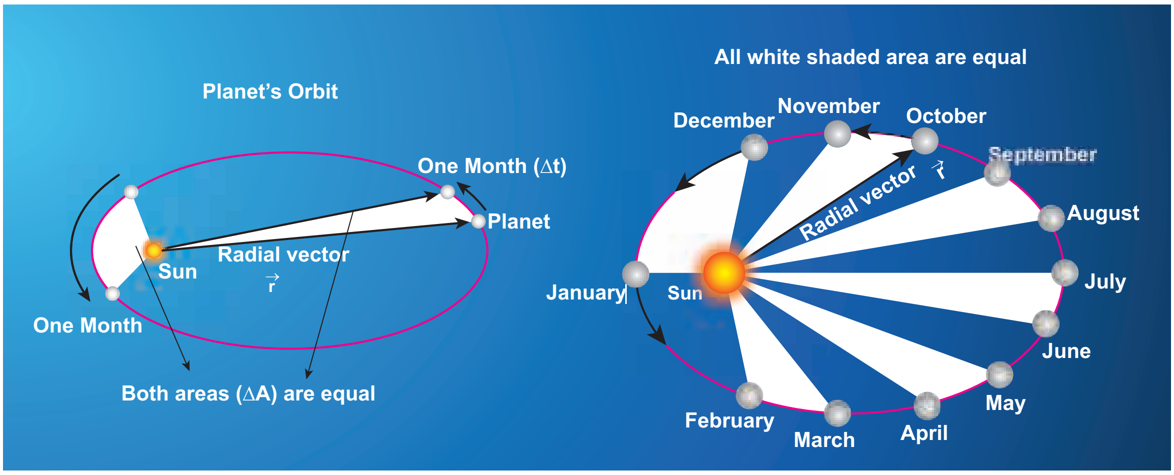


Figure 6.2 Motion of a planet around the Sun depicting 'law of area'.

2. **Law of area:**

The radial vector (line joining the Sun to a planet) sweeps equal areas in equal intervals of time.

In Figure 6.2, the white shaded portion is the area ΔA swept in a small interval of time Δt, by a planet around the Sun. Since the Sun is not at the center of the ellipse, *the planets travel faster when they are nearer to the Sun and slower when they are farther from it, to cover equal area in equal intervals of time.* Kepler discovered the law of area by carefully noting the variation in the speed of planets.

3. **Law of period:**

The square of the time period of revolution of a planet around the Sun in its elliptical orbit is directly proportional to the cube of the semi-major axis of the ellipse. It can be written as:

$$T^2 \propto a^3 \qquad (6.1)$$

$$\frac{T^2}{a^3} = constant \qquad (6.2)$$

where, T is the time period of revolution for a planet and a is the semi-major axis. Physically this law implies that as the distance of the planet from the Sun increases, the time period also increases but not at the same rate.

In Table 6.1, the time period of revolution of planets around the Sun along with their semi-major axes are given. From column four, we can realize that $\frac{T^2}{a^3}$ is nearly a constant endorsing Kepler's third law.

Table 6.1 The time period of revolution of the planets revolving around the Sun and their semi-major axes.

Planet	a ($10^{10}\,m$)	T (years)	$\frac{T^2}{a^3}$
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Points to Contemplate

DATA			PROBLEM
Planet	a	T	What is the law connecting *a* and T?
A	1	3	
B	2	6	
C	4	18	

Comment on the relation between *a* and T for these imaginary planets

6.1.2 Universal Law of Gravitation

Even though Kepler's laws were able to explain the planetary motion, they failed to explain the forces responsible for it. It was Isaac Newton who analyzed Kepler's laws, Galileo's observations and deduced the law of gravitation.

Newton's law of gravitation states that a particle of mass M_1 attracts any other particle of mass M_2 in the universe with an attractive force. The strength of this force of attraction was found to be directly proportional to the product of their masses and is inversely proportional to the square of the distance between them. In mathematical form, it can be written as:

$$\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r} \qquad (6.3)$$

where $\hat{r}$ is the unit vector from M_1 towards M_2 as shown in Figure 6.3, and G is the Gravitational constant that has the value of $6.626\times10^{-11}\,N\,m^2kg^{-2}$, and r is the distance between the two masses M_1 and M_2. In Figure 6.3, the vector $\vec{F}$ denotes the gravitational force experienced by M_2 due to M_1. Here the negative sign indicates that the gravitational force is always attractive in nature and the direction of the force is along the line joining the two masses.

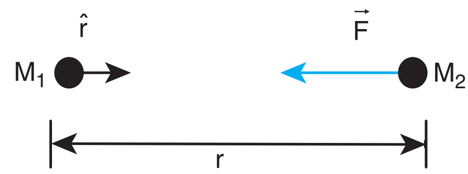
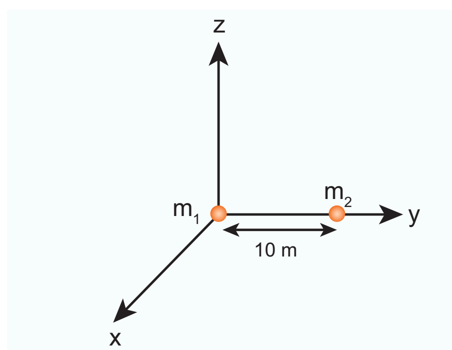


Figure 6.3 Attraction of two masses towards each other.

In cartesian coordinates, the square of the distance is expressed as $r^2 = (x^2 + y^2 + z^2)$. This is dealt in unit 2.

EXAMPLE 6.1

Consider two point masses m_1 and m_2 which are separated by a distance of 10 meter as shown in the following figure. Calculate the force of attraction between them and draw the directions of forces on each of them. Take m_1 = 1 kg and m_2 = 2 kg



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution

The force of attraction is given by

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

From the figure, r =10 m.

First, we can calculate the magnitude of the force

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{100}$$
$$= 13.34\times10^{-13}N.$$

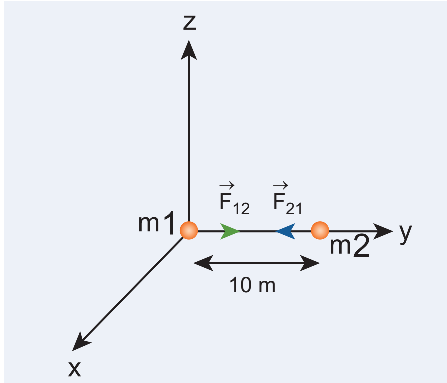
It is to be noted that this force is very small. This is the reason we do not feel the gravitational force of attraction between each other. The small value of G plays a very crucial role in deciding the strength of the force.

The force of attraction ($\vec{F}_{21}$) experienced by the mass m_2 due to m_1 is in the negative 'y' direction ie., $\hat{r} = -\hat{j}$. According to Newton's third law, the mass m_2 also exerts equal and opposite force on m_1. So the force of attraction ($\vec{F}_{12}$) experienced by m_1 due to m_2 is in the direction of positive 'y' axis ie., $\hat{r} = \hat{j}$.

$$\vec{F}_{21} = -13.34\times10^{-13}\hat{j}$$

$$\vec{F}_{12} = 13.34\times10^{-13}\hat{j}$$

The direction of the force is shown in the figure,



Gravitational force of attraction between m_1 and m_2

$\vec{F}_{12} = -\vec{F}_{21}$ which confirms Newton's third law.

Important features of gravitational force:

- As the distance between two masses increases, the strength of the force tends to decrease because of inverse dependence on r^2. Physically it implies that the planet Uranus experiences less gravitational force from the Sun than the Earth since Uranus is at larger distance from the Sun compared to the Earth.

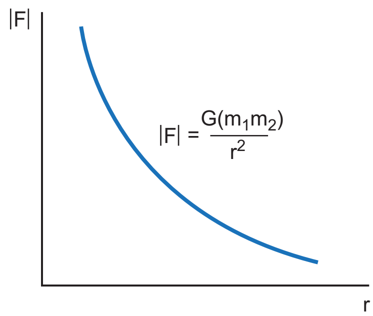


Figure 6.4 Variation of gravitational force with distance

- The gravitational forces between two particles always constitute an action-reaction pair. It implies that the gravitational force exerted by the Sun on the Earth is always towards the Sun. The reaction-force is exerted by the Earth on the Sun. The direction of this reaction force is towards Earth.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- The torque experienced by the Earth due to the gravitational force of the Sun is given by

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left(-\frac{GM_S M_E}{r^2} \hat{r} \right) = 0$$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$

So $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$. It implies that angular momentum $\vec{L}$ is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion. It is true for all the planets. In fact, this constancy of angular momentum leads to the Kepler's second law.

- The expression $\vec{F} = -\frac{GM_1 M_2}{r^2} \hat{r}$ has one inherent assumption that both M_1 and M_2 are treated as point masses. When it is said that Earth orbits around the Sun due to Sun's gravitational force, we assumed Earth and Sun to be point masses. This assumption is a good approximation because the distance between the two bodies is very much larger than their diameters. For some irregular and extended objects separated by a small distance, we cannot directly use the equation (6.3). Instead, we have to invoke separate mathematical treatment which will be brought forth in higher classes.
- However, this assumption about point masses holds even for small distance for one special case. To calculate force of attraction between a hollow sphere of mass M with uniform density and point mass m kept outside the hollow sphere, we can replace the hollow sphere of mass M as equivalent to a point mass M located at the center of the hollow sphere. The force of attraction between the hollow sphere of mass M and point mass m can be calculated by treating the hollow sphere also as another point mass. Essentially the entire mass of the hollow sphere appears to be concentrated at the center of the hollow sphere. It is shown in the Figure 6.5(a).
- There is also another interesting result. Consider a hollow sphere of mass M. If we place another object of mass 'm' inside this hollow sphere as in Figure 6.5(b), the force experienced by this mass 'm' will be zero. This calculation will be dealt with in higher classes.

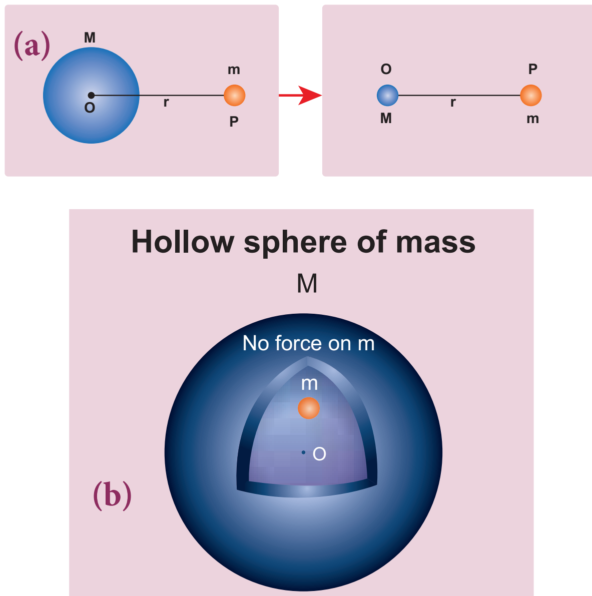


Figure 6.5 A mass placed in a hollow sphere.

- The triumph of the law of gravitation is that it concludes that the mango that is falling down and the Moon orbiting the Earth are due to the same gravitational force.

Newton's inverse square Law:

Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the center is

$$a = -\frac{v^2}{r} \qquad (6.4)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

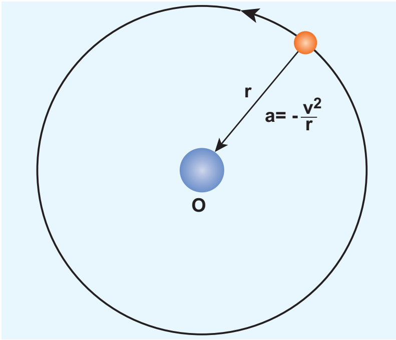


Figure 6.6 Point mass orbiting in a circular orbit.

Here v is the velocity and r, the distance of the planet from the center of the orbit (Figure 6.6).

The velocity in terms of known quantities r and T, is

$$v = \frac{2\pi r}{T} \quad (6.5)$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (6.4) we get,

$$a = -\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = -\frac{4\pi^2 r}{T^2} \quad (6.6)$$

Substituting the value of 'a' from (6.6) in Newton's second law, $F = ma$, where 'm' is the mass of the planet.

$$F = -\frac{4\pi^2 mr}{T^2} \quad (6.7)$$

From Kepler's third law,

$$\frac{r^3}{T^2} = k(constant) \quad (6.8)$$

$$\frac{r}{T^2} = \frac{k}{r^2} \quad (6.9)$$

By substituting equation 6.9 in the force expression, we can arrive at the law of gravitation.

$$F = -\frac{4\pi^2 mk}{r^2} \quad (6.10)$$

Here negative sign implies that the force is attractive and it acts towards the center. In equation (6.10), mass of the planet 'm' comes explicitly. But Newton strongly felt that according to his third law, if Earth is attracted by the Sun, then the Sun must also be attracted by the Earth with the same magnitude of force. So he felt that the Sun's mass (M) should also occur explicitly in the expression for force (6.10). From this insight, he equated the constant $4\pi^2 k$ *to* GM which turned out to be the law of gravitation

$$F = -\frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

In the above discussion we assumed that the orbit of the planet to be circular which is not true as the orbit of the planet around the Sun is elliptical. But this circular orbit assumption is justifiable because planet's orbit is very close to being circular and there is only a very small deviation from the circular shape.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Points to Contemplate

If Kepler's third law was "r^3T^2 = constant" instead of "$\frac{r^3}{T^2}$ = constant" what would be the new law of gravitation? Would it still be an inverse square law? How would the gravitational force change with distance? In this new law of gravitation, will Neptune experience greater gravitational force or lesser gravitational force when compared to the Earth?

EXAMPLE 6.2

Moon and an apple are accelerated by the same gravitational force due to Earth. Compare the acceleration of the two.

The gravitational force experienced by the apple due to Earth

$$F = -\frac{GM_E M_A}{R^2}$$

Here M_A– Mass of the apple, M_E– Mass of the Earth and R – Radius of the Earth.

Equating the above equation with Newton's second law,

$$M_A a_A = -\frac{GM_E M_A}{R^2}.$$

Simplifying the above equation we get,

$$a_A = -\frac{GM_E}{R^2}$$

Here a_A is the acceleration of apple that is equal to 'g'.

Similarly the force experienced by Moon due to Earth is given by

$$F = -\frac{GM_E M_m}{R_m^2}.$$

Here R_m- distance of the Moon from the Earth, M_m – Mass of the Moon

The acceleration experienced by the Moon is given by

$$a_m = -\frac{GM_E}{R_m^2}.$$

The ratio between the apple's acceleration to Moon's acceleration is given by

$$\frac{a_A}{a_m} = \frac{R_m^2}{R^2}.$$

From the Hipparchrus measurement, the distance to the Moon is 60 times that of Earth radius. R_m = 60R.

$$a_A / a_m = \frac{(60R)^2}{R^2} = 3600.$$

The apple's acceleration is 3600 times the acceleration of the Moon.

The same result was obtained by Newton using his gravitational formula. The apple's acceleration is measured easily and it is 9.8 m s^{-2}. Moon orbits the Earth once in 27.3 days and by using the centripetal acceleration formula, (Refer unit 3).

$$\frac{a_A}{a_m} = \frac{9.8}{0.00272} = 3600$$

which is exactly what he got through his law of gravitation.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

> **Note** The above calculation depends on knowing the distance between the Earth and the Moon and the radius of the Earth. The radius of the Earth was measured by Greek librarian Eratosthenes and distance between the Earth and the Moon was measured by Greek astronomer Hipparchrus 2400 years ago. It is very interesting to note that in order to measure these distances he used only high school geometry and trigonometry. These details are discussed in the astronomy section (6.5).

6.1.3 Gravitational Constant

In the law of gravitation, the value of gravitational constant G plays a very important role. The value of G explains why the gravitational force between the Earth and the Sun is so great while the same force between two small objects (for example between two human beings) is negligible.

The force experienced by a mass 'm' which is on the surface of the Earth (Figure 6.7) is given by

$$F = -\frac{GM_E m}{R_E^2} \quad (6.11)$$

M_E-mass of the Earth, m - mass of the object, R_E- radius of the Earth.

Equating Newton's second law, $F = mg$, to equation (6.11) we get,

$$mg = -\frac{GM_E m}{R_E^2}$$

$$g = -\frac{GM_E}{R_E^2} \quad (6.12)$$

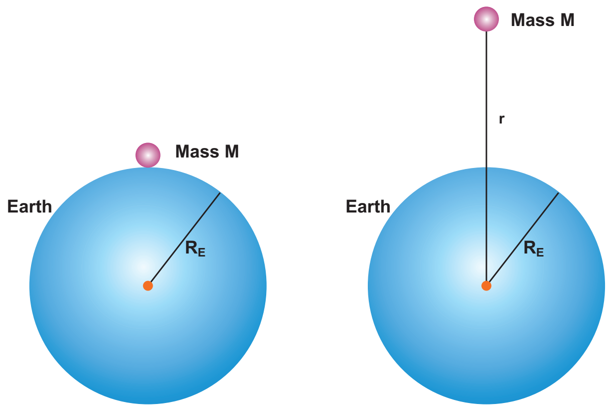


Figure 6.7 Force experienced by a mass on the (i) surface of the Earth (ii) at a distance from the centre of the Earth

Now the force experienced by some other object of mass M at a distance r from the center of the Earth is given by,

$$F = -\frac{GM_E M}{r^2}$$

Using the value of g in equation (6.12), the force F will be,

$$F = -gM\frac{R_E^2}{r^2} \quad (6.13)$$

From this it is clear that the force can be calculated simply by knowing the value of g. It is to be noted that in the above calculation G is not required.

> **DO YOU KNOW?** In the year 1798, Henry Cavendish experimentally determined the value of gravitational constant 'G' by using a torsion balance. He calculated the value of 'G' to be equal to $6.75 \times 10^{-11}\, N\, m^2 kg^{-2}$. Using modern techniques a more accurate value of G could be measured. The currently accepted value of G is $6.67259 \times 10^{-11}\, N\, m^2 kg^{-2}$.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

6.2 GRAVITATIONAL FIELD AND GRAVITATIONAL POTENTIAL

6.2.1 Gravitational field

Force is basically due to the interaction between two particles. Depending upon the type of interaction we can have two kinds of forces: Contact forces and Non-contact forces (Figure 6.8).

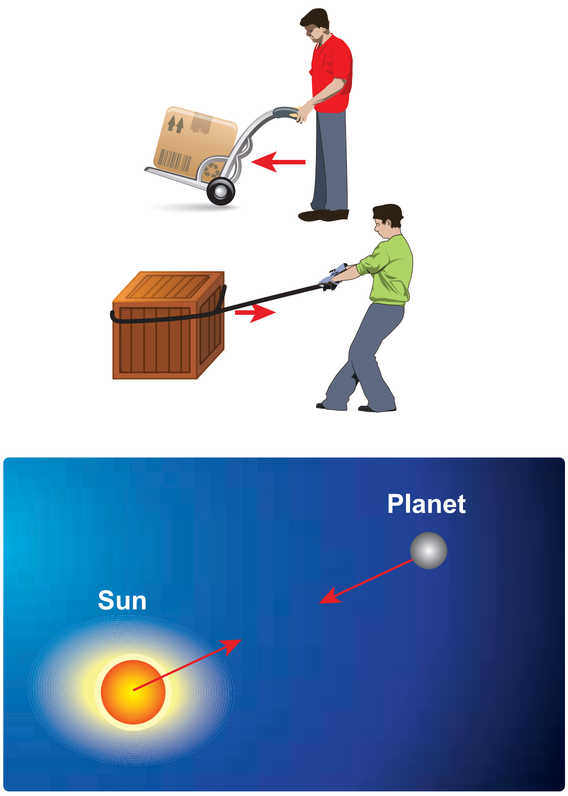


Figure 6.8 Depiction of contact and non-contact forces

Contact forces are the forces applied where one object is in physical contact with the other. The movement of the object is caused by the physical force exerted through the contact between the object and the agent which exerts force.

Consider the case of Earth orbiting around the Sun. Though the Sun and the Earth are not physically in contact with each other, there exists an interaction between them. This is because of the fact that the Earth experiences the gravitational force of the Sun. This gravitational force is a non-contact force.

It sounds mysterious that the Sun attracts the Earth despite being very far from it and without touching it. For contact forces like push or pull, we can calculate the strength of the force since we can feel or see. But how do we calculate the strength of non-contact force at different distances? To understand and calculate the strength of non-contact forces, the concept of 'field' is introduced.

The gravitational force on a particle of mass 'm_2' due to a particle of mass 'm_1' is

$$\vec{F}_{21} = -\frac{Gm_1m_2}{r^2}\hat{r} \qquad (6.14)$$

where $\hat{r}$ is a unit vector that points from m_1 to m_2 along the line joining the masses m_1 and m_2.

The gravitational field intensity $\vec{E}_1$ (here after called as gravitational field) at a point which is at a distance r from m_1 is defined as the gravitational force experienced by unit mass placed at that point. It is given by the ratio $\frac{\vec{F}_{21}}{m_2}$ (where m_2 is the mass of the object on which $\vec{F}_{21}$ acts)

Using $\vec{E}_1 = \frac{\vec{F}_{21}}{m_2}$ in equation (6.14) we get,

$$\vec{E}_1 = -\frac{Gm_1}{r^2}\hat{r} \qquad (6.15)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$\vec{E}_1$ is a vector quantity that points towards the mass m_1 and is independent of mass m_2, Here m_2 is taken to be of unit magnitude . The unit $\hat{r}$ is along the line between m_1 and the point in question. The field $\vec{E}_1$ is due to the mass m_1.

In general, the gravitational field intensity due to a mass M at a distance r is given by

$$\vec{E} = -\frac{GM}{r^2}\hat{r} \qquad (6.16)$$

Now in the region of this gravitational field, a mass 'm' is placed at a point P (Figure 6.9). Mass 'm' interacts with the field $\vec{E}$ and experiences an attractive force due to M as shown in Figure 6.9. The gravitational force experienced by 'm' due to 'M' is given by

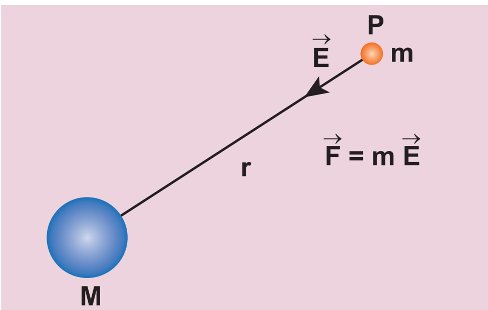


Figure 6.9 Gravitational Field intensity measured with an object of unit mass

$$\vec{F}_m = m\vec{E} \qquad (6.17)$$

Now we can equate this with Newton's second law $\vec{F} = m\vec{a}$

$$m\vec{a} = m\vec{E} \qquad (6.18)$$

$$\vec{a} = \vec{E} \qquad (6.19)$$

In other words, equation (6.18) implies that the gravitational field at a point is equivalent to the acceleration experienced by a particle at that point. However, it is to be noted that $\vec{a}$ and $\vec{E}$ are separate physical quantities that have the same magnitude and direction. The gravitational field $\vec{E}$ is the property of the source and acceleration $\vec{a}$ is the effect experienced by the test mass (unit mass) which is placed in the gravitational field $\vec{E}$. The non-contact interaction between two masses can now be explained using the concept of "Gravitational field".

Points to be noted:

i) The strength of the gravitational field decreases as we move away from the mass M as depicted in the Figure 6.10. The magnitude of $\vec{E}$ decreases as the distance r increases.

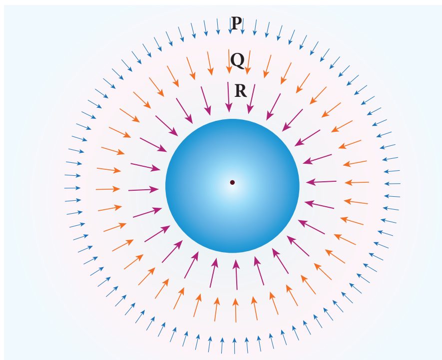


Figure 6.10 Strength of the Gravitational field lines decreases with distance

Figure 6.10 shows that the strength of the gravitational field at points P, Q, and R is given by $|\vec{E}_P| < |\vec{E}_Q| < |\vec{E}_R|$. It can be understood by comparing the length of the vectors at points P, Q, and R.

ii) The "field" concept was introduced as a mathematical tool to calculate gravitational interaction. Later it was found that field is a real physical quantity and it carries energy and momentum in

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

space. The concept of field is inevitable in understanding the behavior of charges.

iii) The unit of gravitational field is Newton per kilogram (N/kg) or m s^{-2}.

6.2.2 Superposition principle for Gravitational field

Consider 'n' particles of masses $m_1, m_2, \ldots m_n$, distributed in space at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots$ etc, with respect to point P. The total gravitational field at a point P due to all the masses is given by the vector sum of the gravitational field due to the individual masses (Figure 6.11). This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\hat{r}_1 - \frac{Gm_2}{r_2^2}\hat{r}_2 - \ldots - \frac{Gm_n}{r_n^2}\hat{r}_n$$

$$= -\sum_{i=1}^{n} \frac{Gm_i}{r_i^2}\hat{r}_i. \quad (6.20)$$

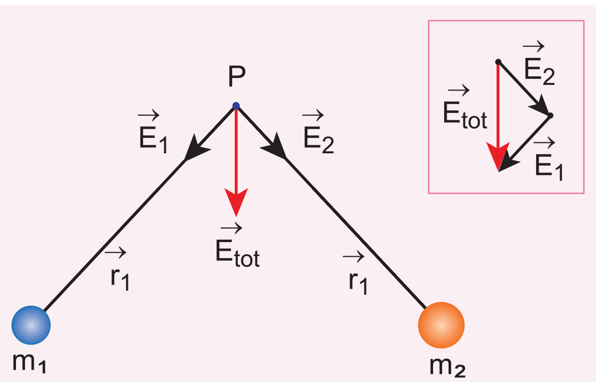
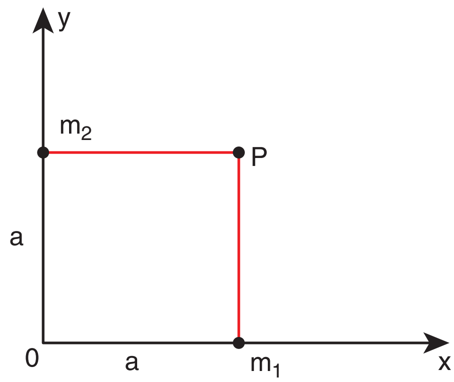


Figure 6.11 Superposition of two gravitational field intensities giving resultant field.

Instead of discrete masses, if we have continuous distribution of a total mass M, then the gravitational field at a point P is calculated using the method of integration.

EXAMPLE 6.3

(a) Two particles of masses m_1 *and* m_2 are placed along the x and y axes respectively at a distance 'a' from the origin. Calculate the gravitational field at a point P shown in figure below.



Solution

Gravitational field due to m_1 at a point P is given by,

$$\vec{E}_1 = -\frac{Gm_1}{a^2}\hat{j}$$

Gravitational field due to m_2 at the point p is given by,

$$\vec{E}_2 = -\frac{Gm_2}{a^2}\hat{i}$$

$$\vec{E}_{total} = -\frac{Gm_1}{a^2}\hat{j} - \frac{Gm_2}{a^2}\hat{i}$$

$$= -\frac{G}{a^2}(m_1\hat{j} + m_2\hat{i})$$

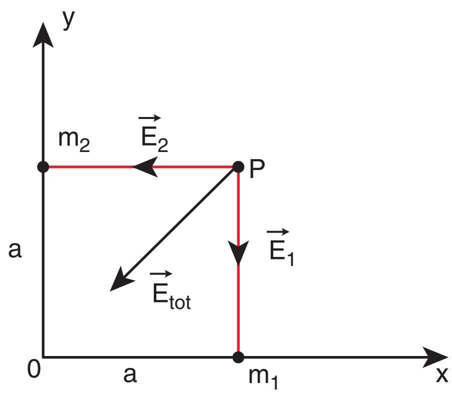
The direction of the total gravitational field is determined by the relative value of m_1 and m_2.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

When $m_1 = m_2 = m$

$$\vec{E}_{total} = -\frac{Gm}{a^2}(\hat{i}+\hat{j})$$

$(\hat{i}+\hat{j}=\hat{j}+\hat{i}$ as vectors obeys commutation law).

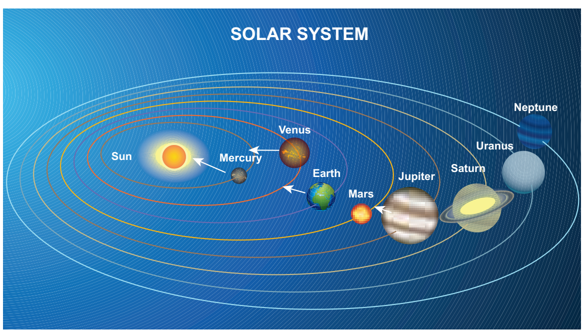


$\vec{E}_{total}$ points towards the origin of the co-ordinate system and the magnitude of $\vec{E}_{total}$ is $\frac{Gm}{a^2}$.

EXAMPLE 6.4

Qualitatively indicate the gravitational field of Sun on Mercury, Earth, and Jupiter shown in figure.

Since the gravitational field decreases as distance increases, Jupiter experiences a weak gravitational field due to the Sun. Since Mercury is the nearest to the Sun, it experiences the strongest gravitational field.



Solar System

6.2.3 Gravitational Potential Energy

The concept of potential energy and its physical meaning were dealt in unit 4. The gravitational force is a conservative force and hence we can define a gravitational potential energy associated with this conservative force field.

Two masses m_1 and m_2 are initially separated by a distance r'. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r as shown in Figure 6.12(a)

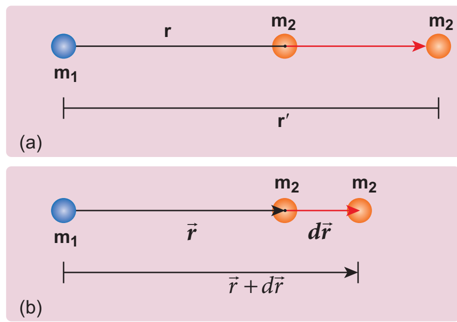


Figure 6.12 Two distant masses changing the linear distance

To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from $\vec{r}$ *to* $\vec{r}+d\vec{r}$ (shown in the Figure 6.12(b)), work has to be done externally. This infinitesimal work is given by

$$dW = \vec{F}_{ext}.d\vec{r} \qquad (6.21)$$

The work is done against the gravitational force, therefore,

$$\left|\vec{F}_{ext}\right| = \left|\vec{F}_G\right| = \frac{Gm_1m_2}{r^2} \qquad (6.22)$$

Substituting Equation (6.22) in 6.21, we get

$$dW = \frac{Gm_1m_2}{r^2}\hat{r}.d\vec{r} \qquad (6.23)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Also we know,

$$d\vec{r} = dr\,\hat{r} \qquad (6.24)$$

$$\Rightarrow dW = \frac{Gm_1m_2}{r^2}\hat{r}.\left(dr\,\hat{r}\right) \qquad (6.25)$$

$$\hat{r}.\hat{r} = 1\left(since\ both\ are\ unit\ vectors\right)$$

$$\therefore \quad dW = \frac{Gm_1m_2}{r^2}dr \qquad (6.26)$$

Thus the total work done for displacing the particle from r' to r is

$$W = \int_{r'}^{r} dW = \int_{r'}^{r} \frac{Gm_1m_2}{r^2}dr \qquad (6.27)$$

$$W = -\left(\frac{Gm_1m_2}{r}\right)_{r'}^{r}$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'} \qquad (6.28)$$

$$W = U(r) - U(r')$$

where $U(r) = \frac{-Gm_1m_2}{r}$

This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the separation between them are r and r' respectively.

Case 1: If $r < r'$

Since gravitational force is attractive, m_2 is attracted by m_1.Then m_2 can move from r to r' without any external work (Figure 6.13). Here work is done by the system spending its internal energy and hence the work done is said to be negative.

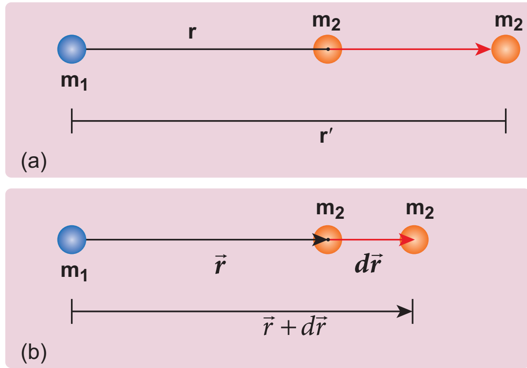


Figure 6.13 Cases for calculation of work done by gravity

Case 2: If $r > r'$

Work has to be done against gravity to move the object from r' to r. Therefore work is done on the body by external force and hence work done is positive.

It is to be noted that only potential energy difference has physical significance. Now gravitational potential energy can be discussed by choosing one point as the reference point.

Let us choose $r' = \infty$. Then the second term in the equation (6.28) becomes zero.

$$W = -\frac{Gm_1m_2}{r} + 0 \qquad (6.29)$$

Now we can define gravitational potential energy of a system of two masses m_1 and m_2 separated by a distance r as the amount of work done to bring the mass m_2 from infinity to a distance r assuming m_1 to be fixed in its position and is written as $U(r) = -\frac{Gm_1m_2}{r}$. It is to be noted that the gravitational potential energy of the system consisting of two masses m_1 and m_2 separated by a distance r, is the gravitational potential energy difference of the system when the masses are

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

separated by an infinite distance and by distance r. $U(r) = U(r) - U(\infty)$. Here we choose $U(\infty) = 0$ as the reference point. The gravitational potential energy $U(r)$ is always negative because when two masses come together slowly from infinity, work is done by the system.

The unit of gravitational potential energy $U(r)$ is Joule and it is a scalar quantity. The gravitational potential energy depends upon the two masses and the distance between them.

6.2.4 Gravitational potential energy near the surface of the Earth

It is already discussed in chapter 4 that when an object of mass m is raised to a height h, the potential energy stored in the object is mgh (Figure 6.14). This can be derived using the general expression for gravitational potential energy.

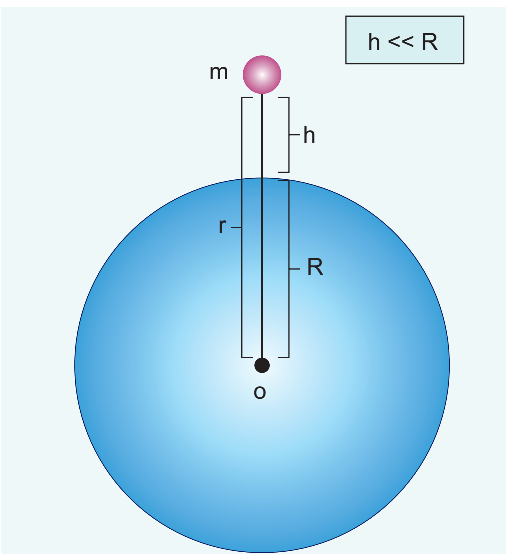


Figure 6.14 Mass placed at a distance r from the center of the Earth

Consider the Earth and mass system, with *r*, the distance between the mass m and the Earth's centre. Then the gravitational potential energy,

$$U = -\frac{GM_e m}{r} \qquad (6.30)$$

Here r = R_e+h, where R_e is the radius of the Earth. h is the height above the Earth's surface

$$U = -G\frac{M_e m}{(R_e + h)} \qquad (6.31)$$

If h << R_e, equation (6.31) can be modified as

$$U = -G\frac{M_e m}{R_e(1 + h/R_e)}$$

$$U = -G\frac{M_e m}{R_e}(1 + h/R_e)^{-1} \qquad (6.32)$$

By using Binomial expansion and neglecting the higher order terms, we get

$$U = -G\frac{M_e m}{R_e}\left(1 - \frac{h}{R_e}\right). \qquad (6.33)$$

We know that, for a mass m on the Earth's surface,

$$G\frac{M_e m}{R_e} = mgR_e \qquad (6.34)$$

Substituting equation (6.34) in (6.33) we get,

$$U = -mgR_e + mgh \qquad (6.35)$$

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

height h_1 to h_2 then the potential energy at h_1 is

$$U(h_1) = -mgR_e + mgh_1 \quad (6.36)$$

and the potential energy at h_2 is

$$U(h_2) = -mgR_e + mgh_2 \quad (6.37)$$

The potential energy difference between h_1 and h_2 is

$$U(h_2) - U(h_1) = mg(h_1 - h_2). \quad (6.38)$$

The term mgR_e in equations (6.36) and (6.37) plays no role in the result. Hence in the equation (6.35) the first term can be omitted or taken to zero. Thus it can be stated that The gravitational potential energy stored in the particle of mass m at a height h from the surface of the Earth is $U = mgh$. On the surface of the Earth, $U = 0$, since h is zero.

It is to be noted that mgh is the work done on the particle when we take the mass m from the surface of the Earth to a height h. This work done is stored as a gravitational potential energy in the mass m. Even though mgh is gravitational potential energy of the system (Earth and mass m), we can take mgh as the gravitational potential energy of the mass m since Earth is stationary when the mass moves to height h.

6.2.5 Gravitational potential V(r)

It is explained in the previous sections that the gravitational field $\vec{E}$ depends only on the source mass which creates the field. It is a vector quantity. We can also define a scalar quantity called "gravitational potential" which depends only on the source mass.

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r and it is denoted as V(r). In other words, the gravitational potential at distance r is equivalent to gravitational potential energy per unit mass at the same distance r. It is a scalar quantity and its unit is J kg^{-1}

We can determine gravitational potential from gravitational potential energy. Consider two masses m_1 and m_2 separated by a distance r which has gravitational potential energy $U(r)$ (Figure 6.15). The gravitational potential due to mass m_1 at a point P which is at a distance r from m_1 is obtained by making m_2 equal to unity (m_2 = 1kg). Thus the gravitational potential $V(r)$ due to mass m_1 at a distance r is

$$V(r) = -\frac{Gm_1}{r} \quad (6.39)$$

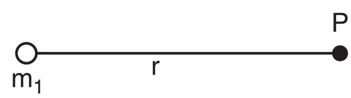


Figure 6.15 Point mass placed at a distance

Gravitational field and gravitational force are vector quantities whereas the gravitational potential and gravitational potential energy are scalar quantities. The motion of particles can be easily analyzed using scalar quantities than vector quantities. Consider the example of a falling apple:

Figure 6.16 shows an apple which falls on Earth due to Earth's gravitational force. This can be explained using the concept of gravitational potential $V(r)$ as follows.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

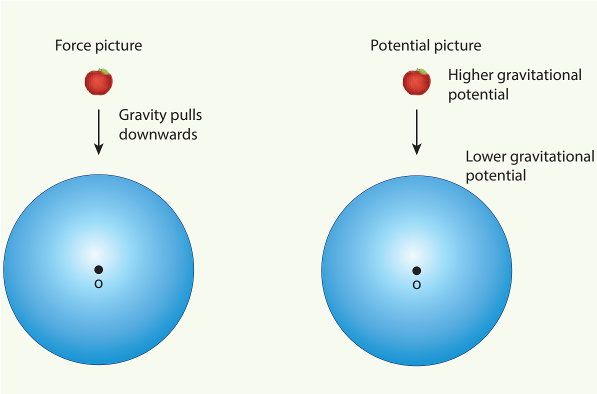


Figure 6.16 Apple falling freely under gravity

The gravitational potential $V(r)$ at a point of height h from the surface of the Earth is given by,

$$V(r=R+h)=-\frac{GM_e}{(R+h)} \quad (6.40)$$

The gravitational potential $V(r)$ on the surface of Earth is given by,

$$V(r=R)=-\frac{GM_e}{R} \quad (6.41)$$

Thus we see that

$$V(r=R)<V(r=R+h). \quad (6.42)$$

It is already discussed in the previous section that the gravitational potential energy near the surface of the Earth at height h is mgh. The gravitational potential at this point is simply $V(h)=U(h)/m$ = gh. In fact, the gravitational potential on the surface of the Earth is zero since h is zero. So the apple falls from a region of a higher gravitational potential to a region of lower gravitational potential. In general, the mass will move from a region of higher gravitational potential to a region of lower gravitational potential.

EXAMPLE 6.5

Water falls from the top of a hill to the ground. Why?

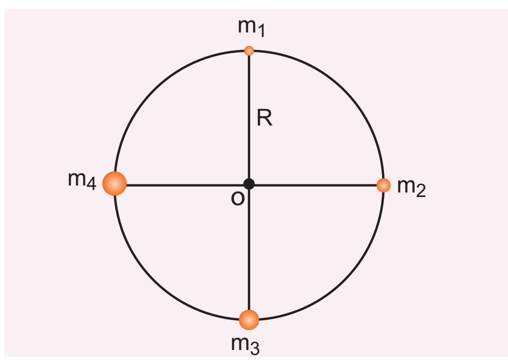
This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth i.e. $V_{hill} > V_{ground}$

Water falling from hill top

The motion of particles can be analyzed more easily using scalars like $U(r)$ *or* $V(r)$ than vector quantities like $\vec{F}$ *or* $\vec{E}$. In modern theories of physics, the concept of potential plays a vital role.

EXAMPLE 6.6

Consider four masses m_1, m_2, m_3, and m_4 arranged on the circumference of a circle as shown in figure below



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Calculate

(a) The gravitational potential energy of the system of 4 masses shown in figure.

(b) The gravitational potential at the point O due to all the 4 masses.

Solution

The gravitational potential energy $U(r)$ can be calculated by finding the sum of gravitational potential energy of each pair of particles.

$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_1m_4}{r_{14}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_2m_4}{r_{24}} - \frac{Gm_3m_4}{r_{34}}$$

Here r_{12}, r_{13} ... are distance between pair of particles

$$r_{14}{}^2 = R^2 + R^2 = 2R^2$$

$$r_{14} = \sqrt{2}R = r_{12} = r_{23} = r_{34}$$

$$r_{13} = r_{24} = 2R$$

$$U = -\frac{Gm_1m_2}{\sqrt{2}R} - \frac{Gm_1m_3}{2R} - \frac{Gm_1m_4}{\sqrt{2}R} - \frac{Gm_2m_3}{\sqrt{2}R} - \frac{Gm_2m_4}{2R} - \frac{Gm_3m_4}{\sqrt{2}R}$$

$$U = -\frac{G}{R}\left[\frac{m_1m_2}{\sqrt{2}} + \frac{m_1m_3}{2} + \frac{m_1m_4}{\sqrt{2}} + \frac{m_2m_3}{\sqrt{2}} + \frac{m_2m_4}{2} + \frac{m_3m_4}{\sqrt{2}}\right]$$

If all the masses are equal, then $m_1 = m_2 = m_3 = m_4 = M$

$$U = -\frac{GM^2}{R}\left[\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}}\right]$$

$$U = -\frac{GM^2}{R}\left[1 + \frac{4}{\sqrt{2}}\right]$$

$$U = -\frac{GM^2}{R}\left[1 + 2\sqrt{2}\right]$$

The gravitational potential V(r) at a point O is equal to the sum of the gravitational potentials due to individual mass. Since potential is a scalar, the net potential at point O is the algebraic sum of potentials due to each mass.

$$V_O(r) = -\frac{Gm_1}{R} - \frac{Gm_2}{R} - \frac{Gm_3}{R} - \frac{Gm_4}{R}$$

If $m_1 = m_2 = m_3 = m_4 = M$

$$V_O(r) = -\frac{4GM}{R}$$

6.3 ACCELERATION DUE TO GRAVITY OF THE EARTH

When objects fall on the Earth, the acceleration of the object is towards the Earth. From Newton's second law, an object is accelerated only under the action of a force. In the case of Earth, this force is the gravitational pull of Earth. This force produces a constant acceleration near the Earth's surface in all bodies, irrespective of their masses. The gravitational force exerted by Earth on the mass m near the surface of the Earth is given by

$$\vec{F} = -\frac{GmM_e}{R_e{}^2}\hat{r}$$

Now equating Gravitational force to Newton's second law,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$m\vec{a} = -\frac{GmM_e}{R_e^2}\hat{r}$$

hence, acceleration is,

$$\vec{a} = -\frac{GM_e}{R_e^2}\hat{r} \quad (6.43)$$

The acceleration experienced by the object near the surface of the Earth due to its gravity is called acceleration due to gravity. It is denoted by the symbol g. The magnitude of acceleration due to gravity is

$$|g| = \frac{GM_e}{R_e^2}. \quad (6.44)$$

It is to be noted that the acceleration experienced by any object is independent of its mass. The value of g depends only on the mass and radius of the Earth. Infact, Galileo arrived at the same conclusion 400 years ago that *all objects fall towards the Earth with the same acceleration* through various quantitative experiments. The acceleration due to gravity g is found to be 9.8 m s^{-2} on the surface of the Earth near the equator.

6.3.1 Variation of g with altitude, depth and latitude

Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is

$$g' = \frac{GM}{(R_e + h)^2} \quad (6.45).$$

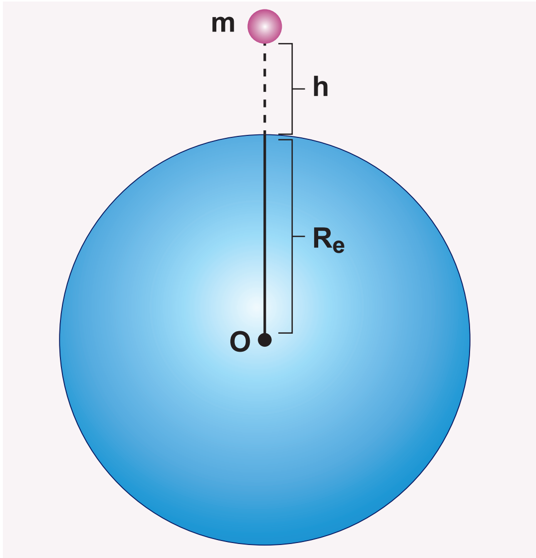


Figure 6.17(a) Mass at a height h from the center of the Earth

$$g' = \frac{GM}{R_e^2\left(1 + \frac{h}{R_e}\right)^2}$$

$$g' = \frac{GM}{R_e^2}\left(1 + \frac{h}{R_e}\right)^{-2}$$

If $h \ll R_e$

We can use Binomial expansion. Taking the terms upto first order

$$g' = \frac{GM}{R_e^2}\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = g\left(1 - 2\frac{h}{R_e}\right) \quad (6.46)$$

We find that $g' < g$. This means that as altitude h increases the acceleration due to gravity g decreases.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 6.7

1. Calculate the value of g in the following two cases:

   (a) If a mango of mass ½ kg falls from a tree from a height of 15 meters, what is the acceleration due to gravity when it begins to fall?

Solution

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = 9.8\left(1 - \frac{2 \times 15}{6400 \times 10^3}\right)$$

$$g' = 9.8\left(1 - 0.469 \times 10^{-5}\right)$$

But $1 - 0.00000469 \cong 1$

Therefore $g' = g$

   (b) Consider a satellite orbiting the Earth in a circular orbit of radius 1600 km above the surface of the Earth. What is the acceleration experienced by the satellite due to Earth's gravitational force?

Solution

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = g\left(1 - \frac{2 \times 1600 \times 10^3}{6400 \times 10^3}\right)$$

$$g' = g\left(1 - \frac{2}{4}\right)$$

$$g' = g\left(1 - \frac{1}{2}\right) = g/2$$

The above two examples show that the acceleration due to gravity is a constant near the surface of the Earth.

Note Can we substitute h = R_e in the equation 6.46? No. To get equation 6.46 we assumed that h << R_e. However h = R_e can be substituted in equation 6.45.

Variation of g with depth:

Consider a particle of mass m which is in a deep mine on the Earth. (Example: coal mines in Neyveli). Assume the depth of the mine as d. To calculate g' at a depth d, consider the following points.

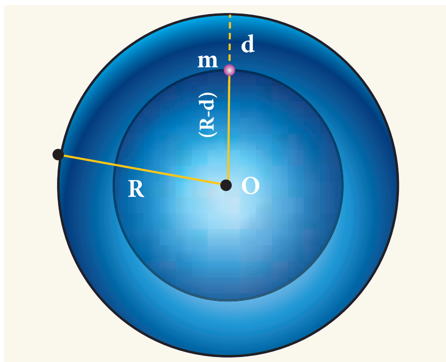


Figure 6.17(b) Particle in a mine

The part of the Earth which is above the radius $(R_e - d)$ do not contribute to the acceleration. The result is proved earlier and is given as

$$g' = \frac{GM'}{(R_e - d)^2} \quad (6.47)$$

Here M' is the mass of the Earth of radius $(R_e - d)$

Assuming the density of Earth ρ to be constant,

$$\rho = \frac{M}{V} \quad (6.48)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

where M is the mass of the Earth and V its volume, Thus,

$$\rho = \frac{M'}{V'}$$

$$\frac{M'}{V'} = \frac{M}{V} \text{ and } M' = \frac{M}{V}V'$$

$$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^3}\right)\left(\frac{4}{3}\pi (R_e - d)^3\right)$$

$$M' = \frac{M}{R_e^3}(R_e - d)^3 \qquad (6.49)$$

$$g' = G\frac{M}{R_e^3}(R_e - d)^3 . \frac{1}{(R_e - d)^2}$$

$$g' = GM\frac{R_e\left(1 - \frac{d}{R_e}\right)}{R_e^3}$$

$$g' = GM\frac{\left(1 - \frac{d}{R_e}\right)}{R_e^2}$$

Thus

$$g' = g\left(1 - \frac{d}{R_e}\right) \qquad (6.50)$$

Here also $g' < g$. As depth increases, g' decreases. It is very interesting to know that acceleration due to gravity is maximum on the surface of the Earth but decreases when we go either upward or downward.

Variation of g with latitude:

Whenever we analyze the motion of objects in rotating frames [explained in chapter 3] we must take into account the centrifugal force. Even though we treat the Earth as an inertial frame, it is not exactly correct because the Earth spins about its own axis. So when an object is on the surface of the Earth, it experiences a centrifugal force that depends on the latitude of the object on Earth. If the Earth were not spinning, the force on the object would have been *mg*. However, the object experiences an additional centrifugal force due to spinning of the Earth.

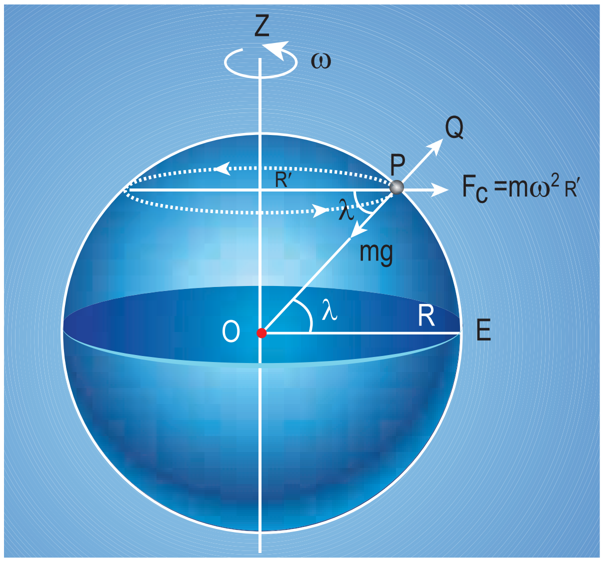


Figure 6.18 Variation of g with latitude

This centrifugal force is given by $m\omega^2 R'$.

$$R' = R\cos\lambda \qquad (6.51)$$

where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is

$$a_{PQ} = \omega^2 R' \cos\lambda = \omega^2 R\cos^2\lambda$$

$$since\ R' = R\cos\lambda$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Therefore,

$$g' = g - \omega^2 R \cos^2 \lambda \qquad (6.52)$$

From the expression (6.52), we can infer that at equator, $\lambda = 0$; $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles $\lambda = 90$; $g' = g$, it is maximum. At the equator, g' is minimum.

EXAMPLE 6.8

Find out the value of g' in your school laboratory?

Solution

Calculate the latitude of the city or village where the school is located. The information is available in Google search. For example, the latitude of Chennai is approximately 13 degree.

$$g' = g - \omega^2 R \cos^2 \lambda$$

Here $\omega^2 R = (2x3.14/86400)^2 x (6400x10^3) = 3.4x10^{-2}$ m s^{-2}.

It is to be noted that the value of λ should be in radian and not in degree. 13 degree is equivalent to 0.2268 rad.

$$g' = 9.8 - \left(3.4 \times 10^{-2}\right) \times \left(\cos 0.2268\right)^2$$

$$g' = 9.7677 \text{ m s}^{-2}$$

Points to Contemplate

Suppose you move towards east-west along the same latitude. Will the value of g' change?

6.4 ESCAPE SPEED AND ORBITAL SPEED

Hydrogen and helium are the most abundant elements in the universe but Earth's atmosphere consists mainly of nitrogen and oxygen. The following discussion brings forth the reason why hydrogen and helium are not found in abundance on the Earth's atmosphere. When an object is thrown up with some initial speed it will reach a certain height after which it will fall back to Earth. If the same object is thrown again with a higher speed, it reaches a greater height than the previous one and falls back to Earth. This leads to the question of what should be the speed of an object thrown vertically up such that it escapes the Earth's gravity and would never come back.

Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i, the initial total energy of the object is

$$E_i = \frac{1}{2} M v_i^2 - \frac{GMM_E}{R_E} \qquad (6.53)$$

where, M_E is the mass of the Earth and R_E- the radius of the Earth. The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero $[U(\infty) = 0]$ and the kinetic energy becomes zero as well. Therefore the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be nonzero.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$E_f = 0$$

According to the law of energy conservation,

$$E_i = E_f \qquad (6.54)$$

Substituting (6.53) in (6.54) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \qquad (6.55)$$

Consider the escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace v_i *with* v_e. i.e,

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

Using $g = \frac{GM_E}{R_e^2}$,

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \qquad (6.56)$$

From equation (6.56) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object. By substituting the values of g ($9.8\ m\,s^{-2}$) and $R_e = 6400\,km$, the escape speed of the Earth is $v_e = 11.2\,km\,s^{-1}$. The escape speed is independent of the direction in which the object is thrown. Irrespective of whether the object is thrown vertically up, radially outwards or tangentially it requires the same initial speed to escape Earth's gravity. It is shown in Figure 6.19

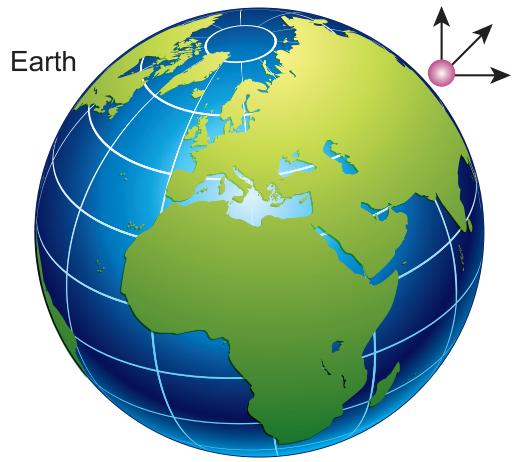


Figure 6.19 Escape speed independent of angle

Lighter molecules such as hydrogen and helium have enough speed to escape from the Earth, unlike the heavier ones such as nitrogen and oxygen. (The average speed of hydrogen and helium atoms compaired with the escape speed of the Earth,is presented in the kinetic theory of gases, unit 9).

6.4.1 Satellites, orbital speed and time period

We are living in a modern world with sophisticated technological gadgets and are able to communicate to any place on Earth. This advancement was made possible because of our understanding of solar system. Communication mainly depends on the satellites that orbit the Earth (Figure 6.20). Satellites revolve around the Earth just like the planets revolve around the Sun. Kepler's laws are applicable to man-made satellites also.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

For a satellite of mass M to move in a circular orbit, centripetal force must be acting on the satellite. This centripetal force is provided by the Earth's gravitational force.

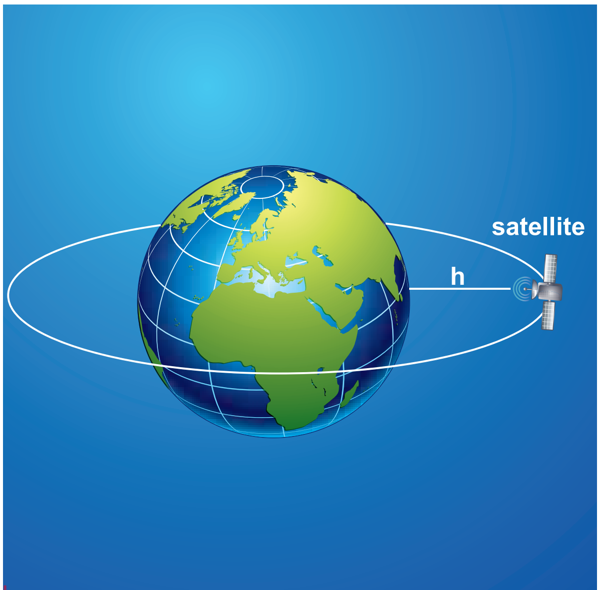


Figure 6.20 Satellite revolving around the Earth.

$$\frac{Mv^2}{(R_E + h)} = \frac{GMM_E}{(R_e + h)^2} \quad (6.57)$$

$$v^2 = \frac{GM_E}{(R_E + h)}$$

$$v = \sqrt{\frac{GM_E}{(R_E + h)}} \quad (6.58)$$

As h increases, the speed of the satellite decreases.

Time period of the satellite:

The distance covered by the satellite during one rotation in its orbit is equal to $2\pi(R_E + h)$ and time taken for it is the time period, T. Then

$$Speed\ v = \frac{Distance\ travelled}{Time\ taken} = \frac{2\pi(R_E + h)}{T}$$

From equation (6.58)

$$\sqrt{\frac{GM_E}{(R_E + h)}} = \frac{2\pi(R_E + h)}{T} \quad (6.59)$$

$$T = \frac{2\pi}{\sqrt{GM_E}}(R_E + h)^{3/2} \quad (6.60)$$

Squaring both sides of the equation (6.60), we get

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$\frac{4\pi^2}{GM_E} = constant\ say\ c$$

$$T^2 = c(R_E + h)^3 \quad (6.61)$$

Equation (6.61) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E. Then,

$$T^2 = \frac{4\pi^2}{GM_E}R_E^3$$

$$T^2 = \frac{4\pi^2}{GM_E/R_E^2}R_E$$

$$T^2 = \frac{4\pi^2}{g}R_E$$

$$since\ GM_E/R_E^2 = g$$

$$T = 2\pi\sqrt{\frac{R_E}{g}} \quad (6.62)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

By substituting the values of $R_E = 6.4 \times 10^6 m$ and g = 9.8 m s^{-2}, the orbital time period is obtained as T $\cong$ 85 minutes.

EXAMPLE 6.9

Moon is the natural satellite of Earth and it takes 27 days to go once around its orbit. Calculate the distance of the Moon from the surface of the Earth assuming the orbit of the Moon as circular.

Solution

We can use Kepler's third law,

$$T^2 = c(R_E + h)^3$$

$$T^{2/3} = c^{1/3}(R_E + h)$$

$$\left(\frac{T^2}{c}\right)^{1/3} = (R_E + h)$$

$$\left(\frac{T^2 GM_E}{4\pi^2}\right)^{\frac{1}{3}} = (R_E + h);$$

$$c = \frac{4\pi^2}{GM_E}$$

$$h = \left(\frac{T^2 GM_E}{4\pi^2}\right)^{1/3} - R_E$$

Here h is the distance of the Moon from the surface of the Earth. Here,

R_E – radius of the Earth = $6.4 \times 10^6 m$

M_E – mass of the Earth = $6.02 \times 10^{24} kg$

G – Universal gravitational constant = $6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

By substituting these values, the distance to the Moon from the surface of the Earth is calculated to be $3.77 \times 10^5 km$.

6.4.2 Energy of an Orbiting Satellite

The total energy of a satellite orbiting the Earth at a distance h from the surface of Earth is calculated as follows; The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy. The potential energy of the satellite is,

$$U = -\frac{GM_s M_E}{(R_E + h)} \quad (6.63)$$

Here M_s - mass of the satellite, M_E - mass of the Earth, R_E - radius of the Earth.

The Kinetic energy of the satellite is

$$K.E = \frac{1}{2} M_s v^2 \quad (6.64)$$

Here v is the orbital speed of the satellite and is equal to

$$v = \sqrt{\frac{GM_E}{(R_E + h)}} \quad (6.65)$$

Substituting the value of v in (6.64), the kinetic energy of the satellite becomes,

$$K.E = \frac{1}{2}\frac{GM_E M_s}{(R_E + h)}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Therefore the total energy of the satellite is

$$E = \frac{1}{2}\frac{GM_E M_s}{(R_E + h)} - \frac{GM_s M_E}{(R_E + h)}$$

$$E = -\frac{GM_s M_E}{2(R_E + h)} \qquad (6.66)$$

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

As h approaches ∞, the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distances.

EXAMPLE 6.10

Calculate the energy of the (i) Moon orbiting the Earth and (ii) Earth orbiting the Sun.

Solution

Assuming the orbit of the Moon to be circular, the energy of Moon is given by,

$$E_m = -\frac{GM_E M_m}{2R_m}$$

where M_E is the mass of Earth 6.02×10^{24} kg; M_m is the mass of Moon 7.35×10^{22} kg; and R_m is the distance between the Moon and the center of the Earth $3.84 \times 10^5\, km$

$$G = 6.67 \times 10^{-11} \text{N m}^2 \text{kg}^{-2}.$$

$$E_m = -\frac{6.67 \times 10^{-11} \times 6.02 \times 10^{24} \times 7.35 \times 10^{22}}{2 \times 3.84 \times 10^5 \times 10^3}$$

$$E_m = -38.42 \times 10^{-19} \times 10^{46}$$

$$E_m = -38.42 \times 10^{46}\ Joule$$

The negative energy implies that the Moon is bound to the Earth.

Same method can be used to prove that the energy of the Earth is also negative.

6.4.3 Geo-stationary and polar satellite

The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Can we calculate the orbital radius of a satellite if its time period is 24 hours?

Kepler's third law is used to find the radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$(R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$R_E + h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

Substituting for the time period (24 hrs = 86400 seconds), mass, and radius of the Earth, h turns out to be 36,000 km. Such satellites are called "geo-stationary satellites", since they appear to be stationary when seen from Earth.

India uses the INSAT group of satellites that are basically geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

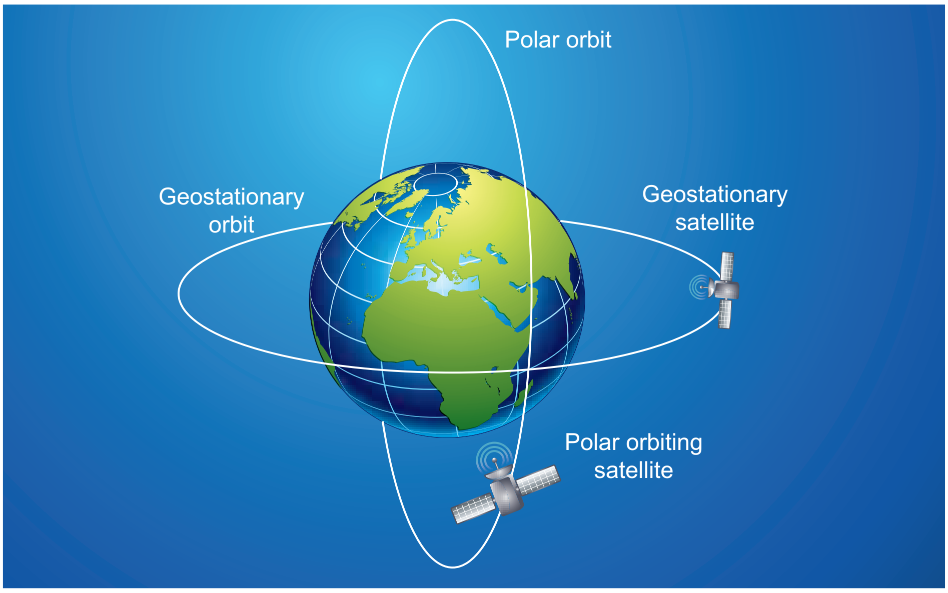


Figure 6.21 Polar orbit and geostationary satellite

of 500 to 800 km from the surface of the Earth orbits the Earth from north to south direction. This type of satellite that orbits Earth from North Pole to South Pole is called a polar satellite. The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day. A Polar satellite covers a small strip of area from pole to pole during one revolution. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

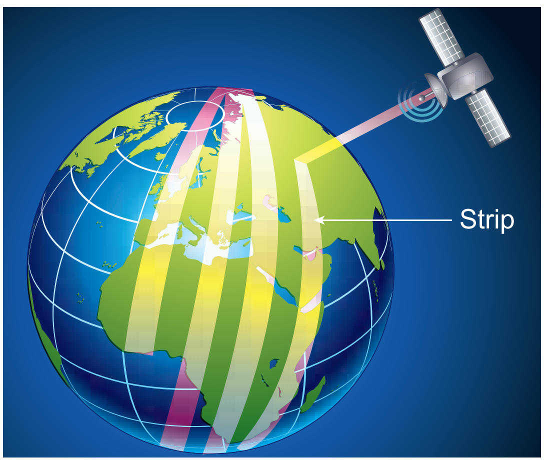


Figure 6.22 Strip of communication region, covered by a polar satellite.

BLC3PC

6.4.4 Weightlessness

Weight of an object

Objects on Earth experience the gravitational force of Earth. The gravitational force acting on an object of mass m is mg. This force always acts downwards towards the center of the Earth. When we stand on the floor, there are two forces acting on us. One is the gravitational force, acting downwards and the other is the normal force exerted by the floor upwards on us to keep us at rest. The weight of an object $\vec{W}$ is defined as the downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth. The direction of weight is in the direction of gravitational force. So the magnitude of

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

weight of an object is denoted as, W=N=mg. Note that even though magnitude of weight is equal to mg, it is not same as gravitational force acting on the object.

Apparent weight in elevators

Everyone who used an elevator would have felt a jerk when the elevator takes off or stops. Why does it happen? Understanding the concept of weight is crucial for explaining this effect. Let us consider a man inside an elevator in the following scenarios.

When a man is standing in the elevator, there are two forces acting on him.

1. Gravitational force which acts downward. If we take the vertical direction as positive y direction, the gravitational force acting on the man is $\vec{F}_G = -mg\hat{j}$
2. The normal force exerted by floor on the man which acts vertically upward, $\vec{N} = N\hat{j}$

Case (i) When the elevator is at rest

The acceleration of the man is zero. Therefore the net force acting on the man is zero. With respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = 0$$
$$-mg\hat{j} + N\hat{j} = 0$$

By comparing the components, we can write

$$N - mg = 0 \text{ (or) } N = mg \quad (6.67)$$

Since weight, W =N, the apparent weight of the man is equal to his actual weight.

Case (ii) When the elevator is moving uniformly in the upward or downward direction

In uniform motion (constant velocity), the net force acting on the man is still zero. Hence, in this case also the apparent weight of the man is equal to his actual weight. It is shown in Figure 6.23(a)

Case (iii) When the elevator is accelerating upwards

If an elevator is moving with upward acceleration ($\vec{a} = a\hat{j}$) with respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = ma\hat{j}$$

By comparing the components,

$$N = m(g + a) \quad (6.68)$$

Therefore, apparent weight of the man is greater than his actual weight. It is shown in Figure 6.23(b)

Case (iv) When the elevator is accelerating downwards

If the elevator is moving with downward acceleration ($\vec{a} = -a\hat{j}$), by applying Newton's second law on the man, we can write

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = -ma\hat{j}$$

By comparing the components,

$$N = m(g - a) \quad (6.69)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Therefore, apparent weight W = N = m(g-a) of the man is lesser than his actual weight. It is shown in Figure 6.23(c)

Weightlessness of freely falling bodies

Freely falling objects experience only gravitational force. As they fall freely, they are not in contact with any surface (by neglecting air friction). The normal force acting on the object is zero. The downward acceleration is equal to the acceleration due to the gravity of the Earth. i.e (a = g). From equation (6.69) we get.

$$a = g \quad \therefore N = m\,(g - g) = 0.$$

This is called the state of weightlessness. When the lift falls (when the lift wire cuts) with downward acceleration a = g, the person inside the elevator is in the state of weightlessness or free fall. It is shown in Figure 6.23(d)

When the apple was falling from the tree it was weightless. As soon as it hit Newton's head, it gained weight! and Newton gained physics!

Weightlessness in satellites:

There is a wrong notion that the astronauts in satellites experience no gravitational force because they are far away from the Earth. Actually the Earth satellites that orbit very close to Earth experience only gravitational force. The astronauts inside the satellite also experience the same gravitational force. Because of this, they cannot exert any force on the floor of the satellite. Thus, the floor of the satellite also cannot exert any normal force on the astronaut. Therefore, the astronauts inside a satellite are in the state of weightlessness. Not only the astronauts, but all the objects in the satellite will be in the state of weightlessness which is similar to that of a free fall. It is shown in the Figure 6.24.

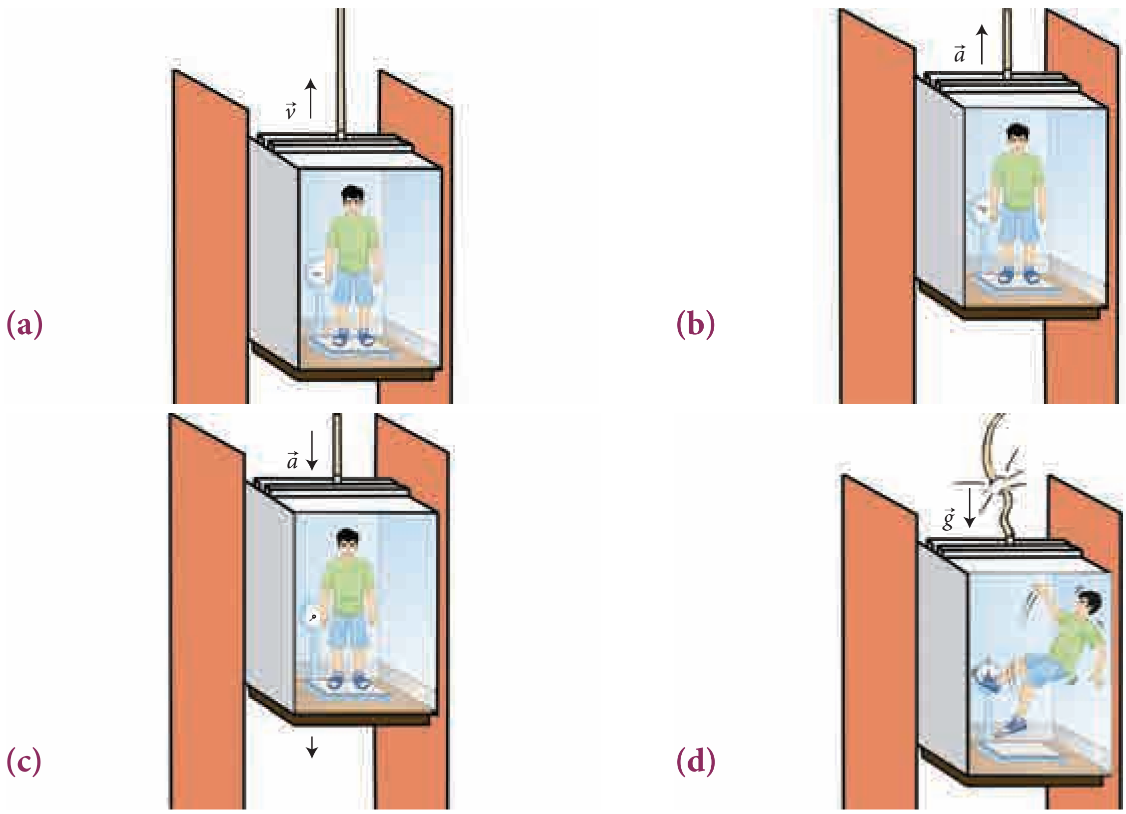


Figure 6.23 Apparent weight in the lift

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Figure 6.24 The well known scientist Stephen Hawking in the state of weightlessness. https://www.youtube.com/watch?v=OCsuHvv_D0s

6.5 ELEMENTARY IDEAS OF ASTRONOMY

Astronomy is one of the oldest sciences in the history of mankind. In the olden days, astronomy was an inseparable part of physical science. It contributed a lot to the development of physics in the 16^{th} century. In fact Kepler's laws and Newton's theory of gravitation were formulated and verified using astronomical observations and data accumulated over the centuries by famous astronomers like Hippachrus, Aristachrus, Ptolemy, Copernicus and Tycho Brahe. Without Tycho Brahe's astronomical observations, Kepler's laws would not have emerged. Without Kepler's laws, Newton's theory of gravitation would not have been formulated.

It was mentioned in the beginning of this chapter that Ptolemy's geocentric model was replaced by Copernicus' heliocentric model. It is important to analyze and explain the shortcoming of the geocentric model over heliocentric model.

6.5.1 Heliocentric system over geocentric system

When the motion of the planets are observed in the night sky by naked eyes over a period of a few months, it can be seen that the planets move eastwards and reverse their motion for a while and return to eastward motion again. This is called "retrograde motion" of planets.

Figure 6.25 shows the retrograde motion of the planet Mars. Careful observation for a period of a year clearly shows that Mars initially moves eastwards (February to June), then reverses its path and moves backwards (July, August, September). It changes its direction of motion once again and continues its forward motion (October onwards). In olden days, astronomers recorded the retrograde motion of all

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

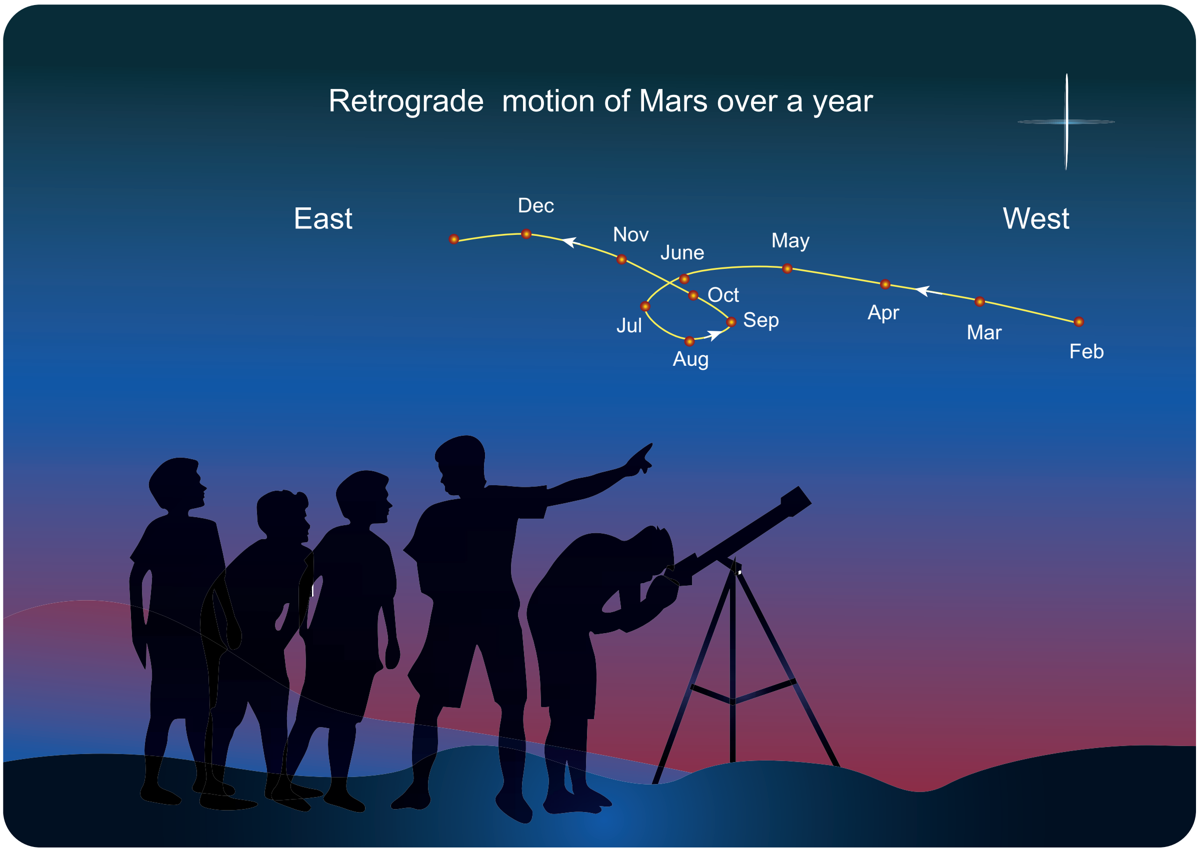


Figure 6.25 Retrograde motion of planets

visible planets and tried to explain the motion. According to Aristotle, the other planets and the Sun move around the Earth in the circular orbits. If it was really a circular orbit it was not known how the planet could reverse its motion for a brief interval. To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle". A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth (Figure 6.26). Essentially Ptolemy retained the Earth centric idea of Aristotle and added the epicycle motion to it.

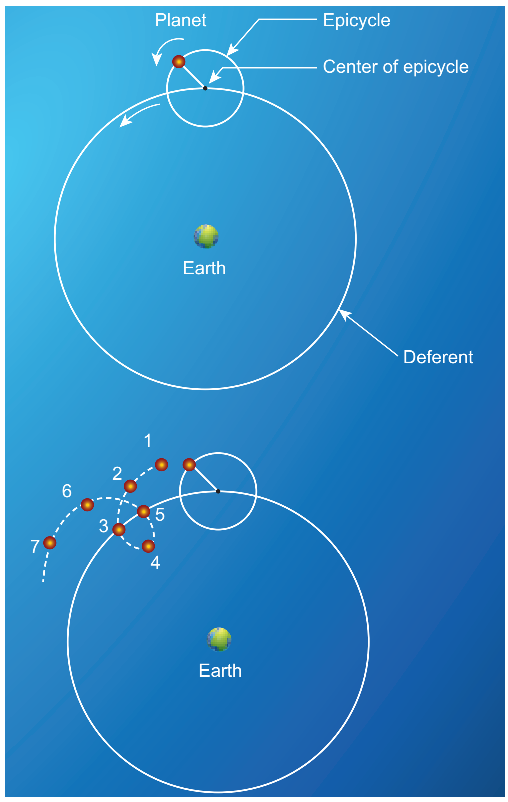


Figure 6.26 "Epicycle" motion of planetary objects around Earth, depicted with respect to months of observation.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

But Ptolemy's model became more and more complex as every planet was found to undergo retrograde motion. In the 15th century, the Polish astronomer Copernicus proposed the heliocentric model to explain this problem in a simpler manner. According to this model, the Sun is at the center of the solar system and all planets orbited the Sun. The retrograde motion of planets with respect to Earth is because of the relative motion of the planet with respect to Earth. The retrograde motion from the heliocentric point of view is shown in Figure 6.27.

Figure 6.27 shows that the Earth orbits around the Sun faster than Mars. Because of the relative motion between Mars and Earth, Mars appears to move backwards from July to October. In the same way the retrograde motion of all other planets was explained successfully by the Copernicus model. It was because of its simplicity, the heliocentric model slowly replaced the geocentric model. Historically, if any natural phenomenon has one or more explanations, the simplest one is usually accepted. Though this was not the only reason to disqualify the geocentric model, a detailed discussion on correctness of the Copernicus model over to Ptolemy's model can be found in astronomy books.

ACTIVITY

Students are encouraged to observe the motion of the planet Mars by naked eye and identify its retrograde motion. As mentioned above, to observe the retrograde motion six to seven months are required. So students may start their observation of Mars from the month of June and continue till April next year. Mars is the little bright planet with reddish color. The position of the planet Mars in the sky can be easily taken from 'Google'.

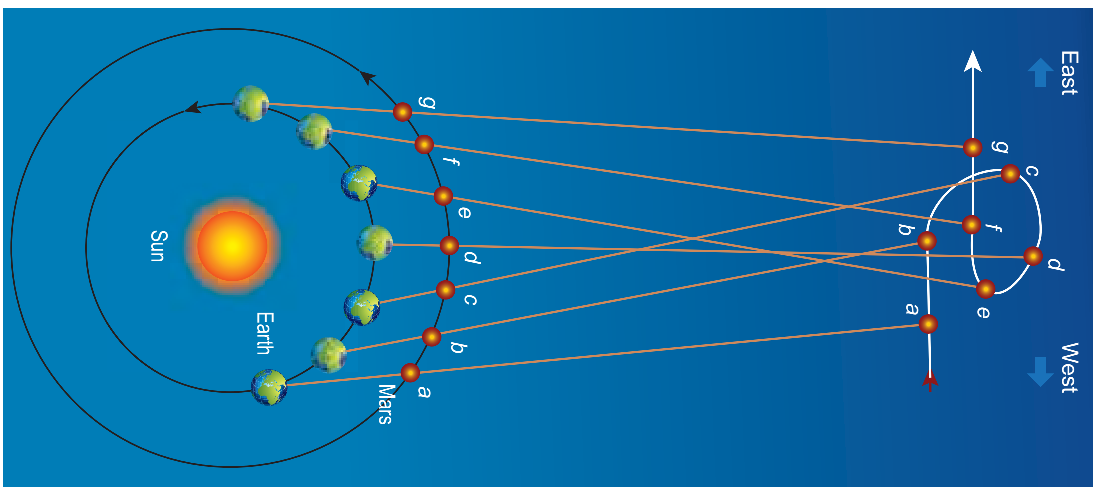


Figure 6.27 'Retrograde motion' in heliocentric model

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

6.5.2 Kepler's Third Law and The Astronomical Distance

When Kepler derived his three laws, he strongly relied on Tycho Brahe's astronomical observation. In his third law, he formulated the relation between the distance of a planet from the Sun to the time period of revolution of the planet. Astronomers cleverly used geometry and trigonometry to calculate the distance of a planet from the Sun in terms of the distance between Earth and Sun. Here we can see how the distance of Mercury and Venus from the Sun were measured. The Venus and Mercury, being inner planets with respect to Earth, the maximum angular distance they can subtend at a point on Earth with respect to the Sun is 46 degree for Venus and 22.5 degree for Mercury. It is shown in the Figure 6.28

Figure 6.29 shows that when Venus is at maximum elongation (i.e., 46 degree) with respect to Earth, Venus makes 90 degree to Sun. This allows us to find the distance between Venus and Sun. The distance between Earth and Sun is taken as one Astronomical unit (1 AU).

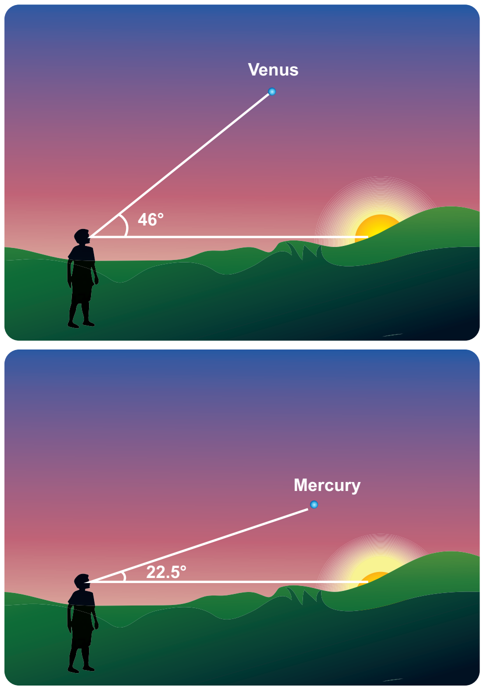


Figure 6.28 Angle of elevation for Venus and Mercury from horizon

The trigonometric relation satisfied by this right angled triangle is shown in Figure 6.29.

$$\sin\theta = \frac{r}{R}$$

where R = 1 AU.

$$r = R\sin\theta = (1AU)(\sin 46^\circ)$$

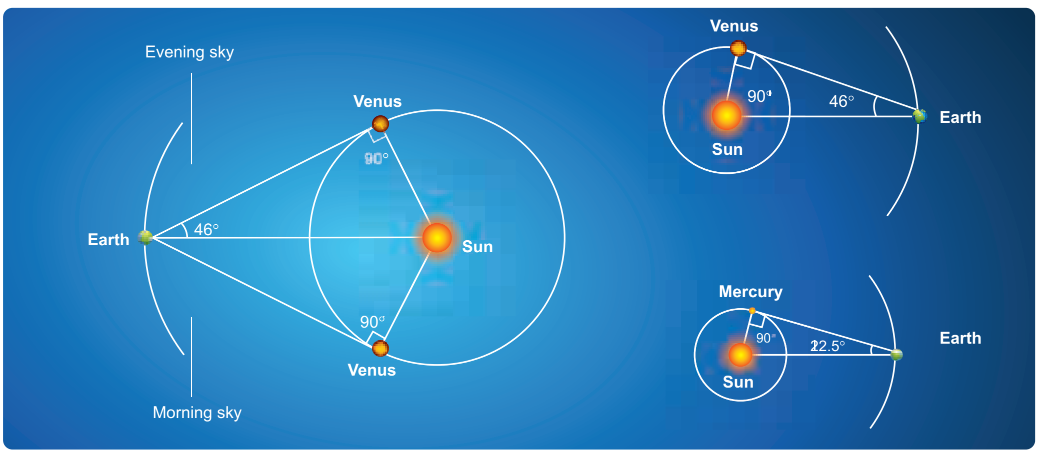


Figure 6.29 Angle of elevation for Mercury from horizon

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Here $\sin 46° = 0.77$. Hence Venus is at a distance of 0.77 AU from Sun. Similarly, the distance between Mercury (θ is 22.5 degree) and Sun is calculated as 0.38 AU. To find the distance of exterior planets like Mars and Jupiter, a slightly different method is used. The distances of planets from the Sun is given in the table below.

Table 6.2 a^3/T^2 for different planets

Planet	semi major axis of the orbit(a)	Period T (years)	a^3/T^2
Mercury	0.389 AU	87.77	7.64
Venus	0.724 AU	224.70	7.52
Earth	1.000 AU	365.25	7.50
Mars	1.524 AU	686.98	7.50
Jupiter	5.200 AU	4332.62	7.49
Saturn	9.510 AU	10,759.20	7.40

It is to be noted that to verify the Kepler's law we need only high school level geometry and trigonometry.

BLKYQZ

ACTIVITY

Venus can be observed with the naked eye. We can see Venus during sunrise or sunset. Students are encouraged to observe the motion of Venus and verify that the maximum elevation is at 46 degree and calculate the distance of Venus from the Sun. As pointed out already Google or Stellarium will be helpful in locating the position of Venus in the sky.

6.5.3 Measurement of radius of the Earth

Around 225 B.C a Greek librarian "Eratosthenes" who lived at Alexandria measured the radius of the Earth with a small error when compared with results using modern measurements. The technique he used involves lower school geometry and

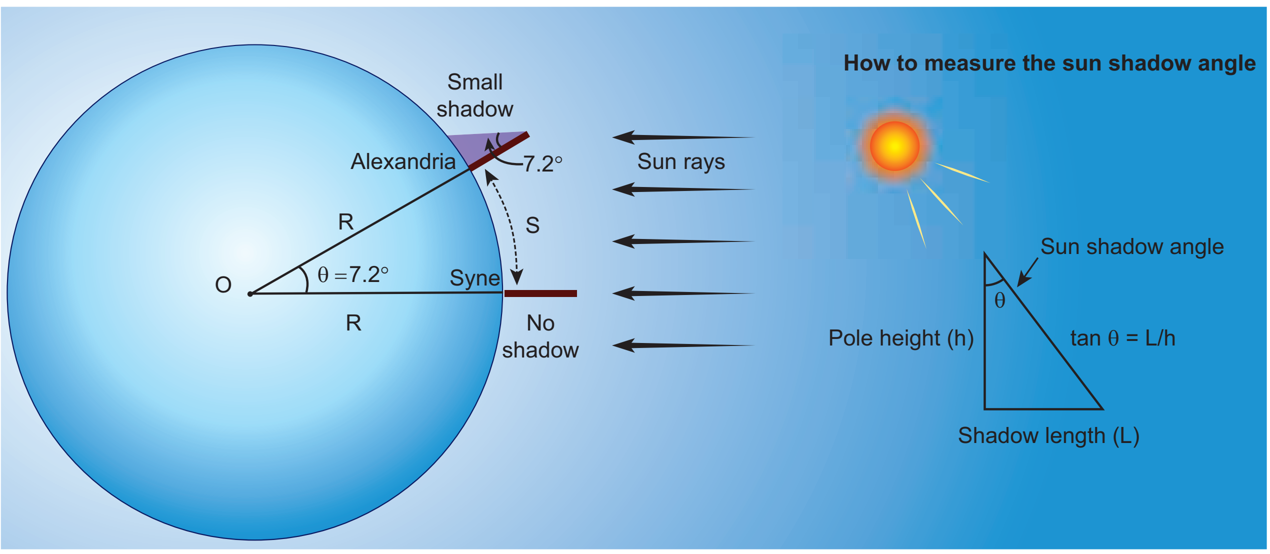


Figure 6.30 Measuring radius of The Earth

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

brilliant insight. He observed that during noon time of summer solstice the Sun's rays cast no shadow in the city Syne which was located 500 miles away from Alexandria. At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical as shown in the Figure 6.30. He realized that this difference of 7.2 degree was due to the curvature of the Earth.

The angle 7.2 degree is equivalent to $\frac{1}{8}$ *radian*. So $\theta = \frac{1}{8} rad$;

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then

$$S = R\theta = 500\,miles,$$

so radius of the Earth

$$R = \frac{500}{\theta} \text{ miles}$$

$$R = 500 \frac{miles}{\frac{1}{8}}$$

R = 4000 miles

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

The distance of the Moon from Earth was measured by a famous Greek astronomer Hipparchus in the 3rd century BC.

BLPLEH

ACTIVITY

To measure the radius of the Earth, choose two different places (schools) that are separated by at least 500 km. It is important to note that these two places have to be along the same longitude of the Earth (For example Hosur and Kanyakumari lie along the same longitude of 77.82° E). Take poles of known length (h) and fix them vertically in the ground (it may be in the school playgrounds) at both the places. At exactly noon in both the places the length of the shadow (L) cast by each pole has to be noted down. Draw the picture like in Figure 6.30. By using the equation $\tan\theta = \frac{L}{h}$, the angle in radian can be found at each place. The difference in angle (θ') is due to the curvature of the Earth. Now the distance between the two schools can be obtained from 'Google maps'. Divide the distance with the angle (θ' in radians) which will give the radius of the Earth.

6.5.4 Interesting Astronomical Facts

1. **Lunar eclipse and measurement of shadow of Earth**

   On January 31, 2018 there was a total lunar eclipse which was observed from various places including Tamil Nadu. It is possible to measure the radius of shadow of the Earth at the point where the Moon crosses. Figure 6.31 illustrates this.

   When the Moon is inside the umbra shadow, it appears red in color. As soon as the Moon exits from the umbra

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

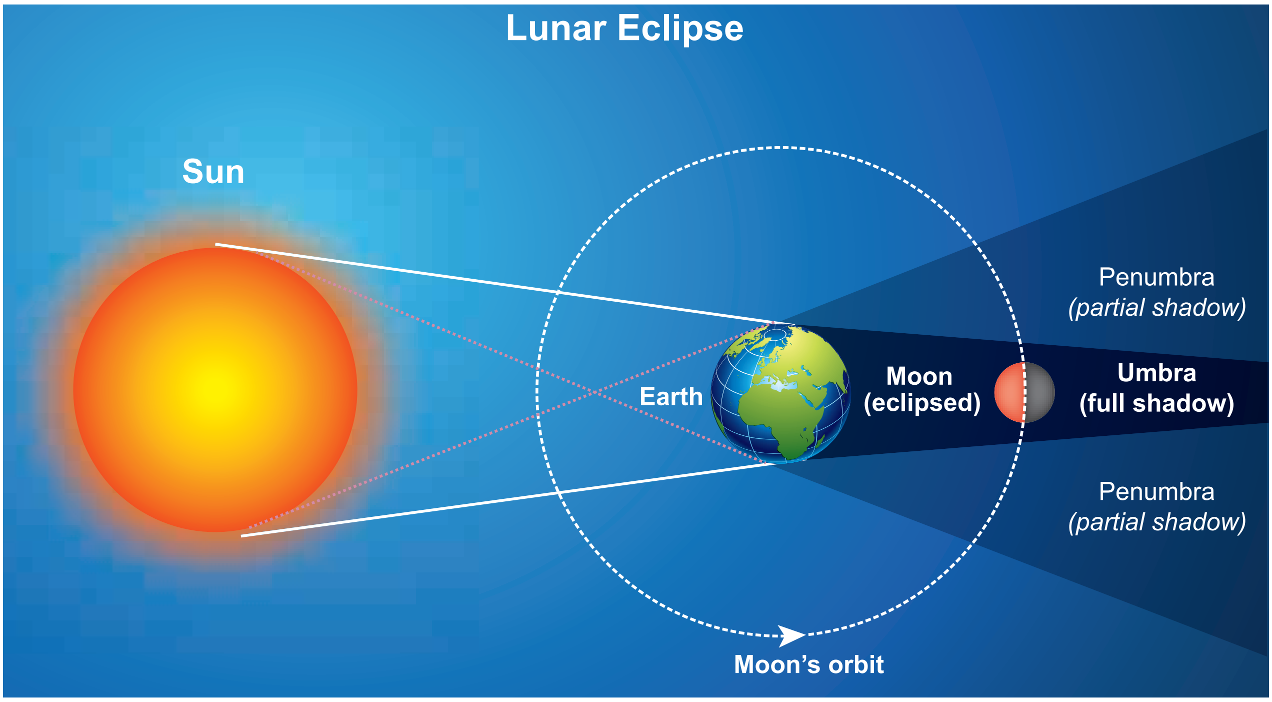


Figure 6.31 Total lunar eclipse

shadow, it appears in crescent shape. Figure 6.32 is the photograph taken by digital camera during Moon's exit from the umbra shadow.

Figure 6.32 Image of the Moon when it exits from umbra shadow

By finding the apparent radii of the Earth's umbra shadow and the Moon, the ratio of the these radii can be calculated. This is shown in Figures 6.33 and 6.34.

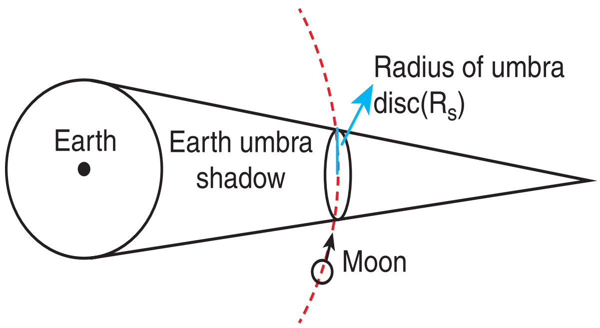


Figure 6.33 Schematic diagram of umbra disk radius

The apparent radius of Earth's umbra shadow = R_s = 13.2 cm

The apparent radius of the Moon = R_m = 5.15 cm

The ratio $\frac{R_s}{R_m} \approx 2.56$

The radius of the Earth's umbra shadow is $R_s = 2.56 \times R_m$

The radius of Moon $R_m = 1737\,km$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

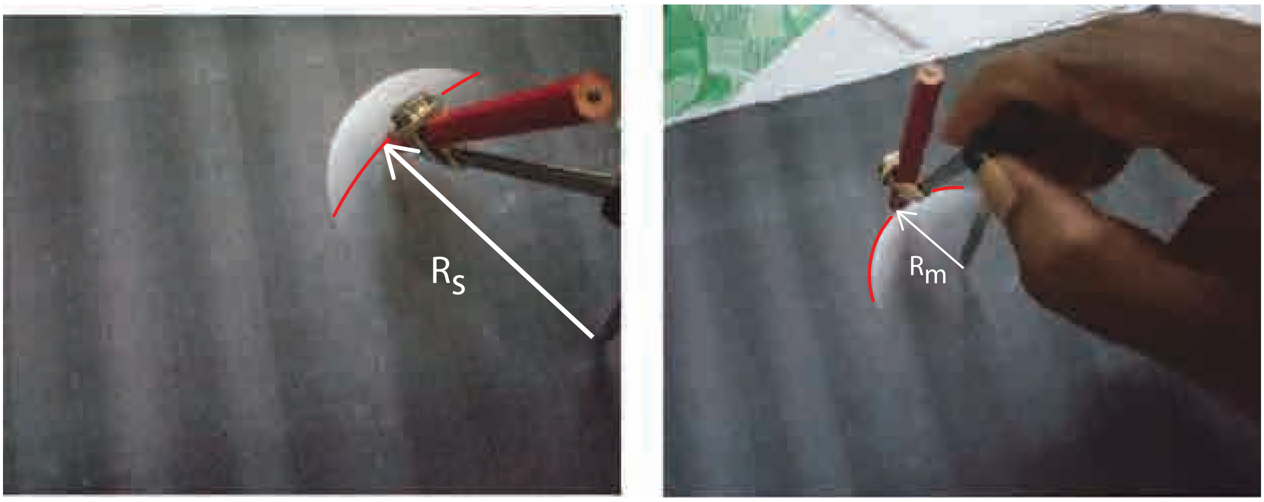


Figure 6.34 Calculation of umbra radius

The radius of the Earth's umbra shadow is $R_s = 2.56 \times 1737 km \cong 4446 km$.

The correct radius is 4610 km.

The percentage of error in the calculation

$$= \frac{4610 - 4446}{4610} \times 100 = 3.5\%.$$

The error will reduce if the pictures taken using a high quality telescope are used. It is to be noted that this calculation is done using very simple mathematics.

Early astronomers proved that Earth is spherical in shape by looking at the shape of the shadow cast by Earth on the Moon during lunar eclipse.

2. **Why there are no lunar eclipse and solar eclipse every month?**
If the orbits of the Moon and Earth lie on the same plane, during full Moon of every month, we can observe lunar eclipse. If this is so during new Moon we can observe solar eclipse. But Moon's orbit is tilted 5° with respect to Earth's orbit. Due to this 5° tilt, only during certain periods of the year, the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment. This is shown in Figure 6.35

3. **Why do we have seasons on Earth?**
The common misconception is that 'Earth revolves around the Sun, so when the Earth is very far away, it is winter and when the Earth is nearer, it is summer'. Actually, the seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. This is shown in Figure 6.36

Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

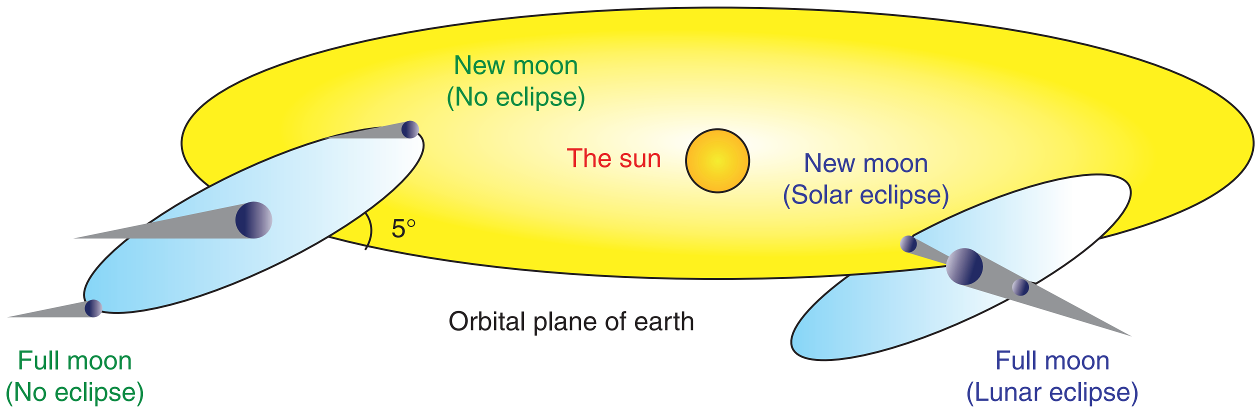


Figure 6.35 Orbital tilt of the Moon

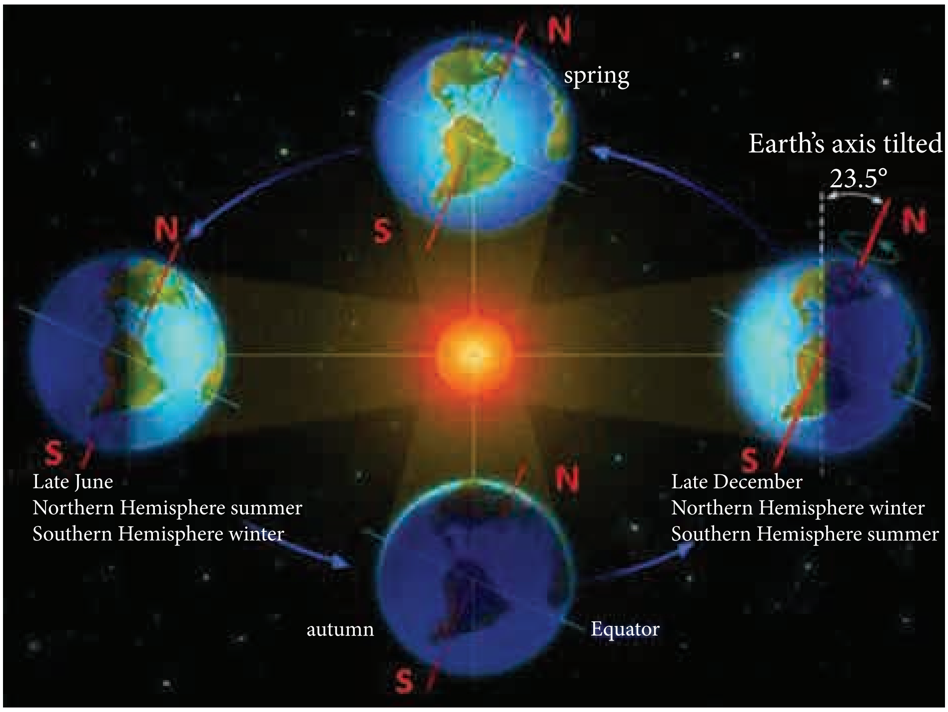


Figure 6.36 Seasons on Earth

> **Note** Pole star is a star located exactly above the Earth's axis of rotation, hence it appears to be stationary. The Star Polaris is our pole star.

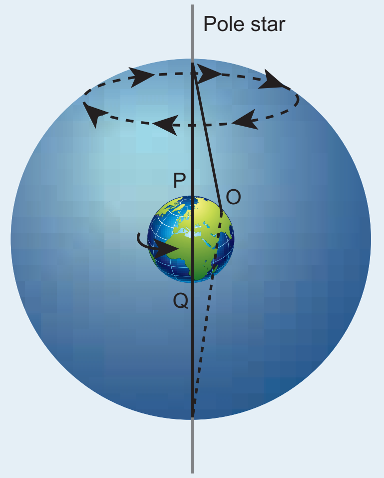
4. **Star's apparent motion and spinning of the Earth**

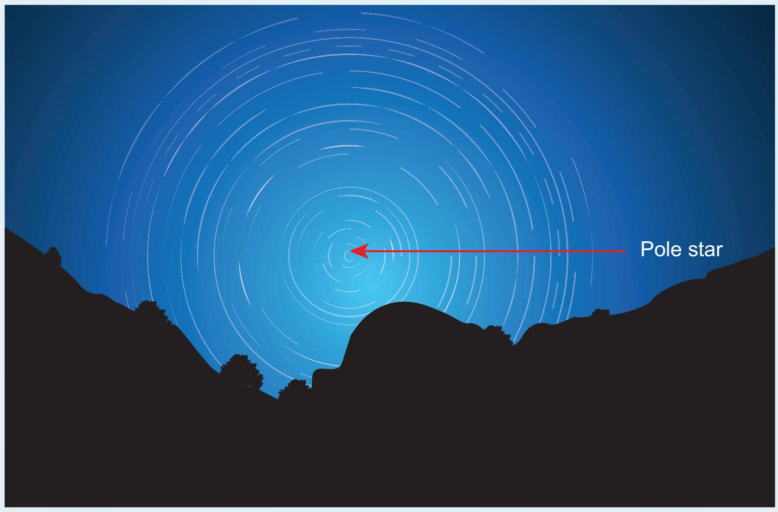
The Earth's spinning motion can be proved by observing star's position over a night. Due to Earth's spinning motion, the stars in sky appear to move in circular motion about the pole star as shown in Figure 6.37

Point to ponder

Using Sun rays and shadows, How will you prove that the Earth's tilt is 23.5° ?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181





If we observe the stars, it slowly changes its position and appears to move in a circle over the night.

Figure 6.37 Star's apparent circular motion due to Earth's rotation.

6.5.5 Recent developments of astronomy and gravitation

Till the 19th century astronomy mainly depended upon either observation with the naked eye or telescopic observation. After the discovery of the electromagnetic spectrum at the end of the 19th century, our understanding of the universe increased enormously. Because of this development in the late 19th century it was found that Newton's law of gravitation could not explain certain phenomena and showed some discrepancies. Albert Einstein formulated his 'General theory of relativity' which was one of the most successful theories of 20th century in the field of gravitation.

In the twentieth century both astronomy and gravitation merged together and have grown in manifold. The birth and death of stars were more clearly understood. Many Indian physicists made very important contributions to the field of astrophysics and gravitation. Subramanian Chandrasekar formulated the theory of black holes and explained the life of stars. These studies brought him the Nobel prize in the year 1983. Another very notable Indian astrophysicist Meghnad Saha discovered the ionization formula which was useful in classifying stars. This formula is now known as "Saha ionization formula". In the field of gravitation Amal Kumar Raychaudhuri solved an equation now known as "Raychaudhuri equation" which was a very important contribution. Another notable Indian Astrophysicist Jayant V Narlikar made pioneering contribution in the field of astrophysics and has written interesting books on astronomy and astrophysics. IUCAA (Inter University Center for Astronomy and Astrophysics) is one of the important Indian research institutes where active research in astrophysics and gravitation are conducted. The institute was founded by Prof. J.V. Narlikar. Students are encouraged to read more about the recent developments in these fields.

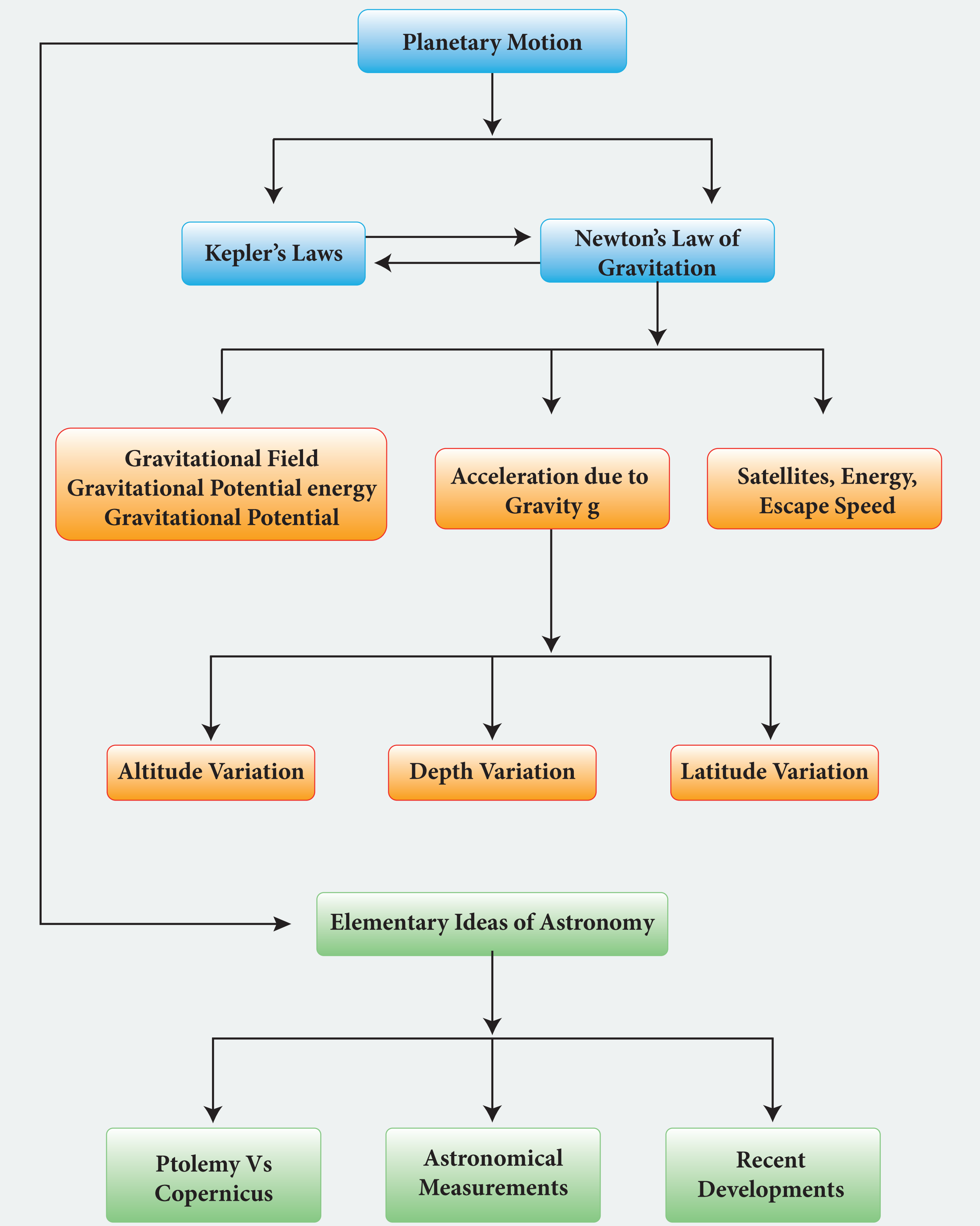
athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SUMMARY

- The motion of planets can be explained using Kepler's laws.
- **Kepler's first law**: All the planets in the solar system orbit the Sun in elliptical orbits with the Sun at one of the foci.
- **Kepler's second law:** The radial vector line joining the Sun to a planet sweeps equal areas in equal intervals of time.
- **Kepler's third law:** The ratio of the square of the time period of planet to the cubic power of semi major axis is constant for all the planets in the solar system.
- **Newton's law of gravitation** states that the gravitational force between two masses is directly proportional to product of masses and inversely proportional to square of the distance between the masses. In vector form it is given by $\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$
- Gravitational force is a central force.
- Kepler's laws can be derived from Newton's law of gravitation.
- The gravitational field due to a mass m at a point which is at a distance r from mass m is given by $\vec{E} = -\frac{Gm}{r^2}\hat{r}$. It is a vector quantity.
- The gravitational potential energy of two masses is given by $U = -\frac{Gm_1m_2}{r}$. It is a scalar quantity.
- The gravitational potential at a point which is at a distance r from mass m is given by $V = -\frac{Gm}{r}$. It is a scalar quantity.
- The acceleration due to Earth's gravity decreases as altitude increases and as depth increases.
- Due to rotation of the Earth, the acceleration due to gravity is maximum at poles and minimum at Earth's equator.
- The (escape) speed of any object required to escape from the Earth's gravitational field is $v_e = \sqrt{2gR_e}$. It is independent of mass of the object.
- The energy of the satellite is negative. It implies that the satellite is bound to Earth's gravitational force.
- Copernicus model explained that retrograde motion is due to relative motion between planets. This explanation is simpler than Ptolemy's epicycle explanation which is complicated
- Copernicus and Kepler measured the distance between a planet and the Sun using simple geometry and trigonometry.
- 2400 years ago, Eratosthenes measured the radius of the Earth using simple geometry and trigonometry.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CONCEPT MAP



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EVALUATION

I. Multiple Choice Questions

1. The linear momentum and position vector of the planet is perpendicular to each other at
   (a) perihelion and aphelion
   (b) at all points
   (c) only at perihelion
   (d) no point

2. If the masses of the Earth and Sun suddenly double, the gravitational force between them will
   (a) remain the same
   (b) increase 2 times
   (c) increase 4 times
   (d) decrease 2 times

3. A planet moving along an elliptical orbit is closest to the Sun at distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are linear speeds at these points respectively. Then the ratio $\frac{v_1}{v_2}$ is

   (NEET 2016)

   (a) $\frac{r_2}{r_1}$ (b) $\left(\frac{r_2}{r_1}\right)^2$

   (c) $\frac{r_1}{r_2}$ (d) $\left(\frac{r_1}{r_2}\right)^2$

4. The time period of a satellite orbiting Earth in a cirular orbit is independent of .
   (a) Radius of the orbit
   (b) The mass of the satellite
   (c) Both the mass and radius of the orbit
   (d) Neither the mass nor the radius of its orbit

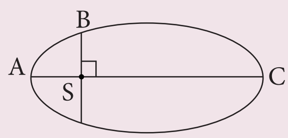
5. If the distance between the Earth and Sun were to be doubled from its present value, the number of days in a year would be
   (a) 64.5 (b) 1032
   (c) 182.5 (d) 730

6. According to Kepler's second law, the radial vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of
   (a) conservation of linear momentum
   (b) conservation of angular momentum
   (c) conservation of energy
   (d) conservation of kinetic energy

7. The gravitational potential energy of the Moon with respect to Earth is
   (a) always positive
   (b) always negative
   (c) can be positive or negative
   (d) always zero

8. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then

   (NEET 2018)

   (a) $K_A > K_B > K_C$
   (b) $K_B < K_A < K_C$
   (c) $K_A < K_B < K_C$
   (d) $K_B > K_A > K_C$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

9. The work done by the Sun's gravitational force on the Earth is
   (a) always zero
   (b) always positive
   (c) can be positive or negative
   (d) always negative

10. If the mass and radius of the Earth are both doubled, then the acceleration due to gravity g'
   (a) remains same (b) $\frac{g}{2}$
   (c) 2 g (d) 4 g

11. The magnitude of the Sun's gravitational field as experienced by Earth is
   (a) same over the year
   (b) decreases in the month of January and increases in the month of July
   (c) decreases in the month of July and increases in the month of January
   (d) increases during day time and decreases during night time.

12. If a person moves from Chennai to Trichy, his weight
   (a) increases
   (b) decreases
   (c) remains same
   (d) increases and then decreases

13. An object of mass 10 kg is hanging on a spring scale which is attached to the roof of a lift. If the lift is in free fall, the reading in the spring scale is
   (a) 98 N (b) zero
   (c) 49 N (d) 9.8 N

14. If the acceleration due to gravity becomes 4 times its original value, then escape speed
   (a) remains same
   (b) 2 times of original value
   (c) becomes halved
   (d) 4 times of original value

15. The kinetic energy of the satellite orbiting around the Earth is
   (a) equal to potential energy
   (b) less than potential energy
   (c) greater than kinetic energy
   (e) zero

Answers

1) a	**2)** c	**3)** a	**4)** b	**5)** b
6) b	**7)** b	**8)** a	**9)** c	**10)** b
11) c	**12)** a	**13)** b	**14)** b	**15)** b

II. Short Answer Questions

1. State Kepler's three laws.
2. State Newton's Universal law of gravitation.
3. Will the angular momentum of a planet be conserved? Justify your answer.
4. Define the gravitational field. Give its unit.
5. What is meant by superposition of gravitational field?
6. Define gravitational potential energy.
7. Is potential energy the property of a single object? Justify.
8. Define gravitational potential.
9. What is the difference between gravitational potential and gravitational potential energy?
10. What is meant by escape speed in the case of the Earth?
11. Why is the energy of a satellite (or any other planet) negative?
12. What are geostationary and polar satellites?
13. Define weight

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

14. Why is there no lunar eclipse and solar eclipse every month?

15. How will you prove that Earth itself is spinning?

III. Long Answer Questions

1. Discuss the important features of the law of gravitation.
2. Explain how Newton arrived at his law of gravitation from Kepler's third law.
3. Explain how Newton verified his law of gravitation.
4. Derive the expression for gravitational potential energy.
5. Prove that at points near the surface of the Earth, the gravitational potential energy of the object is $U = mgh$
6. Explain in detail the idea of weightlessness using lift as an example.
7. Derive an expression for escape speed.
8. Explain the variation of g with lattitude.
9. Explain the variation of g with altitude.
10. Explain the variation of g with depth from the Earth's surface.
11. Derive the time period of satellite orbiting the Earth.
12. Derive an expression for energy of satellite.
13. Explain in detail the geostationary and polar satellites.
14. Explain how geocentric theory is replaced by heliocentric theory using the idea of retrograde motion of planets.
15. Explain in detail the Eratosthenes method of finding the radius of Earth.
16. Describe the measurement of Earth's shadow (umbra) radius during total lunar eclipse

IV. Conceptual Questions

1. In the following, what are the quantities which that are conserved?
   a) Linear momentum of planet
   b) Angular momentum of planet
   c) Total energy of planet
   d) Potential energy of a planet
2. The work done by Sun on Earth in one year will be
   a) Zero b) Non zero
   c) positive d) negative
3. The work done by Sun on Earth at any finite interval of time is
   a) positive, negative or zero
   b) Strictly positive
   c) Strictly negative
   d) It is always zero
4. If a comet suddenly hits the Moon and imparts energy which is more than the total energy of the Moon, what will happen?
5. If the Earth's pull on the Moon suddenly disappears, what will happen to the Moon?
6. If the Earth has no tilt, what happens to the seasons of the Earth?
7. A student was asked a question 'why are there summer and winter for us? He replied as 'since Earth is orbiting in an elliptical orbit, when the Earth is very far away from the Sun(aphelion) there will be winter, when the Earth is nearer to the Sun(perihelion) there will be winter'. Is this answer correct? If not,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

what is the correct explanation for the occurrence of summer and winter?

8. The following photographs are taken from the recent lunar eclipse which occurred on January 31, 2018. Is it possible to prove that Earth is a sphere from these photographs?

V. Numerical Problems

1. An unknown planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is T_1, what is the time period of this unknown planet?

Ans: $T_2 = 2\sqrt{2}T_1$

2. Assume that you are in another solar system and provided with the set of data given below consisting of the planets' semi major axes and time periods. Can you infer the relation connecting semi major axis and time period?

Planet (imaginary)	Time period(T) (in year)	Semi major axis (a) (in AU)
Kurinji	2	8
Mullai	3	18
Marutham	4	32
Neithal	5	50
Paalai	6	72

Ans: $a \propto 2T^2$

3. If the masses and mutual distance between the two objects are doubled, what is the change in the gravitational force between them?

Ans: No change

4. Two bodies of masses m and 4m are placed at a distance r. Calculate the gravitational potential at a point on the line joining them where the gravitional field is zero.

Ans: $V = -\frac{9Gm}{r}$

5. If the ratio of the orbital distance of two planets $\frac{d_1}{d_2}$ =2, what is the ratio of gravitational field experienced by these two planets?

Ans: $E_2 = 4\ E_1$

6. The Moon Io orbits Jupiter once in 1.769 days. The orbital radius of the Moon Io is 421700 km. Calculate the mass of Jupiter?

Ans: 1.898×10^{27} kg

7. If the angular momentum of a planet is given by $\vec{L} = 5t^2\hat{i} - 6t\hat{j} + 3\hat{k}$. What is the torque experienced by the planet? Will the torque be in the same direction as that of the angular momentum?

Ans: $\vec{\tau} = 10t\hat{i} - 6\hat{j}$

8. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. Calculate the speed of each particle

Ans: $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

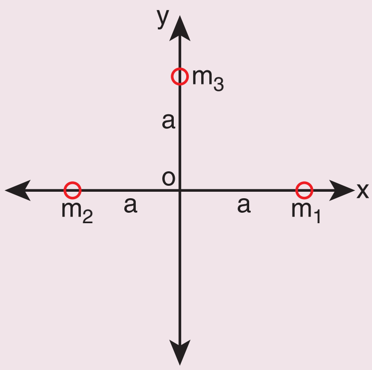
9. Suppose unknowingly you wrote the universal gravitational constant value

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

as $G=6.67\times10^{11}$ instead of the correct value $G=6.67\times10^{-11}$, what is the acceleration due to gravity g' for this incorrect G? According to this new acceleration due to gravity, what will be your weight W'?

Ans: $g' = 10^{22} g$, $W' = 10^{22} W$

10. Calculate the gravitational field at point O due to three masses m_1,m_2 and m_3 whose positions are given by the following figure. If the masses m_1 and m_2 are equal what is the change in gravitational field at the point O?



$$\text{Ans: } \vec{E}=\frac{GM}{a^2}\left[(m_1-m_2)\hat{i}+m_3\hat{j}\right]$$

$$\text{if } m_1 = m_2, \vec{E}=\frac{GM}{a^2}\left[m_3\hat{j}\right]$$

11. What is the gravitational potential energy of the Earth and Sun? The Earth to Sun distance is around 150 million km. The mass of the Earth is 5.9×10^{24} kg and mass of the Sun is 1.9×10^{30} kg.

Ans: V= -49.84×10^{32} Joule

12. Earth revolves around the Sun at 30 km s^{-1}. Calculate the kinetic energy of the Earth. In the previous example you calculated the potential energy of the Earth. What is the total energy of the Earth in that case? Is the total energy positive? Give reasons.

Ans: K.E = 26.5×10^{32} J

E = -23.29×10^{32} J

(−) ve implies that Earth is bounded with Sun

13. An object is thrown from Earth in such a way that it reaches a point at infinity with non-zero kinetic energy $\left[K.E(r=\infty)=\frac{1}{2}Mv_{\infty}^{2}\right]$, with what velocity should the object be thrown from Earth?

Ans: $v_e=\sqrt{v_{\infty}^2+2gR_E}$

14. Suppose we go 200 km above and below the surface of the Earth, what are the g values at these two points? In which case, is the value of g small?

Ans: g_{down} = 0.96 g

g_{up} = 0.94 g

15. Calculate the change in g value in your district of Tamil nadu. (Hint: Get the latitude of your district of Tamil nadu from the Google). What is the difference in g values at Chennai and Kanyakumari?

Ans: $g_{chennai}$ = 9.767 m s^{-2}

$g_{Kanyakumari}$ = 9.798 m s^{-2}

Δg = 0.031 m s^{-2}

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

BOOKS FOR REFERENCE

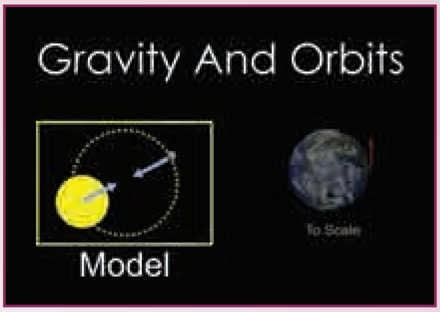
1. Mechanics by Charles Kittel, Walter Knight, Malvin Ruderman, Carl Helmholtz and Moyer
2. Newtonian Mechanics by A.P. French
3. Introduction to Mechanics by Daniel Kepler and Robert Kolenkow
4. Mechanics by Somnath Datta
5. Concepts of Physics volume 1 and Volume 2 by H.C. Verma
6. Physics for Scientist and Engineers with Modern physics by Serway and Jewett
7. Physics for Scientist and Engineers by Paul Tipler and Gene Mosca
8. Physics for the Inquiring Mind by Eric Rogers
9. Fundamental laws of Mechanics by Irodov.
10. Question and Problems in School Physics by Tarasov and Tarasova

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ICT CORNER

Gravitation

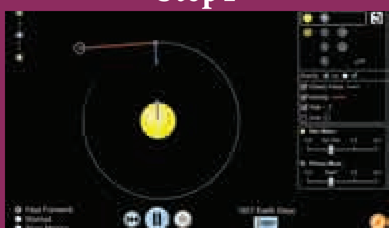
Through this activity you will be able to learn about the gravitational force and orbital paths.



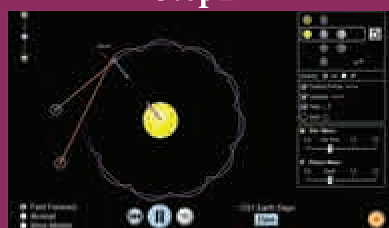
STEPS:

- Click the URL or scan the QR code to launch the activity page. Click on the box labelled "Model" to start the activity.
- In the activity window, a diagram of sun and earth is given. Click the play icon to see the motion of earth.
- We can change the objects by selecting objects from the table given in the right side window.
- The path of the gravity, velocity and the object in motion can be viewed. Check on the relevant boxes given in the table.

Step1

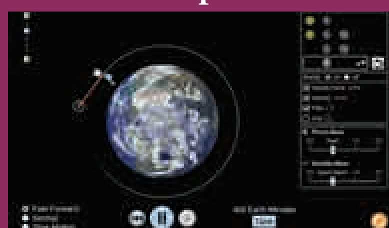


Step2



Step3

Step4



URL:

https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

UNIT 7

PROPERTIES OF MATTER

Many of the greatest advances that have been made from the beginning of the world to the present time have been made in the earnest desire to turn the knowledge of the properties of matter to some purpose useful to mankind– Lord Kelvin

Learning Objectives

In this unit, the student is exposed to

- inter atomic or intermolecular forces in matter
- stress, strain and elastic modulus
- surface tension
- viscosity
- properties of fluids and their applications

BM4QUA

7.1 INTRODUCTION

One of the oldest dams in the world is Kallanai (கல்லணை) located at Trichy. Kallanai was built across river Kaveri for the purpose of irrigation. During heavy floods, the velocity of water is generally very high in river Kaveri. The stability and utility of Kallanai dam reveal the intuitive scientific understanding of Tamils who designed this dam as early as 2nd century AD. The other example known for insightful constructions of the past is the pyramids in Egypt. The flyovers and over bridges are common worldwide today. Heavy vehicles ply over and hence, the bridges are always under stress. Without effective design using suitable materials, the bridges and flyovers will not

Kallanai (கல்லணை)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

be stable. The growth of human civilization is due to the understanding of various forms of matter (solid, liquid, and gas).

The study of properties of matter is very essential in selecting any material for a specific application. For example, in technology, the materials used for space applications should be of lightweight but should be strong. Materials used for artificial human organ replacements should be biocompatible. Artificial body fluids are used as tissue substitute for radiotherapy analysis in medicine. Fluids used as lubricants or fuel should possess certain properties. Such salient macroscopic properties are decided by the microscopic phenomena within matter. This unit deals with the properties of solids and fluids and the laws governing the behaviour of matter.

7.2 MICROSCOPIC UNDERSTANDING OF VARIOUS STATES OF MATTER

Even though various forms of matter such as solid food, liquids like water, and the air that we breathe are familiar in the day – to – day lifestyle for the past several thousand years, the microscopic understanding of solids, liquids, and gases was established only in the 20th century. In the universe, everything is made up of atoms. If so, why the same materials exist in three states? For example, water exists in three forms as solid ice, liquid water, and gaseous steam. Interestingly ice, water, and steam are made up of same types of molecules; two hydrogen atoms and one oxygen atom form a water molecule. Physics helps us to explore this beauty of nature at the microscopic level. The distance between the atoms or molecules determines whether it exist in the solid, liquid or gaseous state.

Solids

In solids, atoms or molecules are tightly fixed. In the solid formation, atoms get bound together through various types of bonding. Due to the interaction between the atoms, they position themselves at a particular interatomic distance. This position of atoms in this bound condition is called their mean positions.

Liquids

When the solid is not given any external energy such as heat, it will remain as a solid due to the bonding between atoms. When heated, atoms of the solid receive thermal energy and vibrate about their mean positions. When the solid is heated above its melting point, the heat energy will break the bonding between atoms and eventually the atoms receive enough energy and wander around. Here also the intermolecular (or interatomic) forces are important, but the molecules will have enough energy to move around, which makes the structure mobile.

Gases

When a liquid is heated at constant pressure to its boiling point or when the pressure is reduced at a constant temperature it will convert to a gas. This process of a liquid changing to a gas is called evaporation. The gas molecules have either very weak bonds or no bonds at all. This enables them to move freely and quickly. Hence, the gas will conform to the shape of its container and also will expand to fill the container. The transition from solid to liquid to gaseous states with the variation in external energy is schematically shown in Figure 7.1.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

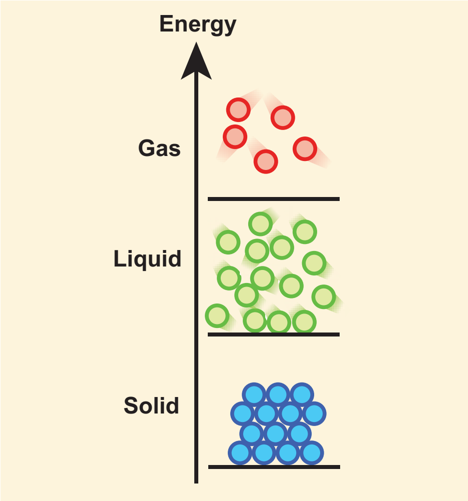


Figure 7.1 Schematic representations of the transition from solid to liquid to gaseous states with a change in external energy

> DO YOU KNOW?
>
> In addition to the three physical states of matter (solid, liquid, and gas), in extreme environments, matter can exist in other states such as plasma, Bose-Einstein condensates. Additional states, such as quark-gluon plasmas are also believed to be possible. A major part of the atomic matter of the universe is hot plasma in the form of rarefied interstellar medium and dense stars.

In the study of Newtonian mechanics (Volume 1), we assumed the objects to be either as point masses or perfect rigid bodies (collection of point masses). Both these are idealized models. In rigid bodies, changes in the shape of the bodies are so small that they are neglected. In real materials, when a force is applied on the objects, there could be some deformations due to the applied force. It is very important to know how materials behave when a deforming force is applied.

7.2.1 Elastic behaviour of materials

In a solid, interatomic forces bind two or more atoms together and the atoms occupy the positions of stable equilibrium. When a deforming force is applied on a body, its atoms are pulled apart or pushed closer. When the deforming force is removed, interatomic forces of attraction or repulsion restore the atoms to their equilibrium positions. If a body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity. The force which changes the size or shape of a body is called a deforming force.

Examples: Rubber, metals, steel ropes.

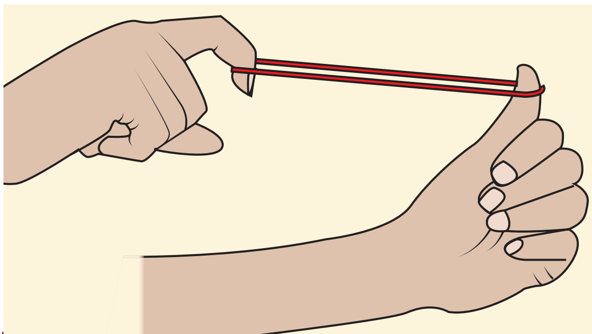


Figure 7.2 Elasticity

Plasticity:

If a body does not regain its original shape and size after removal of the deforming force, it is said to be a plastic body and the property is called plasticity.

Example: Glass

7.2.2 Stress and strain

(a) Stress:

When a force is applied, the size or shape or both may change due to the change in relative positions of atoms or molecules. This deformation may not be noticeable to

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

our naked eyes but it exists in the material itself. When a body is subjected to such a deforming force, internal force is developed in it, called as restoring force. The force per unit area is called as stress.

$$\text{Stress, } \sigma = \frac{Force}{Area} = \frac{F}{A} \quad (7.1)$$

The SI unit of stress is N m^{-2} or pascal (Pa) and its dimension is $[ML^{-1}T^{-2}]$. Stress is a tensor.

(i) Longitudinal stress and shearing stress:

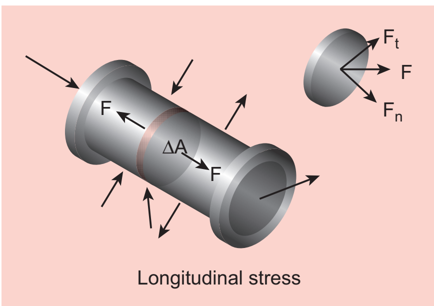


Figure 7.3 Longitudinal stress

Let us consider a body as shown in Figure 7.3. When many forces act on the system (body), the center of mass (defined in unit 5) remains at rest. However, the body gets deformed due to these forces and so the internal forces appear. Let ΔA be the cross sectional area of the body. The parts of the body on the two sides of ΔA exert internal forces $\vec{F}$ and $-\vec{F}$ on each other which is due to deformation. The force can be resolved in two components, F_n normal to the surface ΔA (perpendicular to the surface) and F_t tangential to the surface ΔA (tangent to the surface). The normal stress or longitudinal stress (σ_n) over the area is defined as

$$\sigma_n = \frac{F_n}{\Delta A}$$

Similarly, the tangential stress or shearing stress σ_t over the area is defined as

$$\sigma_t = \frac{F_t}{\Delta A}$$

Longitudinal stress can be classified into two types, tensile stress and compressive stress.

Tensile stress

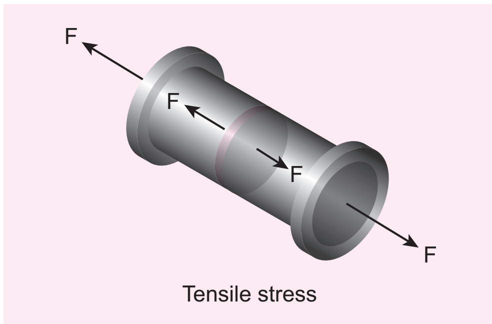


Figure 7.4 Tensile stress

Internal forces on the two sides of ΔA may pull each other, i.e., it is stretched by equal and opposite forces. Then, the longitudinal stress is called tensile stress.

Compressive stress

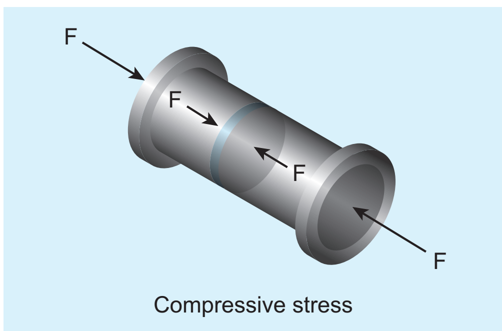


Figure 7.5 Compressive stress

When forces acting on the two sides of ΔA push each other, ΔA is pushed by equal and opposite forces at the two ends. In this case, ΔA is said to be under compression. Then, the longitudinal stress is called compressive stress.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(ii) Volume stress

This happens when a body is acted by forces everywhere on the surface such that the force at any point is normal to the surface and the magnitude of the force on a small surface area is proportional to the area. For instance, when a solid is immersed in a fluid, the pressure at the location of the solid is P, the force on any area ΔA is

$$F = P\,\Delta A$$

Where, F is perpendicular to the area. Thus, force per unit area is called volume stress.

$$\sigma_v = \frac{F}{A}$$

which is the same as the pressure.

(b) Strain:

Strain measures how much an object is stretched or deformed when a force is applied. Strain deals with the fractional change in the size of the object, in other words, strain measures the degree of deformation. As an example, in one dimension, consider a rod of length l when it stretches to a new length Δl then

$$\text{Strain, } \varepsilon = \frac{\text{change in size}}{\text{original size}} = \frac{\Delta l}{l} \qquad (7.2)$$

ε is a dimensionless quantity and has no unit. Strain is classified into three types.

(1) Longitudinal strain

When a rod of length l is pulled by equal and opposite forces, the longitudinal strain is defined as

$$\varepsilon_l = \frac{\text{increase in length of the rod}}{\text{original or natural length of the rod}} = \frac{\Delta l}{l} \qquad (7.3)$$

Longitudinal strain can be classified into two types

(i) Tensile strain: If the length is increased from its natural length then it is known as tensile strain.

(ii) Compressive strain: If the length is decreased from its natural length then it is known as compressive strain.

(2) Shearing strain

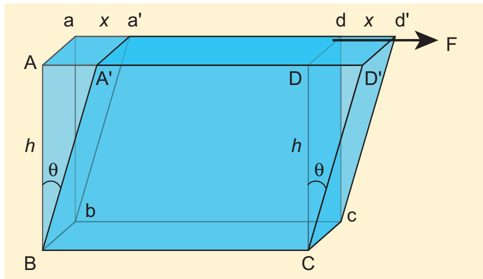


Figure 7.6 Shearing strain

Consider a cuboid as shown in Figure 7.6. Let us assume that the body remains in translational and rotational equilibrium. Let us apply the tangential force F along AD such that the cuboid deforms as shown in Figure 7.6. Hence, shearing strain or shear is (ε_s)

$$\varepsilon_s = \frac{AA'}{BA} = \frac{x}{h} = \tan\theta \qquad (7.4)$$

For small angle, $\tan\theta \approx \theta$

Therefore, shearing strain or shear,

$$\varepsilon_s = \frac{x}{h} = \theta = \text{ Angle of shear}$$

(3) Volume strain

If the body is subjected to a volume stress, the volume will change. Let V be the original volume of the body before stress and $V + \Delta V$ be the change in volume due to stress. The volume strain which measures the fractional change in volume is

$$\text{Volume strain, } \varepsilon_v = \frac{\Delta V}{V} \qquad (7.5)$$

Elastic Limit

The maximum stress within which the body regains its original size and shape after the

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

removal of deforming force is called the elastic limit.

If the deforming force exceeds the elastic limit, the body acquires a permanent deformation. For example, rubber band loses its elasticity if pulled apart too much. It changes its size and becomes misfit to be used again.

7.2.3 Hooke's law and its experimental verification

Hooke's law is for a small deformation, when the stress and strain are proportional to each other. It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed point O. A pan and a pointer are attached at the free end of the wire as shown in Figure 7.7 (a). The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL. It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis. This graph is a straight line passing through the origin as shown in Figure 7.7 (b).

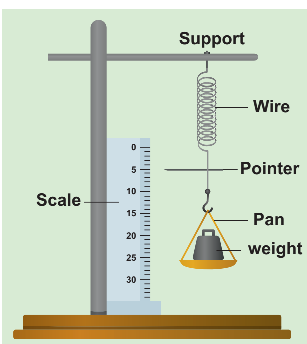


Figure 7.7 (a) Experimental verification of Hooke's law

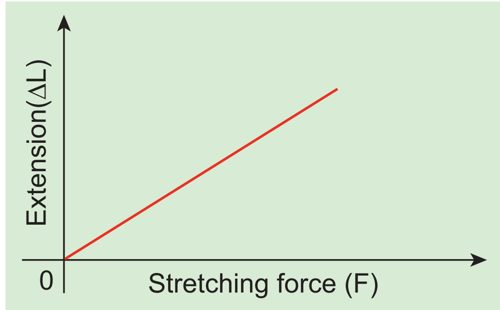


Figure 7.7 (b) Variation of ΔL with F

Therefore,

$$\Delta L = (\text{slope})F$$

Multiplying and dividing by volume,

$$V = A\,L,$$

$$F\,(\text{slope}) = \frac{AL}{AL}\Delta L$$

Rearranging, we get

$$\frac{F}{A} = \left(\frac{L}{A(slope)}\right)\frac{\Delta L}{L}$$

Therefore, $\frac{F}{A} \propto \left(\frac{\Delta L}{L}\right)$

Comparing with equation (7.1) and equation (7.2), we get equation (7.5) as

$$\sigma \propto \varepsilon$$

i.e., the stress is proportional to the strain in the elastic limit.

Stress – Strain profile curve:

The stress versus strain profile is a plot in which stress and strain are noted for each load and a graph is drawn taking strain along the X-axis and stress along the Y-axis. The elastic characteristics of the materials can be analyzed from the stress-strain profile.

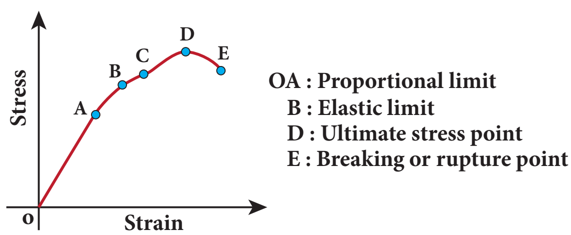


Figure 7.8 Stress-Strain profile

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(a) Portion OA:

In this region, stress is very small such that stress is proportional to strain, which means Hooke's law is valid. The point A is called *limit of proportionality* because above this point Hooke's law is not valid. The slope of the line OA gives the Young's modulus of the wire.

(b) Portion AB:

This region is reached if the stress is increased by a very small amount. In this region, stress is not proportional to the strain. But once the stretching force is removed, the wire will regain its original length. This behaviour ends at point B and hence, the point B is known as *yield point* (elastic limit). The elastic behaviour of the material (here wire) in stress-strain curve is OAB.

(c) Portion BC:

If the wire is stretched beyond the point B (elastic limit), stress increases and the wire will not regain its original length after the removal of stretching force.

(d) Portion CD:

With further increase in stress (beyond the point C), the strain increases rapidly and reaches the point D. Beyond D, the strain increases even when the load is removed and breaks (ruptures) at the point E. Therefore, the maximum stress (here D) beyond which the wire breaks is called *breaking stress* or *tensile strength*. The corresponding point D is known as *fracture point*. The region BCDE represents the plastic behaviour of the material of the wire.

7.2.4 Moduli of elasticity

From Hooke's law, the stress in a body is proportional to the corresponding strain, provided the deformation is very small. In this section, we shall define the elastic modulus of a given material. There are three types of elastic modulus.

(a) Young's modulus

(b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.

Young modulus of a material

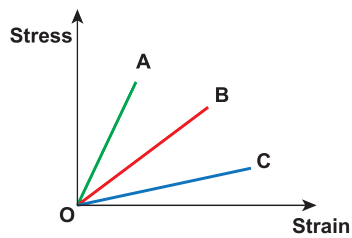
$$= \frac{\textit{Tensile stress or compressive stress}}{\textit{Tensile strain or compressive strain}}$$

$$Y = \frac{\sigma_t}{\varepsilon_t} \quad \text{or} \quad Y = \frac{\sigma_c}{\varepsilon_c} \qquad (7.6)$$

The unit for Young modulus has the same unit of stress because, strain has no unit. So, S.I. unit of Young modulus is N m^{-2} or pascal.

EXAMPLE 7.1

Within the elastic limit, the stretching strain produced in wires A, B, and C due to stress is shown in the figure. Assume the load applied are the same and discuss the elastic property of the material.



BHDLVX

Write down the elastic modulus in ascending order.

Solution

Here, the elastic modulus is Young modulus and due to stretching, stress is tensile stress and strain is tensile strain.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Within the elastic limit, stress is proportional to strain (obey Hooke's law). Therefore, it shows a straight line behavior. So, the modulus of elasticity (here, Young modulus) can be computed by taking slope from this straight line. Hence, calculating the slope for the straight line, we get

Slope of A > Slope of B > Slope of C

Which implies,

Young modulus of C < Young modulus of B < Young modulus of A

Notice that larger the slope, lesser the strain (fractional change in length). So, the material is much stiffer. Hence, the elasticity of wire A is greater than wire B which is greater than C. From this example, we have understood that Young's modulus measures the resistance of solid to a change in its length.

EXAMPLE 7.2

A wire 10 m long has a cross-sectional area 1.25 x 10^{-4} m^2. It is subjected to a load of 5 kg. If Young's modulus of the material is 4 x 10^{10} N m^{-2}, calculate the elongation produced in the wire.

Take g = 10 ms^{-2}.

Solution

We know that $\frac{F}{A} = Y \times \frac{\Delta L}{L}$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

$$= \left(\frac{50}{1.25 \times 10^{-4}}\right)\left(\frac{10}{4 \times 10^{10}}\right) = 10^{-4}\,\text{m}$$

Bulk modulus:

Bulk modulus is defined as the ratio of volume stress to the volume strain.

Bulk modulus, K =

$$\frac{Normal\,(Perpendicular)\,stress\,or\,Pressure}{Volume\,strain}$$

The normal stress or pressure is

$$\sigma_n = \frac{F_n}{\Delta A} = \Delta p$$

The volume strain is

$$\varepsilon_v = \frac{\Delta V}{V}$$

Therefore, Bulk modulus is

$$K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta P}{\frac{\Delta V}{V}} \quad (7.7)$$

The negative sign in the equation (7.7) means that when pressure is applied on the body, its volume decreases. Further, the equation (7.7) implies that a material can be easily compressed if it has a small value of bulk modulus. In other words, bulk modulus measures the resistance of solids to change in their volume. For an example, we know that gases can be easily compressed than solids, which means, gas has a small value of bulk modulus compared to solids. The S.I. unit of K is the same as that of pressure i.e., N m^{-2} or Pa (pascal).

Compressibility

The reciprocal of the bulk modulus is called compressibility. It is defined as the fractional change in volume per unit increase in pressure.

From the equation (7.7) we can say that the compressibility

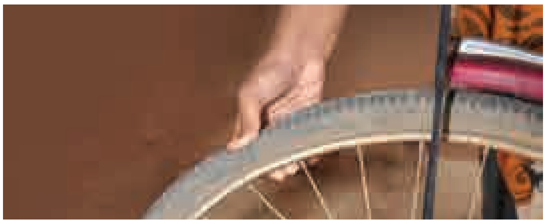
$$C = \frac{1}{K} = -\frac{\varepsilon_v}{\sigma_n} = -\frac{\frac{\Delta V}{V}}{\Delta P} \quad (7.8)$$

Since, gases have small value of bulk modulus than solids, their, values of compressibility is very high.

After pumping the air in the cycle tyre, usually we press the cycle tyre to check whether it has

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

enough air. What is checked here is essentially the compressibility of air. The tyre should be less compressible for its easy rolling



In fact the rear tyre is less compressible than front tyre for a smooth ride

EXAMPLE 7.3

A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 10^6 pascal. If the volume changes by 1.5×10^{-5} m^3, calculate the bulk modulus of the material.

Solution

By definition, $K = \dfrac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\dfrac{V}{\Delta V}$

$$K = \frac{10^6 \times 1}{1.5 \times 10^{-5}} = 6.67 \times 10^{10}\, N\, m^{-2}$$

The rigidity modulus or shear modulus:
The rigidity modulus is defined as
Rigidity modulus or Shear modulus,

$$\eta_R = \frac{\textit{shearing stress}}{\textit{angle of shear or shearing strain}}$$

The shearing stress is

$$\sigma_s = \frac{\text{tangential force}}{\textit{area over which it is applied}} = \frac{F_t}{\Delta A}$$

The angle of shear or shearing strain

$$\varepsilon_s = \frac{x}{h} = \theta$$

Therefore, Rigidity modulus is

$$\eta_R = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{\frac{F_t}{\Delta A}}{\theta} \qquad (7.9)$$

Further, the equation (7.9) implies, that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire, when it is twisted through an angle θ, a restoring torque is developed, that is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since, rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small. The S.I. unit of η_R is the same as that of pressure i.e., N m^{-2} or pascal. For the best understanding, the elastic coefficients of some of the important materials are listed in Table 7.1.

Table 7.1 Elastic coefficients of some of the materials in N m^{-2}

Material	Young's modulus (Y) (10^{10}N m^{-2})	Bulk modulus (K) (10^{10} N m^{-2})	Shear modulus (η_R) (10^{10} N m^{-2})
Steel	20.0	15.8	8.0
Aluminium	7.0	7.0	2.5
Copper	12.0	12.0	4.0
Iron	19.0	8.0	5.0
Glass	7.0	3.6	3.0

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 7.4

A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20 m, F = 4000 N, x = 0.50 cm = 0.005 m and Area A = L² = 0.04 m^2

Therefore,Shear modulus

$$\eta_R = \frac{F}{A} \times \frac{L}{x} = \frac{4000}{0.04} \times \frac{0.20}{0.005} = 4 \times 10^6 \, N\,m^{-2}$$

7.2.5 Poisson's ratio

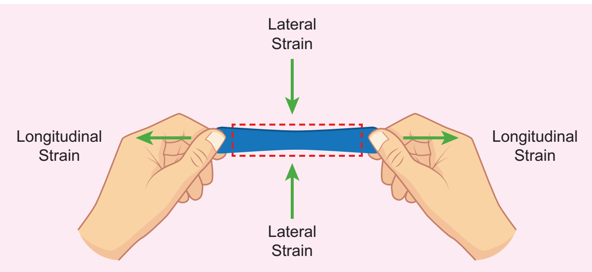


Figure 7.9 Lateral strain verses longitudinal strain

Suppose we stretch a wire, its length increases (elongation) but its diameter decreases (contraction). Similarly, when we stretch a rubber band (elongation), it becomes noticeably thinner (contraction). That is, deformation of the material in one direction produces deformation in another direction. To quantify this, French Physicist S.D. Poisson proposed a ratio, known as Poisson's ratio, which is defined as the ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

$$poisson's\,ratio, \mu = \frac{lateral\,strain}{longitudinal\,strain} \quad (7.10)$$

For a wire of length L with diameter D, due to applied force, wire stretches and hence, increase in length be l and decrease in diameter be d, then

$$\text{Poisson's ratio, } \mu = -\frac{\frac{d}{D}}{\frac{l}{L}} = -\frac{L}{l} \times \frac{d}{D}$$

Negative sign indicates the elongation along longitudinal and contraction along lateral dimension. Further, notice that it is the ratio between quantities of the same dimension. So, Poisson's ratio has no unit and no dimension (dimensionless number). The Poisson's ratio values of some of the materials are listed in Table 7.2.

Table 7.2 The Poisson's ratio of some of the materials

Material	Poisson's ratio
Rubber	0.4999
Gold	0.42 -0.44
Copper	0.33
Stainless steel	0.30-0.31
Steel	0.27-0.30
Cast iron	0.21-0.26
Concrete	0.1-0.2
Glass	0.18-0.3
Foam	0.10-0.50
Cork	0.0

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

7.2.6 Elastic energy

When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy. Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension *l* and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by d*l*, dW = F d*l*

The total work done in stretching the wire from 0 to *l* is

$$W = \int_0^l F\,dl \qquad (7.11)$$

From Young's modulus of elasticity,

$$Y = \frac{F}{A}\times\frac{L}{l} \Rightarrow F = \frac{YAl}{L} \qquad (7.12)$$

Substituting equation (7.12) in equation (7.11), we get

$$W = \int_0^l \frac{YAl}{L}dl$$

Since, *l is* the dummy variable in the integration, we can change *l to l'*(not in limits), therefore

$$W = \int_0^l \frac{YAl'}{L}dl' = \frac{YA}{L}\left(\frac{l'^2}{2}\right)\Bigg|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right)l = \frac{1}{2}Fl$$

$$W = \frac{1}{2}Fl = Elastic\ potential\ energy$$

Energy per unit volume is called energy density,

$$u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2}Fl}{AL}$$

$$\frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}\ (\text{Stress} \times \text{Strain}) \qquad (7.13)$$

EXAMPLE 7.5

A wire of length 2 m with the area of cross-section $10^{-6} m^2$ is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: $Y = 12 \times 10^{10} N\,m^{-2}$.

Solution

(i) stress $= \frac{F}{A} = \frac{980}{10^{-6}} = 98\times10^{7}\,N\,m^{-2}$

(ii) $strain = \frac{stress}{Y} = \frac{98\times10^{7}}{12\times10^{10}} = 8.17\times10^{-3}$ (no unit)

(iii) Since, volume = $2 \times 10^{-6} m^3$

$$Energy = \frac{1}{2}(stress\times strain)\times volume \Rightarrow$$

$$\frac{1}{2}(98\times10^{7})\times(8.17\times10^{-3})\times2\times10^{-6} = 8\ joule$$

7.2.7 Applications of elasticity

The mechanical properties of materials play a very vital role in everyday life. The elastic behavior is one such property which especially decides the structural design of the columns and beams of a building. As far as the structural engineering is concerned, the amount of stress that the design could withstand is a primary safety factor. A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight. The elastic behavior or in other words the bending of beams is a major concern over the stability of the buildings or bridges. For an example, to reduce the bending of a beam for a given load, one should use the material with a higher Young's modulus of elasticity (Y). It is obvious from Table 7.1 that the Young's modulus of steel is greater than aluminium or copper. Iron comes next to steel. This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

There is common misconception that rubber is more elastic. Which one is more elastic? Rubber or steel? Steel is more elastic than rubber. If an equal stress is applied to both steel and rubber, the steel produces less strain. So the Young's modulus is higher for steel than rubber. The object which has higher young's modulus is more elastic.

7.3 FLUIDS

7.3.1 Introduction

Fluids are found everywhere in the world. Earth has about two-thirds of water and one-third of land. Fluids are different from solids. Unlike solid, fluid has no defined shape of its own. As far as fluid is concerned, liquid has fixed volume whereas gas fills the entire volume of the container.

Pressure of a fluid:

Fluid is a substance which begins to flow when an external force is applied on it. It offers a very small resistance to the applied force. If the force acts on a smaller area, then the impact will be more and vice versa. This particular idea decides yet another quantity called '*pressure*'. Assume that an object is submerged in a fluid (say water) at rest. In this case, the fluid will exert a force on the surface of the object. This force is always normal to the surface of the object. If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.

$$P = \frac{F}{A} \qquad (7.14)$$

Pressure is a scalar quantity. It's S.I. unit and dimensions are $N\,m^{-2}$ or pascal (Pa) and $[ML^{-1}T^{-2}]$, respectively. Another common unit of pressure is atmosphere, which is abbreviated as 'atm'. It is defined as the pressure exerted by the atmosphere at sea level. i.e., 1 atm = 1.013×10^5 Pa or $N\,m^{-2}$. Apart from pressure, there are two more parameters such as density and relative density (or specific gravity) which are used to describe the nature of fluids.

Density of a fluid:

The density of a fluid is defined as its mass per unit volume. For a fluid of mass 'm' occupying volume 'V', the density $\rho = \frac{m}{V}$. The dimensions and S.I unit of ρ are $[ML^{-3}]$ and $kg\,m^{-3}$, respectively. It is a positive scalar quantity. Mostly, a liquid is largely incompressible and hence its density is nearly constant at ambient pressure (i.e. at 1 atm. pressure). In the case of gases, there are variations in densities with reference to pressure.

Relative density or specific gravity:

The relative density of a substance is defined as the ratio of the density of a substance to the density of water at 4 °C. It is a dimensionless positive scalar quantity. For example, the density of mercury is $13.6 \times 10^3 kg\,m^{-3}$. Its relative density is equal to $\frac{13.6 \times 10^3 kg\,m^{-3}}{1.0 \times 10^3 kg\,m^{-3}} = 13.6$.

EXAMPLE 7.6

A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution

Radius of the sphere R = 1.5 cm

mass m = 0.038 kg

Volume of the sphere $V=\frac{4}{3}\pi R^3$

$=\frac{4}{3}\times(3.14)\times(1.5\times10^{-2})^3 = 1.413\times10^{-5}m^3$

Therefore, density

$$\rho=\frac{m}{V}=\frac{0.038kg}{1.413\times10^{-5}m^3}=2690\,kg\,m^{-3}$$

Hence, the specific gravity of the sphere

$$=\frac{2690}{1000}=2.69$$

7.3.2 Pressure due to fluid column at rest

A mountaineer climbing the mountain is able to experience a decrease in pressure with altitude. A person jumping into the swimming pool always realizes an increase in pressure with depth below the water surface. In both the cases, the pressure encountered by the mountaineer and diver is usually due to the hydrostatic pressure, because they are due to fluids that are static. In order to understand the increase in pressure with depth below the water surface, consider a water sample of cross sectional area in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder, respectively as shown in Figure 7.10(a). Let F_1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2, such that, $F_1 = P_1 A$ and $F_2 = P_2 A$ Let us assume the mass of the sample to be m and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force (F_1 + mg), in other words, the gravitational force will act downward which is being exactly balanced by the difference between the force $F_2 - F_1$

$$F_2 - F_1 = mg = F_G \qquad (7.15)$$

where m is the mass of the water available in the sample element. Let ρ be the density of the water then, the mass of water available in the sample element is

$$m=\rho V = \rho A (h_2-h_1)$$
$$V=A (h_2-h_1)$$

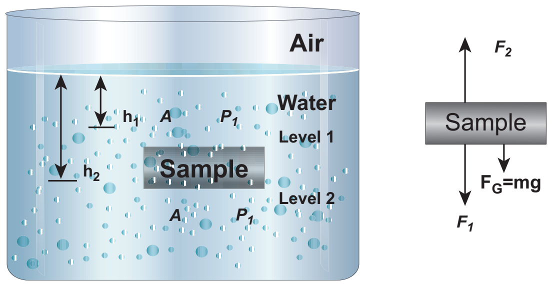


Figure 7.10 (a) A sample of water with base area A in a static fluid with its forces in equilibrium

Hence, gravitational force,

$$F_G = \rho A (h_2 - h_1) g$$

On substituting the value of W in equation (7.15)

$$F_2=F_1+m g \Rightarrow P_2A = P_1A+ \rho A(h_2-h_1)g$$

Cancelling out A on both sides,

$$P_2 = P_1 + \rho(h_2 - h_1)g \qquad (7.16)$$

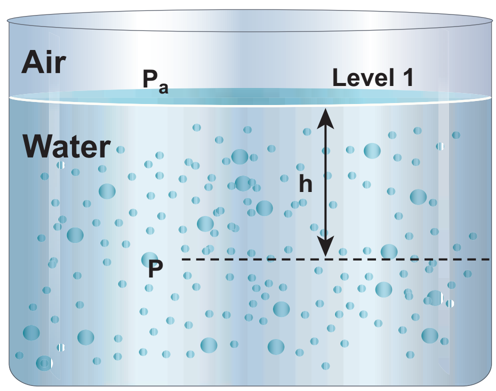


Figure 7.10 (b) The pressure (P) at a depth (h) below the water surface

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface (as shown in Figure 7.10(b)), then the value of h_1 becomes zero ($h_1 = 0$) and in turn P_1 assumes the value of atmospheric pressure (say P_a). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation (7.16), we get

$$P = P_a + \rho g h \qquad (7.17)$$

which means, the pressure at a depth h is greater than the pressure on the surface of the liquid, where P_a is the atmospheric pressure which is equal to 1.013×10^5 Pa.

If the atmospheric pressure is neglected or ignored then

$$P = \rho g h \qquad (7.18)$$

For a given liquid, ρ is fixed and g is also constant, then the pressure due to the fluid column is directly proportional to vertical distance or height of the fluid column. This implies, the height of the fluid column is more important to decide the pressure and not the cross sectional or base area or even the shape of the container.

When we talk about liquid at rest, the liquid pressure is the same at all points at the same horizontal level (or same depth). This statement can be demonstrated by an experiment called 'hydrostatic paradox'. Let us consider three vessels of different shapes A, B, and C as shown in Figure 7.11. These vessels are connected at the bottom by a horizontal pipe. When they are filled with a liquid (say water), it occupies the same level even though the vessels hold different amounts of water. It is true because the liquid at the bottom of each section of the vessel experiences the same pressure.

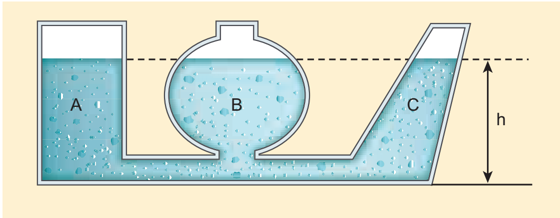
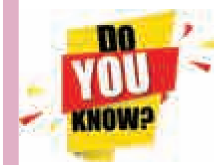


Figure 7.11 Illustration of hydrostatic paradox



The atmospheric pressure at a place is the gravitational force exerted by air above that place per unit surface area. It changes with height and weather conditions (i.e. density of air). In fact, the atmospheric pressure decreases with increasing elevation.

The decrease of atmospheric pressure with altitude has an unwelcome consequence in daily life. For example, it takes longer time to cook at higher altitudes. Nose bleeding is another common experience at higher altitude because of larger difference in atmospheric pressure and blood pressure.

Its value on the surface of the Earth at sea level is 1atm.

ACTIVITY

Take a metal container with an opening. Connect a vacuum pump to the opening. Evacuate the air from inside the container. Why the shape of the metal container gets crumbled?

Inference:

Due to the force of atmospheric pressure acting on its outer surface, the shape of the container crumbles.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ACTIVITY

Take a glass tumbler. Fill it with water to the brim. Slide a cardboard on its rim so that no air remains in between the cardboard and the tumbler. Invert the tumbler gently. The water does not fall down.

Inference:

This is due to the fact that the weight of water in the tumbler is balanced by the upward thrust caused due to the atmospheric pressure acting on the lower surface of the cardboard that is exposed to air.

7.3.3 Pascal's law and its applications

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. Statement of Pascal's law is *If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.*

Application of Pascal's law

Hydraulic lift

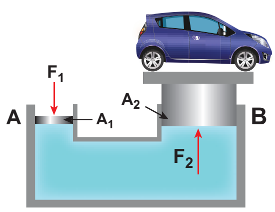


Figure 7.12 Hydraulic lift

A practical application of Pascal's law is the hydraulic lift which is used to lift a heavy load with a small force. It is a force multiplier. It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure 7.12). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to $P\left(where, P = \frac{F_1}{A_1}\right)$. But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2 \Rightarrow F_2 = \frac{A_2}{A_1} \times F_1 \quad (7.19)$$

Therefore by changing the force on the smaller piston A, the force on the piston B has been increased by the factor $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

EXAMPLE 7.7

Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?

Solution

Since, the diameter of the pistons are given, we can calculate the radius of the piston

$$r = \frac{D}{2}$$

Area of smaller piston, $A_1 = \pi\left(\frac{5}{2}\right)^2 = \pi(2.5)^2$

Area of larger piston, $A_2 = \pi\left(\frac{60}{2}\right)^2 = \pi(30)^2$

$$F_2 = \frac{A_2}{A_1} \times F_1 = (50N) \times \left(\frac{30}{2.5}\right)^2 = 7200N$$

This means, with the force of 50 N, the force of 7200 N can be lifted.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

7.3.4 Buoyancy

When a body is partially or fully immersed in a fluid, it displaces a certain amount of fluid. The displaced fluid exerts an upward force on the body. The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called *upthrust* or *buoyant force* and the phenomenon is called *buoyancy*.

Archimedes principle:

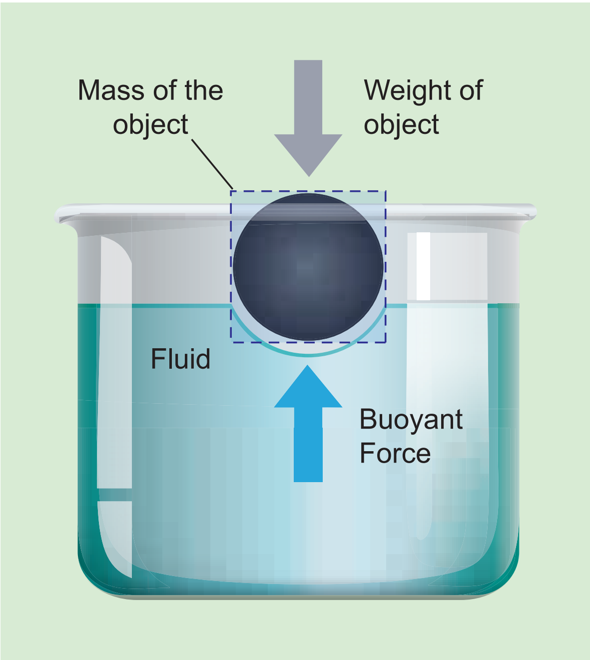


Figure 7.13 Archimedes principle

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

upthrust or buoyant force = weight of liquid displaced.

Law of floatation

It is well-known that boats, ships, and some wooden objects move on the upper part of the water, we say they float. Floatation can be defined as the tendency of an object to rise up to the upper levels of the fluid or to stay on the surface of the fluid.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body. For example, a wooden object weighs 300 kg (about 3000 N) floats in water displaces 300 kg (about 3000 N) of water.

Note If an object floats, the volume of fluid displaced is equal to the volume of the object submerged and the percentage of the volume of the object submerged is equal to the relative density of an object with respect to the density of the fluid in which it floats.

For example, if an ice cube of density 0.9 g cm^{-3} floats in the fresh water of density 1.0 g cm^{-3} then the percentage volume of an object submerged in fresh water is. $\frac{0.9\,g\,cm^{-3}}{1.0\,g\,cm^{-3}} \times 100\% = 90\%$.

Conversely, if the same ice cube floats in sea water of density 1.3 g cm^{-3}, then the percentage volume of the object submerged in seawater would be $\frac{0.9\,g\,cm^{-3}}{1.3\,g\,cm^{-3}} \times 100\% = 69.23\%$ only.

EXAMPLE 7.8

A cube of wood floating in water supports a 300 g mass at the centre of its top face. When the mass is removed, the cube rises by 3 cm. Determine the volume of the cube.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution

Let each side of the cube be l. The volume occupied by 3 cm depth of cube,

$$V=(3cm)\times l^2 = 3l^2 cm$$

According to the principle of floatation, we have

$$V\rho g = mg \Rightarrow V\rho = m$$

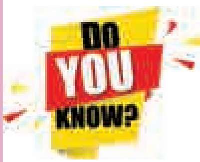
ρ is density of water = 1000 $kg\ m^{-3}$

$$\Rightarrow (3l^2\times 10^{-2}m)\times(1000\ kgm^{-3}) = 300\times 10^{-3}kg$$

$$l^2 = \frac{300\times10^{-3}}{3\times10^{-2}\times1000}m^2 \Rightarrow l^2 = 100\times10^{-4}m^2$$

$$l = 10\times 10^{-2}m = 10\ cm$$

Therefore, volume of cube $V = l^3 = 1000\ cm^3$



Submarines can sink or rise in the depth of water by controlling its buoyancy. To achieve this, the submarines have ballast tanks that can be filled with water or air, alternatively, when the ballast tanks are filled with air, the overall density of the submarine becomes lesser than the surrounding water, and it surfaces (positive buoyancy). If the tanks are flooded with water replacing air, the overall density becomes greater than the surrounding water and submarine begins to sink (negative buoyancy). To keep the submarine at any depth, tanks are filled with air and water (neutral buoyancy).

Examples of floating bodies:

i) A person can swim in sea water more easily than in river water.

ii) Ice floats on water.

iii) The ship is made of steel but its interior is made hollow by giving it a concave shape.

7.4 VISCOSITY

7.4.1 Introduction

In section 7.3, the behavior of fluids at rest is discussed. Successive discussions will bring out the influence of fluid motion on different properties. A fluid in motion is a complex phenomenon, as it possesses potential, kinetic, and gravitational energy besides causing friction viscous forces to come into play. Therefore, it is necessary to consider the case of an ideal liquid to simplify the task. An ideal liquid is incompressible (i.e., bulk modulus is infinity) and in which no shearing forces can be maintained (i.e., the coefficient of viscosity is zero).

Most of the fluids offer resistance towards motion. A frictional force acts at the contact surface when a fluid moves relative to a solid or when two fluids move relative to each other. This resistance to fluid motion is similar to the friction produced when a solid moves on a surface. The internal friction existing between the layers of a moving fluid is viscosity. So, viscosity is defined as 'the property of a fluid to oppose the relative motion between its layers'.

ACTIVITY

Consider three steel balls of the same size, dropped simultaneously in three tall jars each filled with air, water, and oil respectively. It moves easily in air, but not so easily in water. Moving in oil would be even more difficult. There is a relative motion produced between the different layers of the liquid by the falling ball, which causes a viscous force. This frictional force depends

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

on the density of the liquid. This property of a moving fluid to oppose the relative motion between its layers is called viscosity.

Cause of Viscosity:

Consider a liquid flowing through a horizontal surface with two neighboring layers. The upper layer tends to accelerate the lower layer and in turn, the lower layer tends to retard the upper layer. As a result, a backward tangential force is set-up. This tends to destroy the relative motion. This accounts for the viscous behavior of fluids.

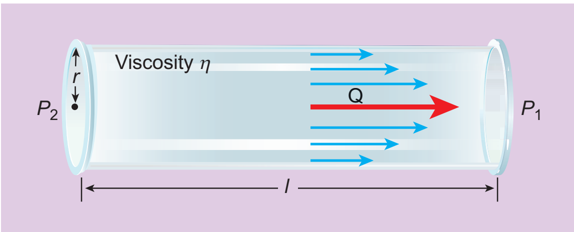


Figure 7.14 Viscosity

Coefficient of Viscosity:

Consider a liquid flowing steadily over a horizontal fixed layer (Figure 7.15). The velocities of the layers increase uniformly as we move away from the fixed layer. Consider any two parallel layers A and B. Let v and $v + dv$ be the velocities of the neighboring layers at distances x and $x + dx$ respectively from the fixed layer.

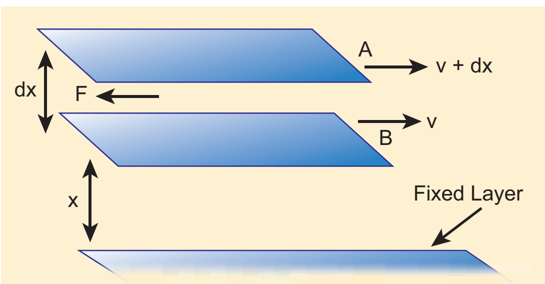


Figure 7.15 Flow of liquid over the horizontal layers

The force of viscosity F acting tangentially between two layers is given by Newton's First law. This force is proportional to (i) area A of the liquid and (ii) the velocity gradient $\frac{dv}{dx}$

$$F \propto A \, and \, F \propto \frac{dv}{dx}$$

$$\Rightarrow F = -\eta A \frac{dv}{dx} \quad (7.20)$$

Where the constant of proportionality η is called the coefficient of viscosity of the liquid and the negative sign implies that the force is frictional and it opposes the relative motion. The dimensional formula for coefficient of viscosity is $\left[ML^{-1}T^{-1}\right]$

Note Viscosity is similar to friction. The kinetic energy of the substance is dissipated as heat energy.

EXAMPLE 7.9

A metal plate of area $2.5\times10^{-4}m^2$ is placed on a $0.25\times10^{-3}m$ thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity $3\times10^{-2}m\,s^{-1}$, calculate the coefficient of viscosity of castor oil.

Given: $A=2.5\times10^{-4}\ m^2$, $dx = 0.25\times10^{-3}m$, $F=2.5N$ and $dv = 3\times10^{-2}\ m\,s^{-1}$

Solution

$$F = -\eta A \frac{dv}{dx}$$

In magnitude, $\eta = \frac{F}{A}\frac{dx}{dv}$

$$= \frac{(2.5N)}{(2.5\times10^{-4}m^2)}\frac{(0.25\times10^{-3}m)}{(3\times10^{-2}ms^{-1})}$$

$= 0.083\times10^3\ Nm^{-2}s$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

7.4.2 Streamlined flow

The flow of fluids occurs in different ways. It can be a steady or streamlined flow, unsteady or turbulent flow, compressible or incompressible flow or even viscous or non-viscous flow. For example, consider a calm flow of water through a river. Careful observation reveals that the velocity of water at different locations of the river is quite different. It is almost faster at the center and slowest near the banks. However, the velocity of the particle at any point is constant. For better understanding, assume that the velocity of the particle is about 4 meter per second at the center of the river. Hence it will be of the same value for all other particles crossing through this point. In a similar way, if the velocity of the particle flowing near the bank of the river is 0.5 meter per second, then the succeeding particles flowing through it will have the same value.

When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a *streamlined flow*. It is also referred to as steady or laminar flow. The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point as shown in Figure 7.16. It is named so because the flow looks like the flow of a stream or river under ideal conditions.

If we assume a bundle of streamlines having the same velocity over any cross section perpendicular to the direction of flow then such bundle is called a '*tube of flow*'. Thus, it is important to note that any particle in a tube of flow always remains in the tube throughout its motion and cannot mix with liquid in another tube. Always the axis of the tube of flow gives the streamline. The streamlines always represent the trajectories of the fluid particles. The flow of fluid is streamlined up to a certain velocity called critical velocity. This means a steady flow can be achieved at low flow speeds below the critical speed.

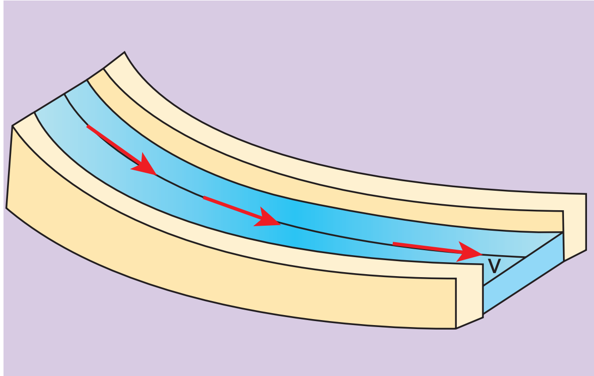


Figure 7.16 Flow is steady velocity at any point in the liquid remains constant

7.4.3 Turbulent flow

When the speed of the moving fluid exceeds the critical speed, v_c the motion becomes turbulent. In this case, the velocity changes both in magnitude and direction from particle to particle and hence the individual particles do not move in a streamlined path. Hence, the path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies (Figure 7.17 (a) and (b)). The flow of water just behind a boat or a ship and the air flow behind a moving bus are a few examples of turbulent flow.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

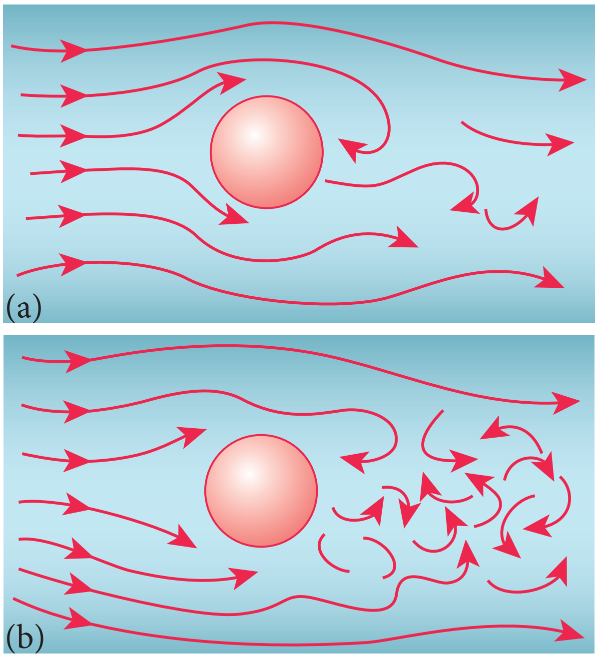


Figure 7.17 (a) turbulent flow around a sphere (when $v = v_c$) (b) turbulent flow around a sphere (when $v > v_c$)

The distinction between the two types of motion can be easily demonstrated by injecting a jet of ink axially in a wide tube through which water flows. When the velocity of the fluid is small the ink will move in a straight line path. Conversely, when the velocity is increased beyond a certain value, the ink will spread out showing the disorderliness and hence the motion becomes turbulent. The zig-zag motion results in the formation of eddy currents and as a consequence, much energy is dissipated.

7.4.4 Reynold's number

We have learnt that the flow of a fluid becomes steady or laminar when the velocity of flow is less than the critical velocity v_c otherwise, the flow becomes turbulent. Osborne Reynolds (1842-1912) formulated an equation to find out the nature of the flow of fluid, whether it is streamlined or turbulent.

$$R_c = \frac{\rho v D}{\eta} \qquad (7.21)$$

It is a dimensionless number called '*Reynold's number*'. It is denoted by the symbol R_c or K. In the equation, ρ denotes the density of the fluid, v the velocity of the flowing fluid, D is the diameter of the pipe in which the fluid flow, and η is the coefficient of viscosity of the fluid. The value of R_c remains the same in any system of units.

Table 7.3 To understand the flow of liquid, Reynold has estimated the value of R_c as follows

S. No.	Reynold's number	Flow
1	Rc < 1000	Streamline
2	1000 < Rc < 2000	Unsteady
3	Rc > 2000	Turbulent

Hence, Reynold's number R_c is a critical variable, which decides whether the flow of a fluid through a cylindrical pipe is streamlined or turbulent. In fact, the critical value of R_c at which the turbulent sets is found to be the same for geometrically similar flows. For example, when two liquids (say oil and water) of different densities and viscosities flow in pipes of same shapes and sizes, the turbulence sets in at almost the same value of R_c. The above fact leads to the *Law of similarity* which states that when there are two geometrically similar flows, both are essentially equal to each other, as long as they embrace the same Reynold's number. The *Law of similarity* plays a very important role in technological applications.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The shape of ships, submarines, racing cars, and airplanes are designed in such a way that their speed can be maximized.

7.4.5 Terminal velocity

To understand terminal velocity, consider a small metallic sphere falling freely from rest through a large column of a viscous fluid.

The forces acting on the sphere are (i) gravitational force of the sphere acting vertically downwards, (ii) upthrust U due to buoyancy and (iii) viscous drag acting upwards (viscous force always acts in a direction opposite to the motion of the sphere).

Initially, the sphere is accelerated in the downward direction so that the upward force is less than the downward force. As the velocity of the sphere increases, the velocity of the viscous force also increases. A stage is reached when the net downward force balances the upward force and hence the resultant force on the sphere becomes zero. It now moves down with a constant velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity V_T. In the Figure 7.18, a graph is drawn with velocity along y- axis and time along x- axis. It is evident from the graph that the sphere is accelerated initially and in course of time it becomes constant, and attains terminal velocity (V_T).

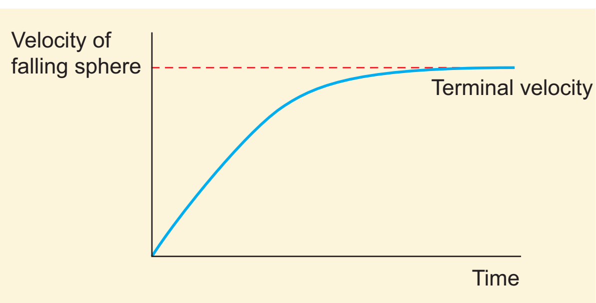


Figure 7.18 Velocity verses time graph

Expression for terminal velocity:

Consider a sphere of radius r which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

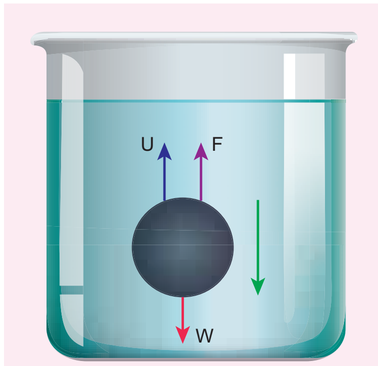


Figure 7.19 Forces acting on the sphere when it falls in a viscous liquid

Gravitational force acting on the sphere,

$F_G = mg = \frac{4}{3}\pi r^3 \rho g$ (downward force)

Up thrust, $U = \frac{4}{3}\pi r^3 \sigma g$ (upward force)

viscous force F = $6\pi\eta r v_t$

At terminal velocity v_t.

downward force = upward force

$$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v_t$$

$$v_t = \frac{2}{9} \times \frac{r^2(\rho - \sigma)}{\eta} g \Rightarrow v_t \propto r^2 \qquad (7.22)$$

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius. If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity. That is why air bubbles rise up through water

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

or any fluid. This is also the reason for the clouds in the sky to move in the upward direction.

Point to ponder

i) The terminal speed of a sphere is directly proportional to the square of the radius of the sphere. Hence, larger raindrops fall with greater speed as compared to the smaller raindrops.
ii) If the density of the material of the sphere is less than the density of the medium, then the sphere shall attain terminal velocity in the upward direction. That is why gas bubbles rise up in soda water.

7.4.6 Stoke's law and its applications

When a body falls through a viscous medium, it drags the layer of the fluid immediately in contact with it. This produces a relative motion between the different layers of the liquid. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force F acting on a spherical body of radius r depends directly on

i) radius (r) of the sphere
ii) velocity (v) of the sphere and
iii) coefficient of viscosity η of the liquid

Therefore $F \propto \eta^x r^y v^z \Rightarrow F = k\eta^x r^y v^z$, where k is a dimensionless constant.

Using dimensions, the above equation can be written as

$$\left[MLT^{-2}\right] = k\left[ML^{-1}T^{-1}\right]^x \times \left[L\right]^y \times \left[LT^{-1}\right]^z$$

On solving, we get x=1, y=1, and z=1

Therefore, F=kη rv

Experimentally, Stoke found that the value of k = 6π

$$F = 6\pi\eta\, rv \qquad (7.23)$$

This relation is known as Stoke's law

Practical applications of Stoke's law

Since the raindrops are smaller in size and their terminal velocities are small, remain suspended in air in the form of clouds. As they grow up in size, their terminal velocities increase and they start falling in the form of rain.

This law explains the following:

a) Floatation of clouds
b) Larger raindrops hurt us more than the smaller ones
c) A man coming down with the help of a parachute acquires constant terminal velocity.

7.4.7 Poiseuille's equation

Poiseuille analyzed the steady flow of liquid through a capillary tube. He derived an expression for the volume of the liquid flowing per second through the capillary tube.

As per the theory, the following conditions must be retained while deriving the equation.

- The flow of liquid through the tube is streamlined.
- The tube is horizontal so that gravity does not influence the flow
- The layer in contact with the wall of the tube is at rest
- The pressure is uniform over any cross section of the tube

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

We can derive Poiseuille's equation using dimensional analysis. Consider a liquid flowing steadily through a horizontal capillary tube. Let $v = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient $\left(\frac{P}{l}\right)$. Then,

$$v \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$v = k\eta^a r^b \left(\frac{P}{l}\right)^c \qquad (7.24)$$

where, k is a dimensionless constant. Therefore,

$$[v] = \frac{volume}{time} = [L^3T^{-1}], \left[\frac{dP}{dx}\right] = \frac{Pressure}{distance} =$$

$$[ML^{-2}T^{-2}], [\eta] = [ML^{-1}T^{-1}] \, and \, [r] = [L]$$

Substituting in equation (7.24)

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

$$M^0L^3T^{-1} = M^{a+b}L^{-a+b-2c}T^{-a-2c}$$

So, equating the powers of M, L, and T on both sides, we get

$$a + c = 0, \; -a + b - 2c = 3, \text{ and } -a - 2c = -1$$

We have three unknowns a, b, and c. We have three equations, on solving, we get

$$a = -1, \; b = 4, \text{ and } c = 1$$

Therefore, equation (7.24) becomes,

$$v = k\eta^{-1}r^4\left(\frac{P}{l}\right)^1$$

Experimentally, the value of k is shown to be $\frac{\pi}{8}$, we have

$$v = \frac{\pi r^4 P}{8\eta l} \qquad (7.25)$$

The above equation is known as *Poiseuille's equation* for the flow of liquid through a narrow tube or a capillary tube. This relation holds good for the fluids whose velocities are lesser than the critical velocity (v_c).

7.4.8 Applications of viscosity

The importance of viscosity can be understood from the following examples.

(1) The oil used as a lubricant for heavy machinery parts should have a high viscous coefficient. To select a suitable lubricant, we should know its viscosity and how it varies with temperature [Note: As temperature increases, the viscosity of the liquid decreases]. Also, it helps to choose oils with low viscosity used in car engines (light machinery).

(2) The highly viscous liquid is used to damp the motion of some instruments and is used as brake oil in hydraulic brakes.

(3) Blood circulation through arteries and veins depends upon the viscosity of fluids.

(4) Millikan conducted the oil drop experiment to determine the charge of an electron. He used the knowledge of viscosity to determine the charge.

7.5 SURFACE TENSION

7.5.1 Intermolecular forces

Different liquids do not mix together due to their physical properties such as density, surface tension force, etc. For example, water and kerosene do not mix together. Mercury does not wet the glass but water sticks to

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

it. Water rises up to the leaves through the stem. They are mostly related to the free surfaces of liquids. Liquids have no definite shape but have a definite volume. Hence they acquire a free surface when poured into a container. Therefore, the surfaces have some additional energy, called as surface energy. The phenomenon behind the above fact is called surface tension. Laplace and Gauss developed the theory of surface and motion of a liquid under various situations.

The molecules of a liquid are not rigidly fixed like in a solid. They are free to move about. The force between the like molecules which holds the liquid together is called '*cohesive force*'. When the liquid is in contact with a solid, the molecules of the these solid and liquid will experience an attractive force which is called '*adhesive force*'.

These molecular forces are effective only when the distance between the molecules is very small about 10^{-9} m (i.e., 10 Å). The distance through which the influence of these molecular forces can be felt in all directions constitute a range and is called *sphere of influence*. The forces outside this range are rather negligible.

Consider three different molecules A, B, and C in a given liquid as shown in Figure 7.20. Let a molecule 'A' be considered well inside the liquid within the sphere of influence. Since this molecule interacts with all other molecules in all directions, the net force experienced by A is zero. Now consider a molecule 'B' in which three-fourth lies below the liquid surface and one–fourth on the air. Since B has more molecules towards its lower side than the upper side, it experiences a net force in the downward direction. In a similar way, if another molecule 'C' is chosen on the liquid surface (i.e, upper half in air and lower half in liquid), it experiences a maximum downward force due to the availability of more number of liquid molecules on the lower part. Hence it is obvious that all molecules of the liquid that falls within the molecular range inside the liquid interact with the molecule and hence experience a downward force.

BHHHXK

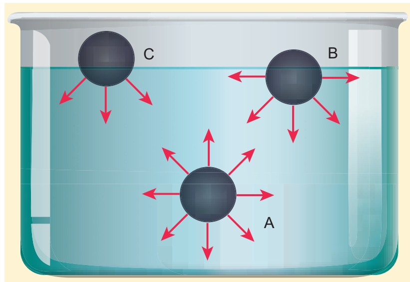


Figure 7.20 Molecules at different levels of liquid

When any molecule is brought towards the surface from the interior of the liquid, work is done against the cohesive force among the molecules of the surface. This work is stored as potential energy in molecules. So the molecules on the surface will have greater potential energy than that of molecules in the interior of the liquid. But for a system to be under stable equilibrium, its potential energy (or surface energy) must be a minimum. Therefore, in order to maintain stable equilibrium, a liquid always tends to have a minimum number of molecules. In other words, the liquid tends to occupy a minimum surface area. This behaviour of the liquid gives rise to surface tension.

Examples for surface tension.

Water bugs and water striders walk on the surface of water (Figure 7.21). The water molecules are pulled inwards and the surface of water acts like a springy or stretched membrane. This balance the weight of water bugs and enables them to walk on the

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

surface of water. We call this phenomenon as surface tension.

Figure 7.21 Water stiders can walk on water because of the surface tension of water

The hairs of the painting brush cling together when taken out of water. This is because the water films formed on them tends to contract to a minimum area (Figure 7.22).

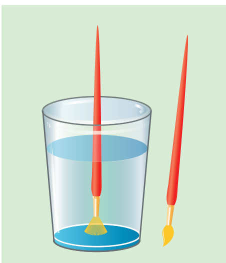


Figure 7.22 Painting brush hairs cling together due to surface tension

Activity 1:Needle floats on water surface

Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating.

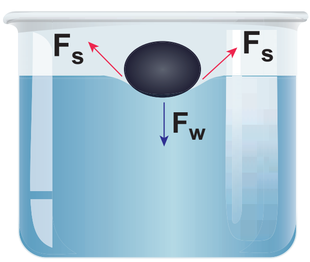


Figure 7.23 Floating needle

The floating needle causes a little depression; the forces F, due to the surface tension of the curved surface are inclined as shown in Figure 7.23. The vertical components of these two forces support the weight of the needle. Now add liquid soap to the water and stir it. We find that the needle sinks.

Activity 2:

Take a plastic sheet and cut out a piece in the shape of a boat (Figure 7.24). A tapering and smooth front with a notch at the back is suggested. Put a piece of camphor into the notch of the boat. Gently release the boat on the surface of the water and we find that the boat is propelled forward when the camphor dissolves. The surface tension is lowered, as the camphor dissolves and produces a difference in surface tension in the water nearby the notch. This causes the water to flow away from the back of the boat, which moves the boat forward.

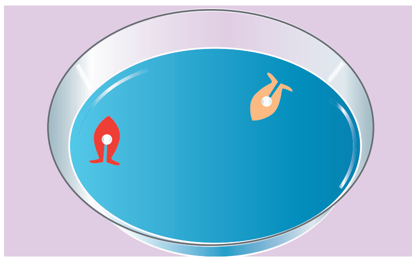


Figure 7.24 Camphor boat

7.5.2 Factors affecting the surface tension of a liquid

Surface tension for a given liquid varies in following situations

(1) ***The presence of any contamination or impurities*** considerably affects the force of surface tension depending upon the degree of contamination.

(2) ***The presence of dissolved substances*** can also affect the value of surface

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

tension. For example, a highly soluble substance like sodium chloride ($NaCl$) when dissolved in water (H_20) increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water.

(3) ***Electrification*** affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases.

(4) ***Temperature*** plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature. For a small range of temperature, the surface tension at T_t at t °C is $T_t = T_0\ (1 - \alpha\ t)$

Where, T_0 is the surface tension at temperature 0°C and α is the temperature coefficient of surface tension. It is to be noted that at the critical temperature, the surface tension is zero as the interface between liquid and vapour disappear. For example, the critical temperature of water is 374°C. Therefore, the surface tension of water is zero at that temperature. van der Wall suggested the important relation between the surface tension and the critical temperature as

$$T_t = T_0\left(1 - \frac{t}{t_c}\right)^{\frac{3}{2}}$$

Generalizing the above relation, we get

$$T_t = T_0\left(1 - \frac{t}{t_c}\right)^{n}$$

which gives more accurate value. Here n, varies for different liquids and t and t_c denotes the temperature and critical temperature in absolute scale (Kelvin scale), respectively.

7.5.3 Surface energy (S.E.) and surface tension (S.T.)

Surface Energy

Consider a sample of liquid in a container. A molecule inside the liquid is being pulled in all direction by other molecules that surround it. However, near the surface, a molecule is pulled down only by the molecules below them and there is a net downward force. As a result, the entire surface of the liquid is being pulled inward. The liquid surface thus tends to have the least surface area. To increase the surface area, some molecules are brought from the interior to the surface. For this reason, work has to be done against the forces of attraction. The amount of work done is stored as potential energy. Thus, the molecules lying on the surface possess greater potential energy than other molecules. This excess energy per unit area of the free surface of the liquid is called '*surface energy*'. In other words, the work done in increasing the surface area per unit area of the liquid against the surface tension force is called the surface energy of the liquid.

$$Surface\ energy = \frac{work\ done\ in\ increasing\ the\ surface\ area}{increase\ in\ surface\ area}$$

$$= \frac{W}{\Delta A} \qquad (7.26)$$

It is expressed in $J\,m^{-2}$ or $N\,m^{-1}$.

Surface tension

The surface tension of a liquid is defined as the energy per unit area of the surface of a liquid

$$T = \frac{F}{l} \qquad (7.27)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The SI unit and dimensions of T are $N m^{-1}$ and MT^{-2}, respectively.

Relation between surface tension and surface energy:

Consider a rectangular frame of wire ABCD in a soap solution (Figure 7.25). Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inward due to surface tension. Let F be the force due to surface tension, then

$$F = (2T)l$$

here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance Δx to new a position $A'B'$. Since the area increases, some work has to be done against the inward force due to surface tension.

Work done = Force × distance = $(2T\, l)\,(\Delta x)$

Increase in area of the film $\Delta A = (2l)(\Delta x) = 2l\,\Delta x$

Therefore,

$$surface\ energy = \frac{work\ done}{increase\ in\ surface\ area}$$

$$= \frac{2Tl\Delta x}{2l\Delta x} = T \qquad (7.28)$$

Hence, the surface energy per unit area of a surface is numerically equal to the surface tension.

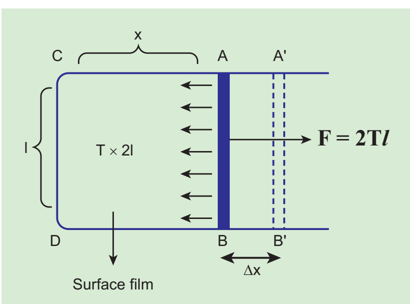


Figure 7.25 A horizontal soap film on a rectangular frame of wire ABCD

Note It should be remembered that a liquid drop has only one free surface. Therefore, the surface area of a spherical drop of radius r is equal to $4\pi r^2$, whereas, a bubble has two free surfaces and hence the surface area of a spherical bubble is equal to $2 \times 4\pi r^2$.

EXAMPLE 7.10

Let $2.4\times10^{-4}\,J$ of work is done to increase the area of a film of soap bubble from $50\ cm^2$ to $100\ cm^2$. Calculate the value of surface tension of soap solution.

Solution:

A soap bubble has two free surfaces, therefore increase in surface area $\Delta A = A_2 - A_1 = 2(100\text{-}50)\times10^{-4} m^2 = 100\times10^{-4} m^2$.

Since, work done $W = T\times\Delta A \Rightarrow T =$

$$\frac{W}{\Delta A} = \frac{2.4\times10^{-4}\,J}{100\times10^{-4}\,m^2} = 2.4\times10^{-2}\,N m^{-1}$$

7.5.4 Angle of contact

When the free surface of a liquid comes in contact with a solid, then the surface of the liquid becomes curved at the point of contact. Whenever the liquid surface becomes a curve, then the angle between the two medium (solid-liquid interface) comes in the picture. For an example, when a glass plate is dipped in water with sides vertical as shown in figure, we can observe that the water is drawn up to the plate. In the same manner, instead of water the glass plate is dipped in mercury, the surface is curved but now the curve is depressed as shown in Figure 7.29

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is known as the *angle of contact between the solid and the liquid*. It is denoted by θ (Read it as "theta" which is Greek alphabet small letter).

Its value is different at interfaces of different pairs of solids and liquids. In fact, it is the factor which decides whether a liquid will spread on the surface of a chosen solid or it will form droplets on it.

Let us consider three interfaces such as liquid-air, solid-air and solid-liquid with reference to the point of contact 'O' and the interfacial surface tension forces T_{sa}, T_{sl} and T_{la} on the respective interfaces as shown in Figure 7.26.

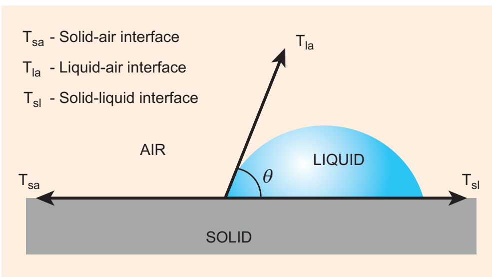


Figure 7.26 Angle of contact of a liquid

Since the liquid is stable under equilibrium, the surface tension forces between the three interfaces must also be in equilibrium. Therefore,

$$T_{sa} = T_{la}\cos\theta + T_{sl} \Rightarrow \cos\theta = \frac{T_{sa} - T_{sl}}{T_{la}} \quad (7.29)$$

From the above equation, there are three different possibilities which can be discussed as follows.

(i) If $T_{sa} > T_{sl}$ and $T_{sa} - T_{sl} > 0$ (water-plastic interface) then the angle of contact θ is acute angle (θ less than 90°) as cosθ is positive.

(ii) If $T_{sa} < T_{sl}$ and $T_{sa} - T_{sl} < 0$ (water-leaf interface) then the angle of contact is obtuse angle (θ less than 180°) and as cosθ is negative.

(iii) If $T_{sa} > T_{la} + T_{sl}$ then there will be no equilibrium and liquid will spread over the solid.

Therefore, the concept of *angle of contact* between the solid-liquid interface leads to some practical applications in real life. For example, soaps and detergents are wetting agents. When they are added to an aqueous solution, they will try to minimize the angle of contact and in turn penetrate well in the cloths and remove the dirt. On the other hand, water proofing paints are coated on the outer side of the building so that it will enhance the angle of contact between the water and the painted surface during the rainfall.

7.5.5 Excess of pressure inside a liquid drop, a soap bubble, and an air bubble

As it is discussed earlier, the free surface of a liquid becomes curved when it has contact with a solid. Depending upon the nature of liquid-air or liquid-gas interface, the magnitude of interfacial surface tension varies. In other words, as a consequence of surface tension, the above such interfaces have energy and for a given volume, the surface will have a minimum energy with least area. Due to this reason, the liquid drop becomes spherical (for a smaller radius).

When the free surface of the liquid is curved, there is a difference in pressure between the inner and outer the side of the surface (Figure 7.27).

i) When the liquid surface is plane, the forces due to surface tension (*T, T*) act tangentially to the liquid surface in

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

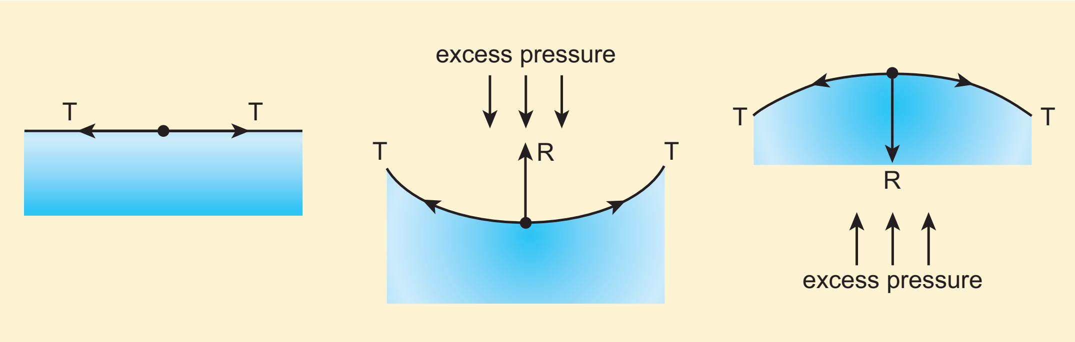


Figure7.27 Excess of pressure across a liquid surface

opposite directions. Hence, the resultant force on the molecule is zero. Therefore, in the case of a plane liquid surface, the pressure on the liquid side is equal to the pressure on the vapour side.

ii) When the liquid surface is curved, every molecule on the liquid surface experiences forces (F_T, F_T) due to surface tension along the tangent to the surface. Resolving these forces into rectangular components, we find that horizontal components cancel out each other while vertical components get added up. Therefore, the resultant force normal to the surface acts on the curved surface of the liquid. Similarly, for a convex surface, the resultant force is directed inwards towards the centre of curvature, whereas the resultant force is directed outwards from the centre of curvature for a concave surface. Thus, for a curved liquid surface in equilibrium, the pressure on its concave side is greater than the pressure on its convex side.

Excess of pressure inside a bubble and a liquid drop:

The small bubbles and liquid drops are spherical because of the forces of surface tension. The fact that a bubble or a liquid drop does not collapse due to the combined effect indicates that the pressure inside a bubble or a drop is greater than that outside it.

1) Excess of pressure inside air bubble in a liquid.

Consider an air bubble of radius R inside a liquid having surface tension T as shown in Figure 7.28 (a). Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is $\Delta P = P_1 - P_2$.

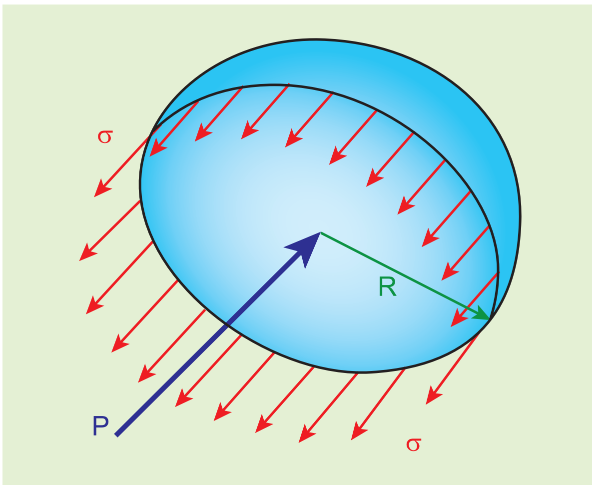


Figure 7.28. (a) Air bubble

In order to find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble. For the hemispherical portion of the bubble, considering the forces acting on it, we get,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

i) The force due to surface tension acting towards right around the rim of length $2\pi R$ is $F_T = 2\pi RT$

ii) The force due to outside pressure P_1 is to the right acting across a cross sectional area of πR^2 is $F_{P_1} = P_1 \pi R^2$

iii) The force due to pressure P_2 inside the bubble, acting to the left is $F_{P_2} = P_2 \pi R^2$.

As the air bubble is in equilibrium under the action of these forces, $F_{P_2} = F_T + F_{P_1}$

$$P_2 \pi R^2 = 2\pi RT + P_1 \pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{2T}{R}$ (7.30)

2) Excess pressure inside a soap bubble

Consider a soap bubble of radius R and the surface tension of the soap bubble be T as shown in Figure 7.28 (b). A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is $2 \times 2\pi RT$. The various forces acting on the soap bubble are,

i) Force due to surface tension $F_T = 4\pi RT$ towards right

ii) Force due to outside pressure, $F_{P_1} = P_1 \pi R^2$ towards right

iii) Force due to inside pressure, $F_{P_2} = P_2 \pi R^2$ towards left

As the bubble is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2 \pi R^2 = 4\pi RT + P_1 \pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{4T}{R}$ (7.31)

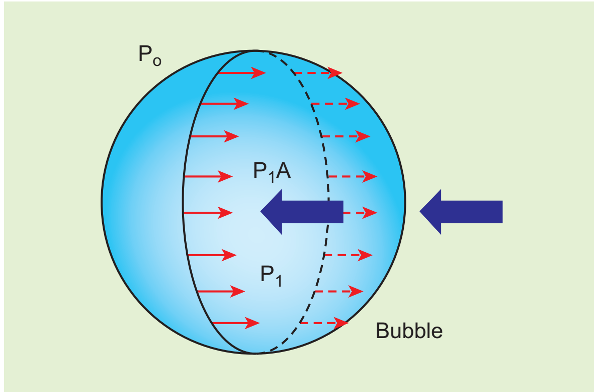


Figure7.28 (b) Soap bubble

3) Excess pressure inside the liquid drop

Consider a liquid drop of radius R and the surface tension of the liquid is T as shown in Figure 7.28 (c).

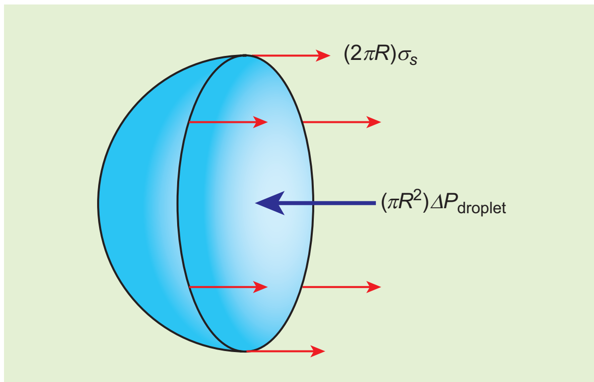


Figure7.28 (c) Liquid drop

The various forces acting on the liquid drop are,

i) Force due to surface tension $F_T = 2\pi RT$ towards right

ii) Force due to outside pressure, $F_{P_1} = P_1 \pi R^2$ towards right

iii) Force due to inside pressure, $F_{P_2} = P_2 \pi R^2$ towards left

As the drop is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2 \pi R^2 = 2\pi RT + P_1 \pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{2T}{R}$ (7.32)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside, the tiny fog droplets are rigid enough to behave like solids.

When an ice-skater skate over the surface of the ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates, the tiny droplets of water act as rigid ball- bearings and help the skaters to run along smoothly.

EXAMPLE 7.11

If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 *mm* high, where R = 2.0 *cm*, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

But $\Delta P = P_2 - P_1 = \rho g h \Rightarrow \rho g h = \frac{4T}{R}$

$\Rightarrow$ Surface tension,

$$T = \frac{\rho g h R}{4} = \frac{(800)(9.8)(4\times10^{-3})(2\times10^{-2})}{4} =$$

$$T = 15.68 \times 10^{-2}\, N\, m^{-1}$$

7.5.6 Capillarity

The word 'capilla' means hair in Latin. If the tubes were hair thin, then the rise would be very large. It means that the tube having a very small diameter is called a 'capillary tube'. When a glass capillary tube open at both ends is dipped vertically in water, the water in the tube will rise above the level of water in the vessel. In case of mercury, the liquid is depressed in the tube below the level of mercury in the vessel (shown in Figure 7.29). In a liquid whose angle of contact with solid is less than 90°, suffers capillary rise. On the other hand, in a liquid whose angle of contact is greater than 90°, suffers capillary fall (Table 7.4). The rise or fall of a liquid in a narrow tube is called capillarity or capillary action. Depending on the diameter of the capillary tube, liquid rises or falls to different heights.

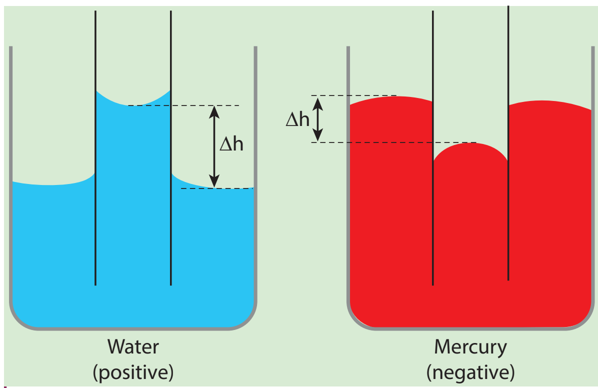


Figure 7.29 Capillary rise or fall

Table 7.4 Capillary rise and fall

Contact angle	Strength of		Degree of wetting	Miniscus	Rise or fall of liquid in the capillary tube
	Cohesive force	Adhesive force			
θ=0 (A)	Weak	Strong	Perfect Wetting	Plane	Neither rises nor is depressed
θ<90 (B)	Weak	Strong	High	Concave	Rise of liquid
θ>90 (C)	Strong	Weak	Low	Convex	Fall of liquid

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

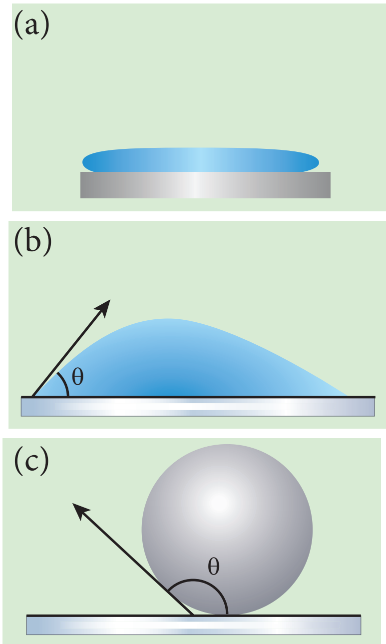


Figure 7.30 (a) water on silver surface (b) glass plate on water (c) glass on mercury

Practical applications of capillarity

- Due to capillary action, oil rises in the cotton within an earthen lamp. Likewise, sap rises from the roots of a plant to its leaves and branches.
- Absorption of ink by a blotting paper
- Capillary action is also essential for the tear fluid from the eye to drain constantly.
- Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for sweat.

7.5.7 Surface Tension by capillary rise method

The pressure difference across a curved liquid-air interface is the basic factor behind the rising up of water in a narrow tube (influence of gravity is ignored). The capillary rise is more dominant in the case of very fine tubes. But this phenomenon is the outcome of the force of surface tension. In order to arrive a relation between the capillary rise (h) and surface tension (T), consider a capillary tube which is held vertically in a beaker containing water; the water rises in the capillary tube to a height h due to surface tension (Figure 7.31).

The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into two components i) Horizontal component $T\sin\theta$ and ii) Vertical component $T\cos\theta$ acting upwards, all along the whole circumference of the meniscus.

Total upward force

$$= (T\cos\theta)(2\pi r) = 2\pi r T\cos\theta$$

where θ is the angle of contact, r is the radius of the tube. Let ρ be the density of water and h be the height to which the liquid rises inside the tube. Then,

$$\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube,V}\end{pmatrix} = \begin{pmatrix}\text{volume of the}\\ \text{liquid column of radius r}\\ \text{height h}\end{pmatrix} + \begin{pmatrix}\text{volume of liquid of radius r}\\ \text{and height r - Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$$

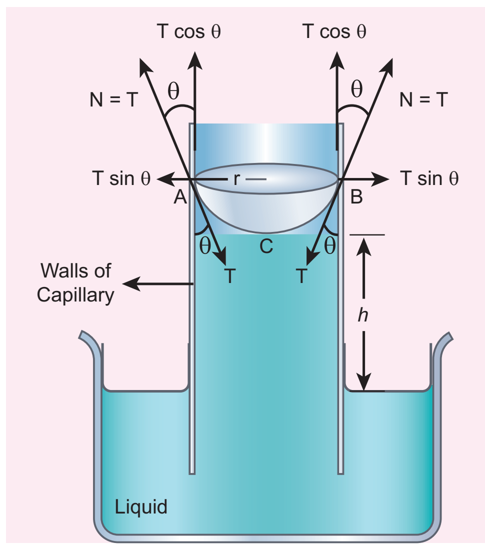


Figure 7.31 Capillary rise by surface tension

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$V=\pi r^2 h+\left(\pi r^2\times r-\frac{2}{3}\pi r^3\right)\Rightarrow V=\pi r^2 h+\frac{1}{3}\pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore,

$$2\pi r T\cos\theta=\pi r^2\left(h+\frac{1}{3}r\right)\rho g\Rightarrow T=\frac{r\left(h+\frac{1}{3}r\right)\rho g}{2\cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then $\frac{r}{3}$ can be neglected when it is compared to the height h. Therefore,

$$T=\frac{r\rho gh}{2\cos\theta} \qquad (7.33)$$

Liquid rises through a height h

$$h=\frac{2T\cos\theta}{r\rho g}\Rightarrow h\propto\frac{1}{r} \qquad (7.34)$$

This implies that the capillary rise (h) is inversely proportional to the radius (r) of the tube. i.e, the smaller the radius of the tube greater will be the capillarity.

EXAMPLE 7.12

Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?

Solution

From equation (7.34), we have

$$h\propto\frac{1}{r}\Rightarrow hr=\text{constant}$$

Consider two capillary tubes with radius r_1 and r_2 which on placing in a liquid, capillary rises to height h_1 and h_2, respectively. Then,

$$h_1 r_1=h_2 r_2=\text{constant}$$

$$\Rightarrow\ h_2=\frac{h_1 r_1}{r_2}=\frac{\left(2\times10^{-2}m\right)\times r}{\frac{r}{3}}\Rightarrow h_2=6\times10^{-2}m$$

EXAMPLE 7.13

Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 2 *mm*, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside?. Surface tension of mercury T=0.456 $N\,m^{-1}$; Density of mercury ρ = 13.6 × 10^3 $kg\,m^{-3}$

Solution

Capillary descent,

$$h=\frac{2T\cos\theta}{r\rho g}=\frac{2\times(0.465\,N\,m^{-1})(\cos 140^0)}{(2\times10^{-3}m)(13.6\times10^3)(9.8\,m\,s^{-2})}$$

$$\Rightarrow h=-6.89\times10^{-4}\,m$$

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

7.5.8 Applications of surface tension

- Mosquitoes lay their eggs on the surface of water. To reduce the surface tension of water, a small amount of oil is poured. This breaks the elastic film of water surface and eggs are killed by drowning.
- Chemical engineers must finely adjust the surface tension of the liquid, so it forms droplets of designed size and so it adheres to the surface without smearing. This is used in desktop printing, to paint automobiles and decorative items.
- Specks of dirt get removed when detergents are added to hot water while washing clothes because surface tension is reduced.
- A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

7.6 BERNOULLI'S THEOREM

7.6.1 Equation of continuity

In order to discuss the mass flow rate through a pipe, it is necessary to assume that the flow of fluid is steady, the flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant with respect to time. Under this condition, the path taken by the fluid particle is a streamline.

Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively as shown in Figure 7.32.

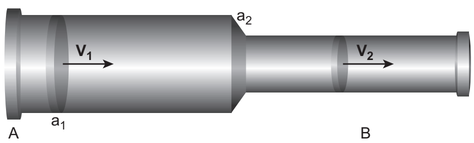


Fig 7.32 A streamlined flow of fluid through a pipe of varying cross sectional area

Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \Delta t)\,\rho$

Let m_2 be the mass of fluid flowing through section B in time Δt, $m_2 = (a_2 v_2 \Delta t)\,\rho$

For an incompressible liquid, mass is conserved $m_1 = m_2$

$$a_1 v_1\, \Delta t\, \rho = a_2 v_2 \Delta t\, \rho$$

$$a_1 v_1 = a_2 v_2 \Rightarrow a\,v = \text{constant} \qquad (7.35)$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, $a\ v$ = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

EXAMPLE 7.14

In a normal adult, the average speed of the blood through the aorta (radius r = 0.8 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

Solution:

$$a_1 v_1 = 30\, a_2\, v_2 \Rightarrow \pi r_1^2 v_1 = 30\, \pi r_2^2 v_2$$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1 \Rightarrow v_2 = \frac{1}{30} \times \left(\frac{0.8\times10^{-2}\,\text{m}}{0.4\times10^{-2}\,\text{m}}\right)^2$$

$$\times (0.33\,\text{ms}^{-1})$$

$$v_2 = 0.044\ m\,s^{-1}$$

7.6.2 Pressure, kinetic and potential energy of liquids

A liquid in a steady flow can possess three kinds of energy. They are (1) Kinetic energy, (2) Potential energy, and (3) Pressure energy, respectively.

i) **Kinetic energy:** The kinetic energy of a liquid of mass m moving with a velocity v is given by

$$KE = \frac{1}{2} m v^2$$

The kinetic energy per unit mass =

$$\frac{KE}{m} = \frac{\frac{1}{2} m v^2}{m} = \frac{1}{2} v^2$$

BN6A1V

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Similarly, the kinetic energy per unit volume

$$= \frac{KE}{\text{volume}} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$$

ii) **Potential energy:** The potential energy of a liquid of mass *m* at a height *h* above the ground level is given by

$$PE = mgh$$

The potential energy per unit mass

$$= \frac{PE}{m} = \frac{mgh}{m} = gh$$

Similarly, the potential energy per unit volume $= \frac{PE}{\text{volume}} = \frac{mgh}{V} = \left(\frac{m}{V}\right)gh = \rho gh$

iii) **Pressure energy:** The energy acquired by a fluid by applying pressure on the fluid. We know that

$$\text{Pressure} = \frac{Force}{Area} \Rightarrow \text{Force} = \text{Pressure} \times \text{Area}$$

$$F \times d = (P\,A) \times d = P\,(A \times d)$$

$$\Rightarrow F \times d = \text{W} = \text{P V} = \text{pressure energy}$$

Therefore, pressure energy, $E_P = PV$

The pressure energy per unit mass =

$$\frac{E_P}{m} = \frac{\text{PV}}{m} = \frac{\text{P}}{\frac{\text{m}}{\text{V}}} = \frac{\text{P}}{\rho}$$

Similarly, the potential energy per unit volume $= \frac{E_P}{\text{volume}} = \frac{PV}{V} = P$

7.6.3 Bernoulli's theorem and its applications

In 1738, the Swiss scientist Daniel Bernoulli developed a relationship for the flow of fluid through a pipe of varying cross section. He proposed a theorem for the streamline flow of a liquid based on the law of conservation of energy.

Bernoulli's theorem

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant. Mathematically,

$$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant} \qquad (7.36)$$

This is known as Bernoulli's equation.

Proof:

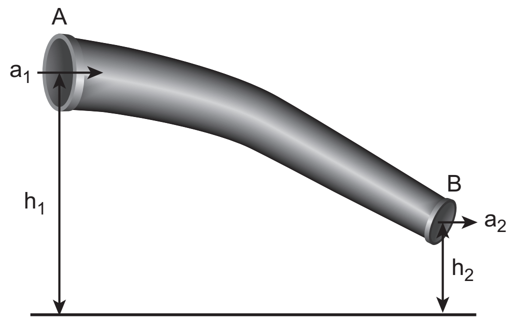


Figure 7.33 Flow of liquid through a pipe AB

Let us consider a flow of liquid through a pipe AB as shown in Figure 7.33. Let *V* be the volume of the liquid when it enters *A* in a time *t* which is equal to the volume of the liquid leaving *B* in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at *A* respectively.

Let the force exerted by the liquid at *A* is

$$F_A = P_A a_A$$

Distance travelled by the liquid in time *t* is

$$d = v_A\, t$$

Therefore, the work done is

$$W = F_A d = P_A a_A v_A\, t$$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at *A*.

Thus, the work done is the pressure energy (at *A*), $W = F_A d = P_A V$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Pressure energy per unit volume at

$$A = \frac{\text{Pressure energy}}{\text{volume}} = \frac{P_A V}{V} = P_A$$

Pressure energy per unit mass at

$$A = \frac{\text{Pressure energy}}{\text{mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is

$$E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$$

Potential energy of the liquid at A,

$$PE_A = mg\,h_A,$$

Due to the flow of liquid, the kinetic energy of the liquid at A,

$$KE_A = \frac{1}{2} m\, v_A^2$$

Therefore, the total energy due to the flow of liquid at A, $E_A = EP_A + KE_A + PE_A$

$$E_A = m\frac{P_A}{\rho} + \frac{1}{2} mv_A^2 + mg\,h_A$$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at *B*. Calculating the total energy at EB, we get

$$E_B = m\frac{P_B}{\rho} + \frac{1}{2} mv_B^2 + mg\,h_B$$

From the law of conservation of energy,

$$EA = EB$$

$$m\frac{P_A}{\rho} + \frac{1}{2} mv_A^2 + mgh_A = m\frac{P_B}{\rho} + \frac{1}{2} mv_B^2 + mgh_B$$

$$\frac{P_A}{\rho} + \frac{1}{2} v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2} v_B^2 + gh_B = \text{constant}$$

Thus, the above equation can be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

The above equation is the consequence of the conservation of energy which is true until there is no loss of energy due to friction. But in practice, some energy is lost due to friction. This arises due to the fact that in a fluid flow, the layers flowing with different velocities exert frictional forces on each other. This loss of energy is generally converted into heat energy. Therefore, Bernoulli's relation is strictly valid for fluids with zero viscosity or non-viscous liquids. Notice that when the liquid flows through a horizontal pipe, then $h = 0 \Rightarrow \frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} =$ constant.

Applications of Bernoulli's Theorem

(a) Blowing off roofs during wind storm

In olden days, the roofs of the huts or houses were designed with a slope as shown in Figure.7.34. One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.

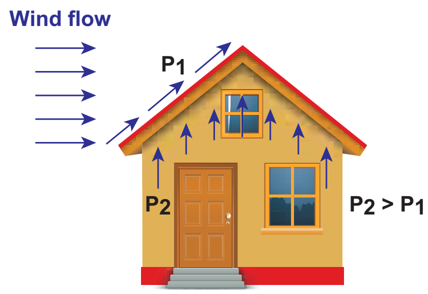


Figure 7.34 Roofs of the huts or houses

During cyclonic condition, the roof is blown off without damaging the other parts of the house. In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1. The pressure under the roof P_2 is greater. Therefore, this pressure difference $(P_2 - P_1)$ creates an up thrust and the roof is blown off.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(b) Aerofoil lift

The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the real edge. As the aircraft moves, the air moves faster above the aerofoil than at the bottom as shown in Figure 7.35.

According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an upthrust called the dynamic lift to the aircraft.

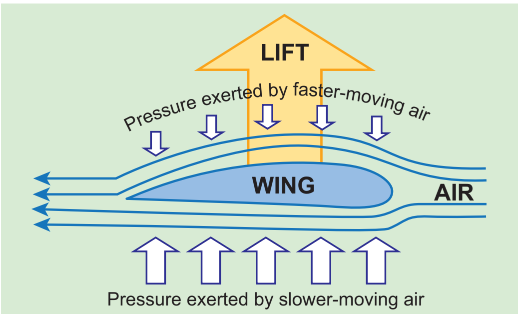


Figure 7.35 Aerofoil lift

(c) Bunsen burner

In this, the gas comes out of the nozzle with high velocity, hence the pressure in the stem decreases. So outside air reaches into the burner through an air vent and the mixture of air and gas gives a blue flame as shown in Figure 7.36.

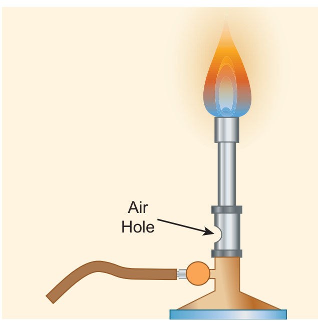


Figure 7.36 Bunsen burner

(d) Venturimeter

This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem. It consists of two wider tubes A and A' (with cross sectional area A) connected by a narrow tube B (with cross sectional area *a*). A manometer in the form of U-tube is also attached between the wide and narrow tubes as shown in Figure7.37. The manometer contains a liquid of density 'ρ_m'.

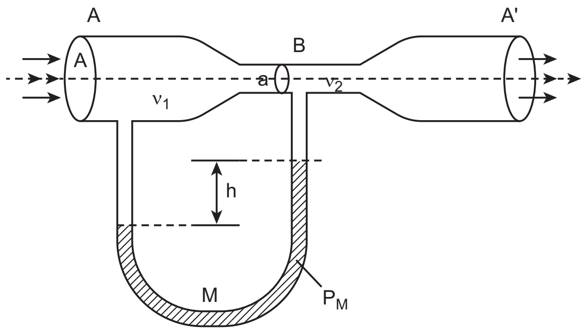


Figure 7.37 A schematic diagram of venturimeter

Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that $Av_1 = a\, v_2$ which means that

$$v_2 = \frac{A}{a}\, v_1.$$

Using Bernoulli's equation,

$$P_1 + \rho\frac{v_1^2}{2} = P_2 + \rho\frac{v_2^2}{2} = P_2 + \rho\frac{1}{2}\left(\frac{A}{a}v_1\right)^2$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

From the above equation, the pressure difference

$$\Delta P = P_1 - P_2 = \rho \frac{v_1^2}{2} \frac{(A^2 - a^2)}{a^2}$$

Thus, the speed of flow of fluid at the wide end of the tube A

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)} \Rightarrow v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$$

The volume of the liquid flowing out per second is

$$V = Av_1 = A\sqrt{\frac{2(\Delta P)a^2}{p(A^2 - a^2)}} = aA\sqrt{\frac{2(\Delta P)}{p(A^2 - a^2)}}$$

(e) Other applications

This Bernoulli's concept is mainly used in the design of carburetor of automobiles, filter pumps, atomizers, and sprayers. For example, the carburetor has a very fine channel called nozzle through which the air is allowed to flow in larger speed. In this case, the pressure is lowered at the narrow neck and in turn, the required fuel or petrol is sucked into the chamber so as to provide the correct mixture of air and fuel necessary for ignition process.

ACTIVITY

A bottle is filled with thermocol balls. One end of a flexible tube is kept inside the bottle immersed inside the balls. The free end is rotated and we find the balls sprayed all around. This explains the working of an atomizer or sprayer.

DO YOU KNOW?

A spider web is much stronger than what we think. A single strand of spider silk can stop flying insects which are tens and thousands times its mass. The young's modulus of the spider web is approximately $4.5 \times 10^9 \, N m^{-2}$. Compare this value with Young's modulus of wood.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SUMMARY

- The force between the atoms of an element is called inter-atomic force whereas the force between the molecules of a compound is called inter-molecular force.
- *Hooke's law:* within the elastic limit, the stress is directly proportional to strain.
- The force per unit area is known as *stress*. If F is the force applied and A is the area of cross section of the body then the magnitude of *stress* is equal to F/A. Tensile or compressional stress can be expressed using a single term called *longitudinal stress*.
- The ratio of change in length to the original length of a cylinder is ΔL/L, which is known as *longitudinal strain*
- Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called the Young' s modulus of the material of the wire.
- Within the elastic limit, the ratio of volume stress to the volume strain is called the *bulk modulus*.
- Within the elastic limit, the ratio of shear stress to the shear strain is called the *rigidity modulus*.
- Poisson's ratio = lateral strain/longitudinal strain
- The elastic potential energy stored in the wire per unit volume is $U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$, where Y denotes Young's modulus of the material.
- If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.
- The total pressure at a depth h below the liquid surface is $P = P_a + \rho gh$, where P_a is the atmospheric pressure which is equal to 1.013×10^5 Pa.
- Pascal's law states that the pressure in a fluid at rest is the same at all points if they are at the same height.
- The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.
- The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.
- When a liquid flows such that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor then the flow of liquid is said to be streamlined flow.
- During the flow of fluid, when the critical velocity is exceeded by the moving fluid, the motion becomes *turbulent*.
- Reynold's number has a significance as it decides which decides whether the flow of fluid through a cylindrical pipe is streamlined or turbulent.

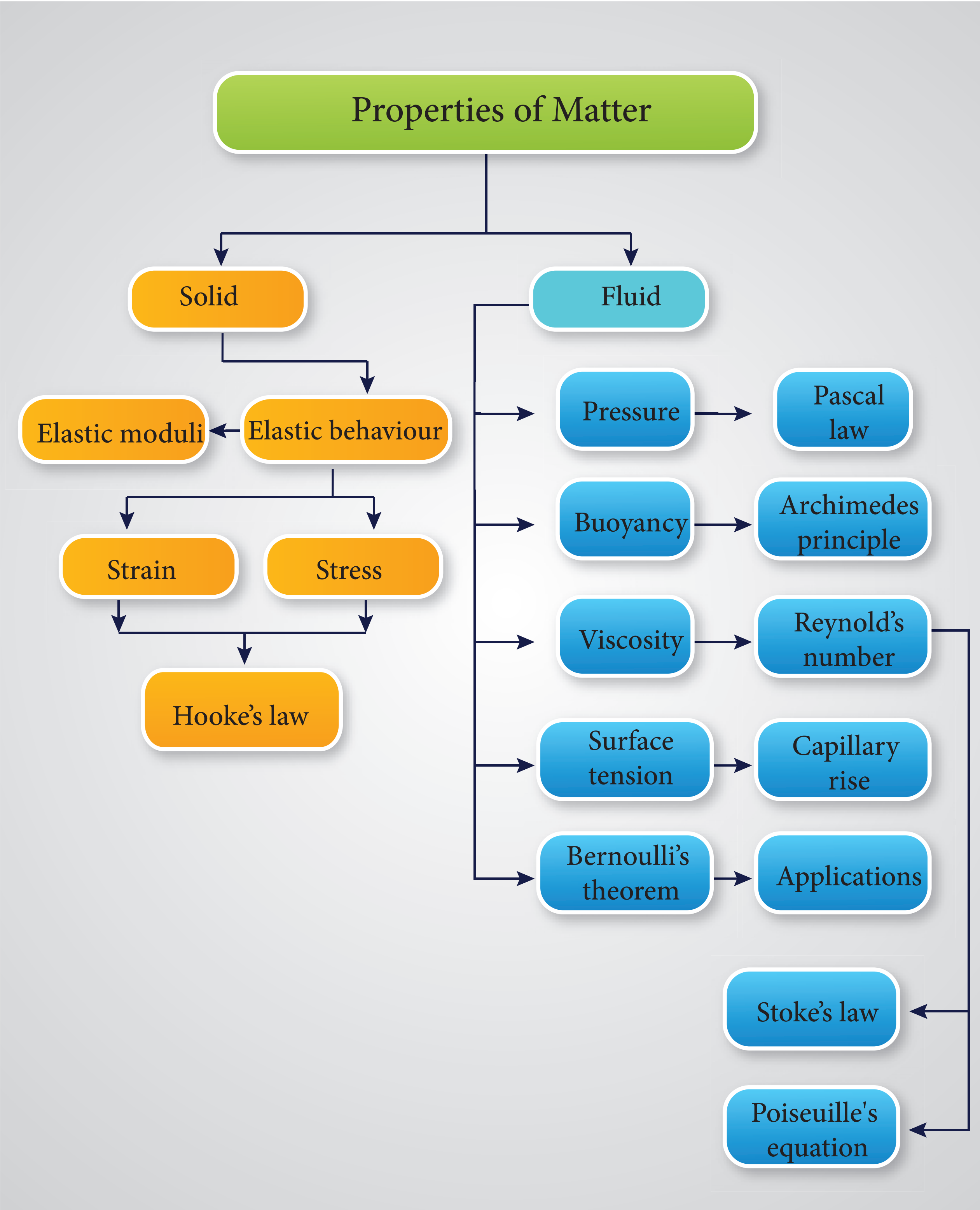
athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SUMMARY (cont.)

- Stokes formula F= 6πηav, where F is the viscous force acting on a sphere of radius a and v is the terminal velocity of the sphere.
- The surface tension of a liquid is defined as the force of tension acting on a unit length of an imaginary line drawn on the free surface of the liquid, the direction of the force being perpendicular to the line so drawn and acting parallel to the surface.
- The angle between tangents drawn at the point of contact to the liquid surface and solid surface inside the liquid is called the *angle of contact* for a pair of solid and liquid.
- The flow of a fluid is said to be steady if, at any given point, the velocity of each passing fluid particle remains constant with respect to time.
- The equation $a_1 v_1 = a_2 v_2$ is called the equation of continuity for a flow of fluid through a tube and it is due to the conservation of mass in the flow of fluids. It states that the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains constant. i.e., $P/\rho + v^2/2 + gh =$ constant.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CONCEPT MAP



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

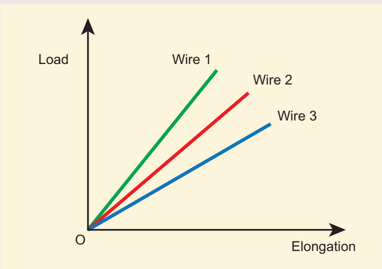
I. Multiple Choice Questions

1. Consider two wires X and Y. The radius of wire X is 3 times the radius of Y. If they are stretched by the same load then the stress on Y is
   (a) equal to that on X
   (b) thrice that on X
   (c) nine times that on X
   (d) Half that on X

BNF63I

2. If a wire is stretched to double of its original length, then the strain in the wire is
   (a) 1 (b) 2
   (c) 3 (d) 4

3. The load – elongation graph of three wires of the same material are shown in figure. Which of the following wire is the thickest?



   (a) wire 1
   (b) wire 2
   (c) wire 3
   (d) all of them have same thickness

4. For a given material, the rigidity modulus is $\left(\frac{1}{3}\right)^{rd}$ of Young's modulus. Its Poisson's ratio is
   (a) 0 (b) 0.25
   (c) 0.3 (d) 0.5

5. A small sphere of radius 2cm falls from rest in a viscous liquid. Heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity is proportional to

   (NEET model 2018)

   (a) 2^2 (b) 2^3
   (c) 2^4 (d) 2^5

6. Two wires are made of the same material and have the same volume. The area of cross sections of the first and the second wires are A and 2A respectively. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount?

   (NEET model 2018)

   (a) 2 (b) 4
   (c) 8 (d) 16

7. With an increase in temperature, the viscosity of liquid and gas, respectively will
   (a) increase and increase
   (b) increase and decrease
   (c) decrease and increase
   (d) decrease and decrease

8. The Young's modulus for a perfect rigid body is
   (a) 0 (b) 1
   (c) 0.5 (d) infinity

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

9. Which of the following is not a scalar?

(a) viscosity

(b) surface tension

(c) pressure

(d) stress

10. If the temperature of the wire is increased, then the Young's modulus will

(a) remain the same

(b) decrease

(c) increase rapidly

(d) increase by very a small amount

11. Copper of fixed volume V is drawn into a wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. If Y represents the Young's modulus, then which of the following graphs is a straight line?

(NEET 2014 model)

(a) Δl verses V

(b) Δl verses Y

(c) Δl verses F

(d) Δl verses $\frac{1}{l}$

12. A certain number of spherical drops of a liquid of radius R coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then

(a) energy $= 4\,V\,T\left(\frac{1}{r}-\frac{1}{R}\right)$ is released

(b) energy $= 3\,V\,T\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed

(c) energy $= 3\,V\,T\left(\frac{1}{r}-\frac{1}{R}\right)$ is released

(d) energy is neither released nor absorbed

13. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?

(a) length = 200 cm, diameter = 0.5 mm

(b) length= 200 cm, diameter = 1 mm

(c) length = 200 cm, diameter = 2 mm

(d) length= 200 cm, diameter = 3 m

14. The wettability of a surface by a liquid depends primarily on

(a) viscosity

(b) surface tension

(c) density

(d) angle of contact between the surface and the liquid

15. In a horizontal pipe of non-uniform cross section, water flows with a velocity of 1 m s^{-1} at a point where the diameter of the pipe is 20 cm. The velocity of water (m s^{-1}) at a point where the diameter of the pipe is

(a) 8 (b) 16

(c) 24 (d) 32

Answers:

1) c	2) a	3) a	4) d
5) d	6) b	7) c	8) d
9) d	10) b	11) c	12) c
13) a	14) d	15) b	

II. Short Answer Questions

1. Define stress and strain.
2. State Hooke's law of elasticity.
3. Define Poisson's ratio.
4. Explain elasticity using intermolecular forces.
5. Which one of these is more elastic, steel or rubber? Why?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

6. A spring balance shows wrong readings after using for a long time. Why?
7. What is the effect of temperature on elasticity?
8. Write down the expression for the elastic potential energy of a stretched wire.
9. State Pascal's law in fluids.
10. State Archimedes principle.
11. What do you mean by upthrust or buoyancy?
12. State the law of floatation.
13. Define coefficient of viscosity of a liquid.
14. Distinguish between streamlined flow and turbulent flow.
15. What is Reynold's number? Give its significance.
16. Define terminal velocity.
17. Write down the expression for the Stoke's force and explain the symbols involved in it.
18. State Bernoulli's theorem.
19. What are the energies possessed by a liquid? Write down their equations.
20. Two streamlines cannot cross each other. Why?
21. Define surface tension of a liquid. Mention its S.I unit and dimension.
22. How is surface tension related to surface energy?
23. Define angle of contact for a given pair of solid and liquid.
24. Distinguish between cohesive and adhesive forces.
25. What are the factors affecting the surface tension of a liquid?
26. What happens to the pressure inside a soap bubble when air is blown into it?
27. What do you mean by capillarity or capillary action?
28. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?
29. State the principle and usage of Venturimeter.

III. Long Answer Questions

1. State Hooke's law and verify it with the help of an experiment.
2. Explain the different types of modulus of elasticity.
3. Derive an expression for the elastic energy stored per unit volume of a wire.
4. Derive an equation for the total pressure at a depth 'h' below the liquid surface.
5. State and prove Pascal's law in fluids.
6. State and prove Archimedes principle.
7. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.
8. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.
9. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.
10. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.
11. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

12. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

13. Describe the construction and working of venturimeter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

IV. Numerical Problems

1. A capillary of diameter d*mm* is dipped in water such that the water rises to a height of 30*mm*. If the radius of the capillary is made $\left(\frac{2}{3}\right)$ of its previous value, then compute the height up to which water will rise in the new capillary?

   (Answer: 45 mm)

2. A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of 4×10^5 N is applied at the other end. If the rigidity modulus of the cylinder is 6×10^{10} N m^{-2} then, calculate the twist produced in the cylinder. (Answer: 45.60)

3. A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

4. A block of Ag of mass *x* kg hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block. (Answer: $x = 4$ kg)

5. The reading of pressure meter attached with a closed pipe is 5×10^5 N m^{-2}. On opening the valve of the pipe, the reading of the pressure meter is 4.5×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

   (Answer: 10 ms^{-1})

V. Conceptual questions

1. Why coffee runs up into a sugar lump (a small cube of sugar) when one corner of the sugar lump is held in the liquid?
2. Why two holes are made to empty an oil tin?
3. We can cut vegetables easily with a sharp knife as compared to a blunt knife. Why?
4. Why the passengers are advised to remove the ink from their pens while going up in an aeroplane?
5. We use straw to suck soft drinks, why?

BOOKS FOR REFERENCE

1. Serway and Jewett, Physics for scientist and Engineers with modern physics, Brook/Coole publishers, Eighth edition
2. Paul Tipler and Gene Mosca, Physics for scientist and engineers with modern physics, Sixth edition, W.H.Freeman and Company
3. H.C.Verma, Concepts of physics volume 1 and Volume 2, Bharati Bhawan Publishers

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ICT CORNER

Properties of Matter

Through this activity you will be able to learn about the Viscosity.

STEPS:

- Use the URL or scan the QR code to open 'Viscosity' activity page.
- In the activity window , 'Select Viscosity' by dragging the ball in the meter in the side.
- Select the 'Start' button , sphere falls from top to bottom of the beaker. We can see the changes in Distance and Time.
- Spheres can be reset to the top of each beaker by clicking the 'Reset' button.

Step1

Step2

Step3

Step4

URL:

http://www.geo.cornell.edu/hawaii/220/PRI/viscosity.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

UNIT 8

HEAT AND THERMODYNAMICS

Classical thermodynamics.... is the only physical theory of universal content which I am convinced... will never be overthrown. – Albert Einstein

LEARNING OBJECTIVES

In this unit, a student is exposed to

- meaning of heat, work and temperature
- ideal gas laws
- concept of specific heat capacity
- thermal expansion of solids, liquids and gases
- various states of matter
- Newton's law of cooling
- Stefan's law and Wien's law
- meaning of thermodynamic equilibrium
- meaning of internal energy
- zeroth and first laws of thermodynamics
- various thermodynamic processes
- work done in various thermodynamic processes
- second law of thermodynamics
- working of carnot engine and refrigerator

BNP256

8.1 HEAT AND TEMPERATURE

8.1.1 Introduction

Temperature and heat play very important role in everyday life. All species can function properly only if its body is maintained at a particular temperature. In fact life on Earth is possible because the Sun maintains its temperature. Understanding the meaning of temperature and heat are very crucial to understand the nature. Thermodynamics is a branch of physics which explains the phenomena of temperature, heat etc. The concepts presented in this chapter will help us to understand the terms 'hot' and 'cold' and also differentiate heat from temperature. In thermodynamics, heat and temperature are two different but closely related parameters.

8.1.2 Meaning of heat

When an object at higher temperature is placed in contact with another object at lower temperature, there will be a spontaneous flow of energy from the object

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

at higher temperature to the one at lower temperature. This energy is called heat. This process of energy transfer from higher temperature object to lower temperature object is called heating. Due to flow of heat sometimes the temperature of the body will increase or sometimes it may not increase.

Note There is a misconception that heat is a quantity of energy. People often talk 'this water has more heat or less heat'. These words are meaningless. Heat is not a quantity. Heat is an energy in transit which flows from higher temperature object to lower temperature object. Once the heating process is stopped we cannot use the word heat. When we use the word 'heat', it is the energy in transit but not energy stored in the body.

EXAMPLE 8.1

a. 'A lake has more rain'.
b. 'A hot cup of coffee has more heat'.

What is wrong in these two statements?

Solution

a. When it rains, lake receives water from the cloud. Once the rain stops, the lake will have more water than before raining. Here 'raining' is a process which brings water from the cloud. Rain is not a quantity rather it is water in transit. So the statement 'lake has more rain' is wrong, instead the 'lake has more water' will be appropriate.
b. When heated, a cup of coffee receives heat from the stove. Once the coffee is taken from the stove, the cup of coffee has more internal energy than before. 'Heat' is the energy in transit and which flows from an object at higher temperature to an object at lower temperature. Heat is not a quantity. So the statement 'A hot cup of coffee has more heat' is wrong, instead 'coffee is hot' will be appropriate.

8.1.3 Meaning of work

When you rub your hands against each other the temperature of the hands increases. You have done some work on your hands by rubbing. The temperature of the hands increases due to this work. Now if you place your hands on the chin, the temperature of the chin increases. This is because the hands are at higher temperature than the chin. In the above example, the temperature of hands is increased due to work and temperature of the chin is increased due to heat transfer from the hands to the chin. It is shown in the Figure 8.1

By doing work on the system, the temperature in the system will increase and sometimes may not. Like heat, work is also

Figure 8.1 Difference between work and heat

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

not a quantity and through the work energy is transferred to the system . So we cannot use the word 'the object contains more work' or 'less work'.

Either the system can transfer energy to the surrounding by doing work on surrounding or the surrounding may transfer energy to the system by doing work on the system. For the transfer of energy from one body to another body through the process of work, they need not be at different temperatures.

8.1.4 Meaning of temperature

Temperature is the degree of hotness or coolness of a body. Hotter the body higher is its temperature. The temperature will determine the direction of heat flow when two bodies are in thermal contact.

The SI unit of temperature is ***kelvin*** **(K)**.

In our day to day applications, ***Celsius*** (°C) and ***Fahrenheit*** (°F) scales are used.

Temperature is measured with a thermometer.
The conversion of temperature from one scale to other scale is given in Table 8.1

Table 8.1 Temperature conversion

Scale	To Kelvin	From Kelvin
Celsius	K=°C + 273.15	°C = K – 273.15
Fahrenheit	K=(°F + 459.67)÷1.8	°F=(K × 1.8)-459.67
Scale	**To Fahrenheit**	**From Fahrenheit**
Celsius	°F=(1.8 × °C) + 32	°C = (°F – 32)÷1.8
Scale	**To Celsius**	**From Celsius**
Fahrenheit	°C=(°F – 32)÷1.8	°F = (1.8 × °C) + 32

8.2 THERMAL PROPERTIES OF MATTER

8.2.1 Boyle's law, Charles' law and ideal gas law

For a given gas at low pressure (density) kept in a container of volume V, experiments revealed the following information.

- When the gas is kept at constant temperature, the pressure of the gas is inversely proportional to the volume. $P \propto \frac{1}{V}$. It was discovered by Robert Boyle (1627-1691) and is known as Boyle's law.
- When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature. $V \propto T$. It was discovered by Jacques Charles (1743-1823) and is known as Charles' law.
- By combining these two equations we have

$PV = CT$. Here C is a positive constant.

We can infer that C is proportional to the number of particles in the gas container by considering the following argument. If we take two containers of same type of gas with same volume V, same pressure P and same temperature T, then the gas in each container obeys the above equation. $PV = CT$. If the two containers of gas is considered as a single system, then the pressure and temperature of this combined system will be same but volume will be twice and number of particles will also be double as shown in Figure 8.2

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

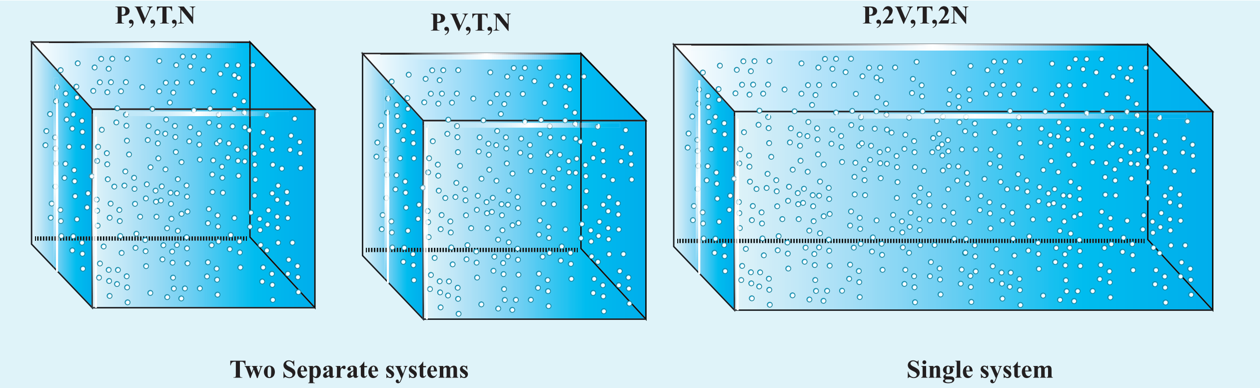


Figure 8.2 Ideal gas law

For this combined system, V becomes 2V, so C should also double to match with the ideal gas equation $\frac{P(2V)}{T} = 2C$. It implies that C must depend on the number of particles in the gas and also should have the dimension of $\left[\frac{PV}{T}\right] = JK^{-1}$. So we can write the constant C as k times the number of particles N.

Here k is the Boltzmann constant (1.381×10^{-23} JK^{-1}) and it is found to be a universal constant.

So the ideal gas law can be stated as follows

$$PV = NkT \quad (8.1)$$

The equation (8.1) can also be expressed in terms of mole.

Mole is the practical unit to express the amount of gas. One mole of any substance is the amount of that substance which contains Avogadro number (N_A) of particles (such as atoms or molecules). The Avogadro's number N_A is defined as the number of carbon atoms contained in exactly 12 g of ^{12}C.

Suppose if a gas contains μ mole of particles then the total number of particles can be written as

$$N = \mu N_A \quad (8.2)$$

where N_A is Avogadro number (6.023 $\times10^{23} mol^{-1}$)

Substituting for N from equation (8.2), the equation (8.1) becomes

$PV = \mu N_A kT$. Here $N_A k = R$ called universal gas constant and its value is 8.314 J /mol. K.

So the ideal gas law can be written for μ mole of gas as

$$PV = \mu RT \quad (8.3)$$

This is called the equation of state for an ideal gas. It relates the pressure, volume and temperature of thermodynamic system at equilibrium.

EXAMPLE 8.2

A student comes to school by a bicycle whose tire is filled with air at a pressure 240 kPa at 27°C. She travels 8 km to reach the school and the temperature of the bicycle tire increases to 39°C. What is the change in pressure in the tire when the student reaches school?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution

We can take air molecules in the tire as an ideal gas. The number of molecules and the volume of tire remain constant. So the air molecules at 27°C satisfies the ideal gas equation $P_1V_1 = NkT_1$ and at 39°C it satisfies $P_2V_2 = NkT_2$

But we know

$$V_1 = V_2 = V$$

$$\frac{P_1V}{P_2V} = \frac{NkT_1}{NkT_2}$$

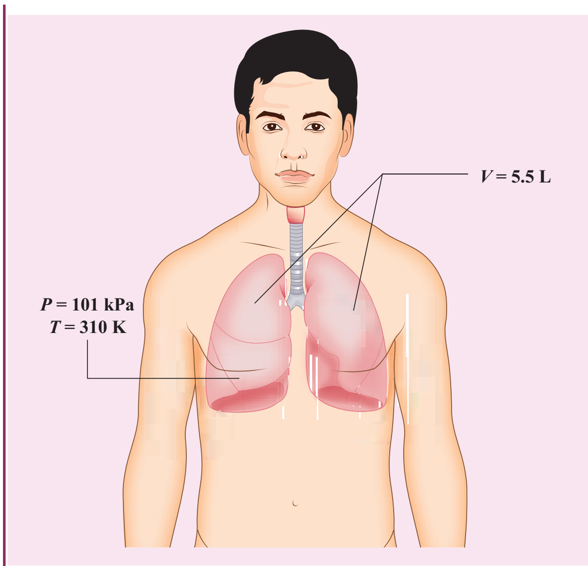
$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$P_2 = \frac{T_2}{T_1}P_1$$

$$P_2 = \frac{312K}{300K}\quad 240\quad 10^3\,Pa = 249.6\ kPa$$

EXAMPLE 8.3

When a person breaths, his lungs can hold up to 5.5 Litre of air at body temperature 37°C and atmospheric pressure (1 atm =101 kPa). This Air contains 21% oxygen. Calculate the number of oxygen molecules in the lungs.



Solution

We can treat the air inside the lungs as an ideal gas. To find the number of molecules, we can use the ideal gas law.

$$PV = NkT$$

Here volume is given in the Litre. 1 Litre is volume occupied by a cube of side 10 cm.
$1Litre = 10cm \times 10cm \times 10cm = 10^{-3}\ m^3$

$$N = \frac{PV}{kT} = \frac{1.01\times10^5\,Pa\times5.5\times10^{-3}\,m^3}{1.38\times10^{-23}\,JK^{-1}\times310K}$$

$= 1.29\times10^{23}$ Molecules

Only 21% of N are oxygen. The total number of oxygen molecules

$$= 1.29 \times 10^{23} \times \frac{21}{100}$$

Number of oxygen molecules
$= 2.7 \times 10^{22}$ molecules

EXAMPLE 8.4

Calculate the volume of one mole of any gas at STP and at room temperature (300K) with the same pressure 1 atm.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution:

Here STP means standard temperature (T=273K or 0°C) and Pressure (P=1 atm or 101.3 kPa)

We can use ideal gas equation $V = \frac{\mu RT}{P}$.

Here $\mu = 1\ mol$ and R =8.314 J/mol.K.

By substituting the values

$$V = \frac{(1mol)(8.314\frac{J}{mol}K)(273K)}{1.013\times10^5\ Nm^{-2}}$$

$=22.4 \times 10^{-3}\ m^3$

We know that 1 Litre (L) = $=10^{-3}m^3$. So we can conclude that 1 mole of any ideal gas has volume 22.4 L.

By multiplying 22.4L by $\frac{300K}{273K}$ we get the volume of one mole of gas at room temperature. It is 24.6 L.

EXAMPLE 8.5

Estimate the mass of air in your class room at NTP. Here NTP implies normal temperature (room temperature) and 1 atmospheric pressure.

Solution

The average size of a class is 6m length, 5 m breadth and 4 m height. The volume of the room V = 6 × 5 × 4 = $120m^3$. We can determine the number of mole. At room temperature 300K, the volume of a gas occupied by any gas is equal to 24.6L.

The number of mole $\mu = \frac{120m^3}{24.6\times10^{-3}m^3}$ $\approx 4878\ mol$.

Air is the mixture of about 20% oxygen, 79% nitrogen and remaining one percent are argon, hydrogen, helium, and xenon. The molar mass of air is 29 $gmol^{-1}$.

So the total mass of air in the room $m = 4878 \times 29 = 141.4kg$.

8.2.2 Heat capacity and specific heat capacity

Take equal amount of water and oil at temperature 27°C and heat both of them till they reach the temperature 50°C. Note down the time taken by the water and oil to reach the temperature 50°C. Obviously these times are not same. We can see that water takes more time to reach 50°C than oil. It implies that water requires more heat energy to raise its temperature than oil. Now take twice the amount of water at 27°C and heat it up to 50°C , note the time taken for this rise in temperature. The time taken by the water is now twice compared to the previous case.

We can define 'heat capacity' as the amount of heat energy required to raise the temperature of the given body from T to T + ΔT .

$$\text{Heat capacity } S = \frac{\Delta Q}{\Delta T}$$

Specific heat capacity of a substance is defined as the amount of heat energy required to raise the temperature of 1kg of a substance by 1 Kelvin or 1°C

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$\Delta Q = m\, s \Delta T$$

Therefore,

$$s = \frac{1}{m}\left(\frac{\Delta Q}{\Delta T}\right)$$

BNXX6T

Where s is known as *specific heat capacity* of a substance and its value depends only on the nature of the substance not amount of substance

ΔQ = Amount of heat energy

ΔT = Change in temperature

m = Mass of the substance

The SI unit for specific heat capacity is $J\ kg^{-1}\ K^{-1}$. Heat capacity and specific heat capacity are always positive quantities.

Table 8.2 Specific heat capacity of some common substances at 1 *atm* (20°C)

Material	Specific heat capacity ($Jkg^{-1}K^{-1}$)
Air	1005
Lead	130
Copper	390
Iron (steel)	450
Glass	840
Aluminium	900
Human body	3470
Water	4186

From the table it is clear that water has the highest value of specific heat capacity. For this reason it is used as a coolant in power stations and reactors.

Note The term heat capacity or specific heat capacity does not mean that object contains a certain amount of heat. Heat is energy transfer from the object at higher temperature to the object at lower temperature. The correct usage is 'internal energy capacity'. But for historical reason the term 'heat capacity' or 'specific heat capacity' are retained.

DO YOU KNOW? When two objects of same mass are *heated* at equal rates, the object with *smaller specific heat capacity* will have a *faster temperature increase.*

When two objects of same mass are *left to cool down*, the temperature of the object with *smaller specific heat capacity* will *drop faster.*

When we study properties of gases, it is more practical to use molar specific heat capacity. Molar specific heat capacity is defined as heat energy required to increase the temperature of one mole of substance by 1K or 1°C. It can be written as follows

$$C = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)$$

Here C is known as *molar specific heat capacity* of a substance and μ is number of moles in the substance.

The SI unit for molar specific heat capacity is $J\ mol^{-1}\ K^{-1}$. It is also a positive quantity.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

8.2.3 Thermal expansion of solids, liquids and gases

Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.

All three states of matter (solid, liquid and gas) expand when heated. When a solid is heated, its atoms vibrate with higher amplitude about their fixed points. The relative change in the size of solids is small. Railway tracks are given small gaps so that in the summer, the tracks expand and do not buckle. Railroad tracks and bridges have expansion joints to allow them to expand and contract freely with temperature changes. It is shown in Figure 8.3

Liquids, have less intermolecular forces than solids and hence they expand more than solids. This is the principle behind the mercury thermometers.

In the case of **gas** molecules, the intermolecular forces are almost negligible and hence they expand much more than solids. For example in hot air balloons when gas particles get heated, they expand and take up more space.

The increase in dimension of a body due to the increase in its temperature is called thermal expansion.

The expansion in length is called **linear expansion**. Similarly the expansion in area is termed as **area expansion** and the

Figure 8.3 Expansion joints for safety

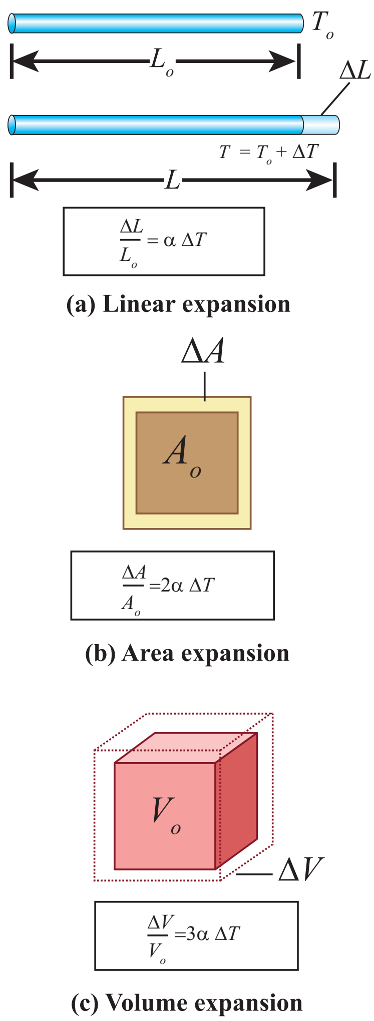


Figure 8.4 Thermal expansions

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

expansion in volume is termed as **volume expansion**. It is shown in Figure 8.4

Linear Expansion

In solids, for a small change in temperature ΔT, the fractional change in length $\left(\frac{\Delta L}{L}\right)$ is directly proportional to ΔT.

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

Therefore, $\alpha_L = \frac{\Delta L}{L\Delta T}$

Where, α_L = coefficient of linear expansion.

ΔL = Change in length

L = Original length

ΔT = Change in temperature.

> **DO YOU KNOW?**
>
> - When the lid of a glass bottle is tight, keep the lid near the hot water which makes it easier to open. It is because the lid has higher thermal expansion than glass.
> - When the hot boiled egg is dropped in cold water, the egg shell can be removed easily. It is because of the different thermal expansions of the shell and egg.

EXAMPLE 8.6

Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is 2°C and in hot summer its average temperature 25°C. Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron $\alpha = 10 \times 10^{-6}$ per °C

Solution

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

$$\Delta L = \alpha_L L\Delta T$$

$\Delta L = 10 \times 10^{-6} \times 300 \times 23 = 0.69$ m=69 cm

Area Expansion

For a small change in temperature ΔT the fractional change in area $\left(\frac{\Delta A}{A}\right)$ of a substance is directly proportional to ΔT and it can be written as

$$\frac{\Delta A}{A} = \alpha_A \Delta T$$

Therefore, $\alpha_A = \frac{\Delta A}{A\Delta T}$

Where, α_A = coefficient of area expansion.

ΔA = Change in area

A = Original area

ΔT = Change in temperature

Volume Expansion

For a small change in temperature ΔT the fractional change in volume $\left(\frac{\Delta V}{V}\right)$ of a substance is directly proportional to ΔT.

$$\frac{\Delta V}{V} = \alpha_V \Delta T$$

Therefore, $\alpha_V = \frac{\Delta V}{V\Delta T}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Where, α_V = coefficient of volume expansion.

ΔV = Change in volume

V = Original volume

ΔT = Change in temperature

Unit of coefficient of linear, area and volumetric expansion of solids is $^{\circ}C^{-1}$ or K^{-1}

Note

For a given specimen,

$\frac{\Delta L}{L} = \alpha_L \Delta T$ (Linear expansion)

$\frac{\Delta A}{A} \approx 2\alpha_L \Delta T$ (Area expansion ≈ 2 × Linear expansion)

$\frac{\Delta V}{V} \approx 3\alpha_L \Delta T$ (Volume expansion ≈ 3 × Linear expansion)

8.2.4 Anomalous expansion of water

Liquids expand on heating and contract on cooling at moderate temperatures. But water exhibits an anomalous behavior. It contracts on heating between 0˚C and 4˚C. The volume of the given amount of water decreases as it is cooled from room temperature, until it reach 4˚C . Below 4˚C the volume increases and so the density decreases. This means that the water has a maximum density at 4˚C . This behavior of water is called anomalous expansion of water. It is shown in the Figure 8.5

In cold countries during the winter season, the surface of the lakes will be at lower temperature than the bottom as shown in the Figure 8.6. Since the solid water (ice) has lower density than its liquid form, below 4°C, the frozen water will be on the top surface above the liquid water (ice floats). This is due to the anomalous expansion of water. As the water in lakes and ponds freeze only at the top the species living in the lakes will be safe at the bottom.

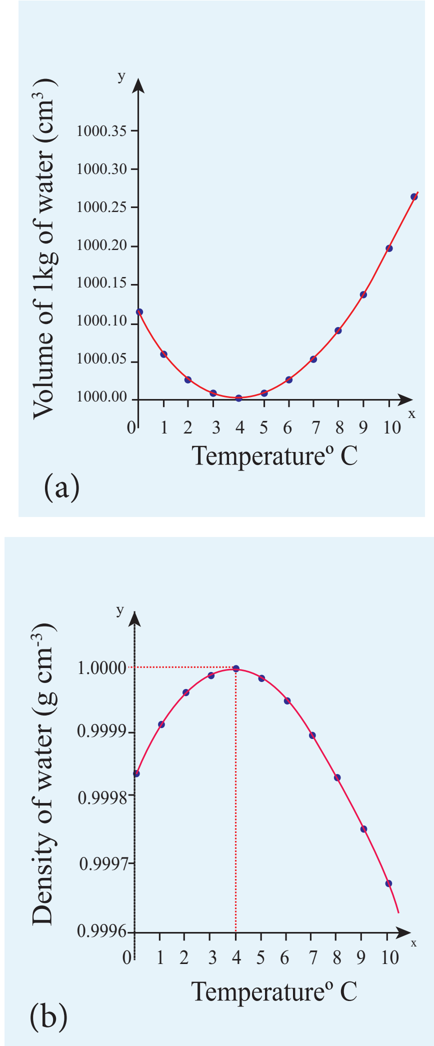


Figure 8.5 Anomalous Expansion of water

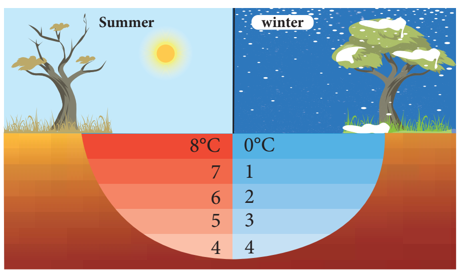


Figure 8.6 Anomalous expansion of water in lakes

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

8.2.5 Change of state

All matter exists normally in three states as solids, liquids or gases. Matter can be changed from one state to another either by heating or cooling.

Examples:

1. Melting (solid to liquid)
2. Evaporation (liquid to gas)
3. Sublimation (solid to gas)
4. Freezing / Solidification (liquid to solid)
5. Condensation (gas to liquid)

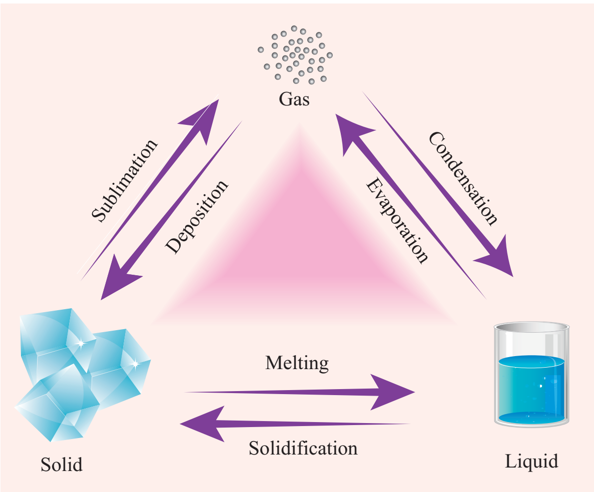


Figure 8.7 Change of states of matter

Latent heat capacity:

While boiling a pot of water, the temperature of the water increases until it reaches 100 °C which is the boiling point of water, and then the temperature remains constant until all the water changes from liquid to gas. During this process heat is continuously added to the water. But the temperature of water does not increase above its boiling point. This is the concept of latent heat capacity.

Latent heat capacity of a substance is defined as the amount of heat energy required to change the state of a unit mass of the material.

$$Q = m \times L$$

Therefore, $L = \frac{Q}{m}$

Where L = Latent heat capacity of the substance

Q = Amount of heat

m = mass of the substance

The SI unit for Latent heat capacity is $J\,kg^{-1}$.

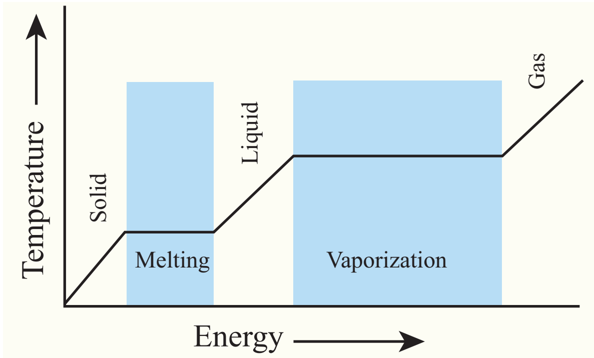


Figure 8.8 Temperature versus heat for water

Note When heat is added or removed during a change of state, the temperature remains constant.

- The latent heat for a solid - liquid state change is called the latent heat of fusion (L_f)
- The latent heat for a liquid - gas state change is called the latent heat of vaporization (L_v)
- The latent heat for a solid - gas state change is called the latent heat of sublimation (L_s)

Triple point

the triple point of a substance is the temperature and pressure at which the three phases (gas, liquid and solid) of that substance coexist in thermodynamic equilibrium.

The triple point of water is at 273.1 K and a partial vapour pressure of 611.657 Pascal.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

8.2.6 Calorimetry

Calorimetry means the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the cold body. No heat is allowed to escape to the surroundings. It can be mathematically expressed as

$$Q_{gain} = -Q_{lost}$$

$$Q_{gain} + Q_{lost} = 0$$

Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water as shown in Figure 8.9.

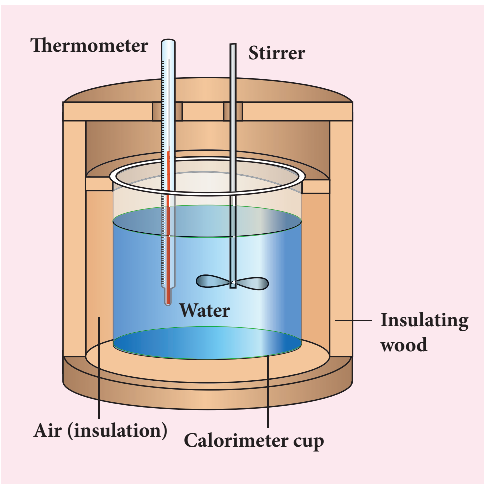


Figure 8.9 Calorimeter

A sample is heated at high temperature (T_1) and immersed into water at room temperature (T_2) in the calorimeter. After some time both sample and water reach a final equilibrium temperature T_f. Since the calorimeter is insulated, heat given by the hot sample is equal to heat gained by the water. It is shown in the Figure 8.10

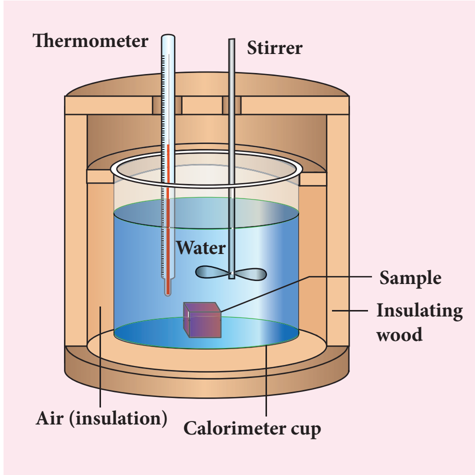


Figure 8.10 Calorimeter with sample of block

$$Q_{gain} = -Q_{lost}$$

Note the sign convention. The heat lost is denoted by negative sign and heat gained is denoted as positive.

From the definition of specific heat capacity

$$Q_{gain} = m_2 s_2 (T_f - T_2)$$

$$Q_{lost} = m_1 s_1 (T_f - T_1)$$

Here s_1 and s_2 specific heat capacity of hot sample and water respectively.

So we can write

$$m_2 s_2 (T_f - T_2) = - m_1 s_1 (T_f - T_1)$$

$$m_2 s_2 T_f - m_2 s_2 T_2 = - m_1 s_1 T_f + m_1 s_1 T_1$$

$$m_2 s_2 T_f + m_1 s_1 T_f = m_2 s_2 T_2 + m_1 s_1 T_1$$

The final temperature

$$T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

EXAMPLE 8.7

If 5 L of water at 50°C is mixed with 4L of water at 30°C, what will be the final temperature of water? Take the specific heat capacity of water as 4184 J kg^{-1} K^{-1}.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution

We can use the equation

$$T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

$m_1 = 5L = 5$kg and $m_2 = 4L = 4$kg, $s_1 = s_2$ and T_1=50°C =323K and T_2 = 30°C=303 K.

So

$$T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = \frac{5\times323+4\times303}{5+4} = 314.11 \text{ K}$$

T_f = 314.11 K-273K ≈ 41°C.

Suppose if we mix equal amount of water ($m_1 = m_2$) with 50°C and 30°C, then the final temperature is average of two temperatures.

$$T_f = \frac{T_1 + T_2}{2} = \frac{323+303}{2} = 313\text{K} = 40°\text{C}$$

Suppose if both the water are at 30°C then the final temperature will also 30°C. It implies that they are at equilibrium and no heat exchange takes place between each other.

Note It is important to note that the final equilibrium temperature of mixing of gas or liquid depends on mass of the substances, their specific heat capacities and their temperatures. Only if we mix the same substances at equal amount, the final temperature will be an average of the individual temperatures.

8.2.7 Heat transfer

As we have seen already heat is a energy in transit which is transferred from one body to another body due to temperature difference. There are three modes of heat transfer: Conduction, Convection and Radiation.

Conduction

Conduction is the process of direct transfer of heat through matter due to temperature difference. When two objects are in direct contact with one another, heat will be transferred from the hotter object to the colder one. The objects which allow heat to travel easily through them are called conductors.

Thermal conductivity

Thermal conductivity is the ability to conduct heat.

The quantity of heat transferred through a unit length of a material in a direction normal to unit surface area due to a unit temperature difference under steady state conditions is known as thermal conductivity of a material.

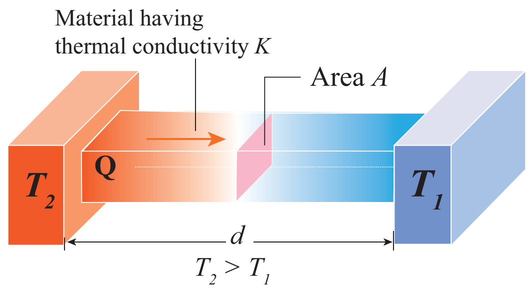


Figure 8.11 Steady state heat flow by conduction.

In steady state, the rate of flow of heat Q is proportional to the temperature difference ΔT and the area of cross section A and is inversely proportional to the length L. So the rate of flow of heat is written as

$$\frac{Q}{t} = \frac{KA\Delta T}{L}$$

Where, K is known as the **coefficient of thermal conductivity.**

(Not to be confused with Kelvin represented by upper case K)

The SI unit of thermal conductivity is J s^{-1} m^{-1} K^{-1} or W m^{-1} K^{-1}.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Table 8.3: Thermal conductivities (in W m^{-1} K^{-1}) of some materials at 1 *atm*

Material	Thermal Conductivity	Material	Thermal Conductivity
Diamond	2300	Water	0.56
Silver	420	Human tissue	0.2
Copper	380	Wood	0.17
Aluminum	200	Helium	0.152
Steel	40	Cork	0.042
Glass	0.84	Air	0.023
Brick	0.84		
Ice	2		

Note Steady state:
The state at which temperature attains constant value everywhere and there is no further transfer of heat anywhere is called steady state.

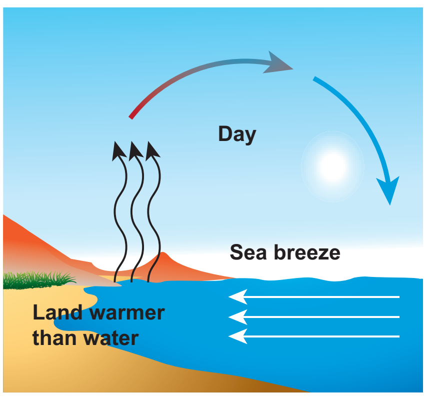
Thermal conductivity depends on the nature of the material. For example silver and aluminum have high thermal conductivities. So they are used to make cooking vessels.

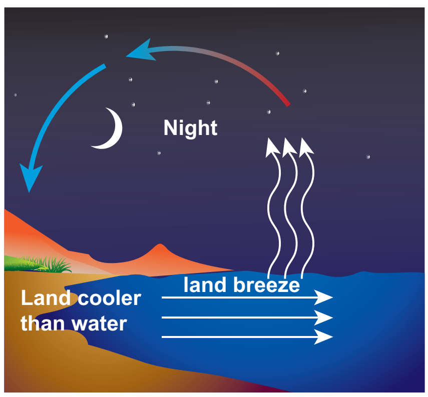
Convection

Convection is the process in which heat transfer is by actual movement of molecules in fluids such as liquids and gases. In convection, molecules move freely from one place to another. It happens naturally or forcefully.

DO YOU KNOW?

During the day, sun rays warm up the land more quickly than sea water. It is because land has less specific heat capacity than water. As a result the air above the land becomes less dense and rises. At the same time the cooler air above the sea flows to land and it is called 'sea breeze'. During the night time the land gets cooled faster than sea due to the same reason (specific heat). The air molecules above sea are warmer than air molecules above the land. So air molecules above the sea are replaced by cooler air molecules from the land. It is called 'land breeze'.





athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Boiling water in a cooking pot is an example of convection. Water at the bottom of the pot receives more heat. Due to heating, the water expands and the density of water decreases at the bottom. Due to this decrease in density, molecules rise to the top. At the same time the molecules at the top receive less heat and become denser and come to the bottom of the pot. This process goes on continuously. The back and forth movement of molecules is called convection current.

To keep the room warm, we use room heater. The air molecules near the heater will heat up and expand. As they expand, the density of air molecules will decrease and rise up while the higher density cold air will come down. This circulation of air molecules is called convection current.

Radiation:

When we keep our hands near the hot stove we feel the heat even though our hands are not touching the hot stove. Here heat transferred from the hot stove to our hands is in the form of radiation. We receive energy from the sun in the form of radiations. These radiations travel through vacuum and reach the Earth. It is the peculiar character of radiation which requires no medium to transfer energy from one object to another. The conduction or convection requires medium to transfer the heat.

Radiation is a form of energy transfer from one body to another by electromagnetic waves.

Example:
1. Solar energy from the Sun.
2. Radiation from room heater.

Note The parameter temperature is generally thought to be associated with matter (solid, liquid and gas). But radiation is also considered as a thermodynamic system which has well defined temperature and pressure. The visible radiation coming from the Sun is at the temperature of 5700 K and the Earth re emits the radiation in the infrared range into space which is at a temperature of around 300K.

8.2.8 Newton's law of cooling

Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in the temperature between that body and its surroundings .

$$\frac{dQ}{dt} \propto -(T - T_s) \qquad (8.4)$$

The negative sign indicates that the quantity of heat lost by liquid goes on decreasing with time. Where,

T = Temperature of the object

T_s = Temperature of the surrounding

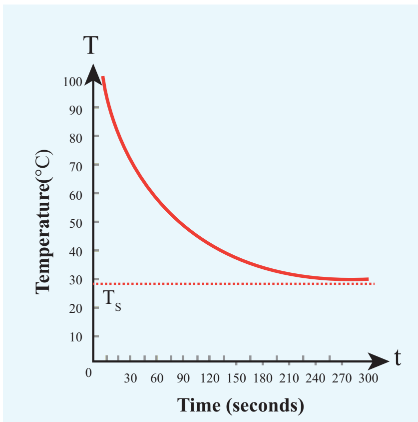


Figure 8.12 Cooling of hot water with time

From the graph in Figure 8.12 it is clear that the rate of cooling is high initially and decreases with falling temperature.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Let us consider an object of mass m and specific heat capacity s at temperature T. Let T_s be the temperature of the surroundings. If the temperature falls by a small amount dT in time dt, then the amount of heat lost is,

$$dQ = msdT \qquad (8.5)$$

Dividing both sides of equation (8.5) by dt

$$\frac{dQ}{dt} = \frac{msdT}{dt} \qquad (8.6)$$

From Newton's law of cooling

$$\frac{dQ}{dt} \propto -(T - T_s)$$

$$\frac{dQ}{dt} = -a(T - T_s) \qquad (8.7)$$

Where a is some positive constant.

From equation (8.6) and (8.7)

$$-a(T - T_s) = ms\frac{dT}{dt}$$

$$\frac{dT}{T - T_s} = -\frac{a}{ms}dt \qquad (8.8)$$

Integrating equation (8.8) on both sides,

$$\int_0^{\infty} \frac{dT}{T - T_s} = -\int_0^t \frac{a}{ms}dt$$

$$\ln(T - T_s) = -\frac{a}{ms}t + b_1$$

Where b_1 is the constant of integration. taking exponential both sides, we get

$$T = T_s + b_2\, e^{-\frac{a}{ms}t} \qquad (8.9)$$

here $b_2 = e^{b_1}$ = constant

EXAMPLE 8.8

A hot water cools from 92°C to 84°C in 3 minutes when the room temperature is 27°C. How long will it take for it to cool from 65°C to 60°C?

The hot water cools 8°C in 3 minutes. The average temperature of 92°C and 84°C is 88°C. This average temperature is 61°C above room temperature. Using equation (8.8)

$$\frac{dT}{T - T_s} = -\frac{a}{ms}dt \quad \text{or} = \frac{dT}{dt} = -\frac{a}{ms}(T - T_s)$$

$$\frac{8^oC}{3\,\text{min}} = -\frac{a}{ms}(61^oC)$$

Similarly the average temperature of 65°C and 60°C is 62.5°C. The average temperature is 35.5°C above the room temperature. Then we can write

$$\frac{5^oC}{dt} = -\frac{a}{ms}(35.5^oC)$$

By diving both the equation, we get

$$\frac{\frac{8^oC}{3\,\text{min}}}{\frac{5^oC}{dt}} = \frac{-\frac{a}{ms}(61^oC)}{-\frac{a}{ms}(35.5^oC)}$$

$$\frac{8 \times dt}{3 \times 5} = \frac{61}{35.5}$$

$$dt = \frac{61 \times 15}{35.5 \times 8} = \frac{915}{284} = 3.22\,\text{min}$$

8.3 LAWS OF HEAT TRANSFER

8.3.1 Prevost theory of heat exchange

Every object emits heat radiations at all finite temperatures (except 0 K) as well as it absorbs radiations from the surroundings. For example, if you touch someone, they might feel your skin as either hot or cold.

A body at high temperature radiates more heat to the surroundings than it receives from it. Similarly, a body at a lower temperature receives more heat from the surroundings than it loses to it.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Prevost applied the idea of 'thermal equilibrium' to radiation. He suggested that all bodies radiate energy but hot bodies radiate more heat than the cooler bodies. At one point of time the rate of exchange of heat from both the bodies will become the same. Now the bodies are said to be in 'thermal equilibrium'.

Only at absolute zero temperature a body will stop emitting. Therefore *Prevost theory states that all bodies emit thermal radiation at all temperatures above absolute zero irrespective of the nature of the surroundings.*

8.3.2 Stefan Boltzmann law

Stefan Boltzmann law states that, *the total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.*

$$E \propto T^4 \text{ or } E = \sigma T^4 \qquad (8.10)$$

Where, σ is known as Stefan's constant. Its value is 5.67×10^{-8} W $m^{-2} k^{-4}$

Note

If a body is not a perfect black body, then

$$E = e\,\sigma\, T^4 \qquad (8.11)$$

Where 'e' is emissivity of surface.

Emissivity is defined as the ratio of the energy radiated from a material's surface to that radiated from a perfectly black body at the same temperature and wavelength.

8.3.3 Wien's displacement law

In the universe every object emits radiation. The wavelengths of these radiations depend on the object's absolute temperature. These radiations have different wavelengths and all the emitted wavelengths will not have equal intensity.

Wien's law states that, *the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body.*

$$\lambda_m \propto \frac{1}{T} \,(or)\, \lambda_m = \frac{b}{T} \qquad (8.12)$$

Where, b is known as Wien's constant. Its value is 2.898×10^{-3} m K

It implies that if temperature of the body increases, maximal intensity wavelength (λ_m) shifts towards lower wavelength (higher frequency) of electromagnetic spectrum. It is shown in Figure 8.13

Graphical representation

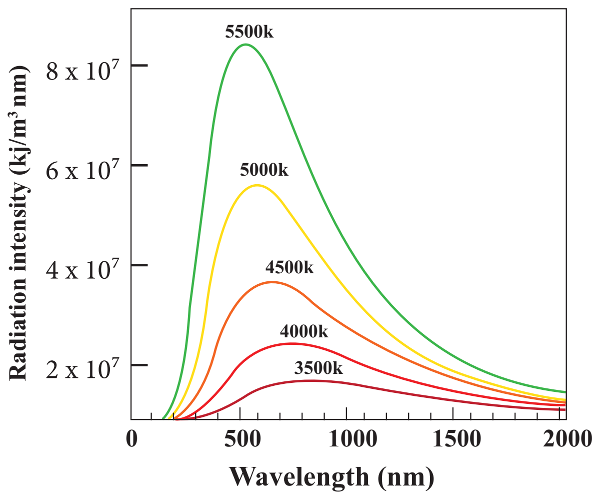


Figure 8.13 Wien's displacement law

From the graph it is clear that the peak of the wavelengths is inversely proportional to temperature. The curve is known as 'black body radiation curve'.

Wien's law and Vision:

Why our eye is sensitive to only visible wavelength (in the range 400 nm to 700nm)?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The Sun is approximately taken as a black body. Since any object above 0 K will emit radiation, Sun also emits radiation. Its surface temperature is about 5700 K. By substituting this value in the equation (8.12),

$$\lambda_m = \frac{b}{T} = \frac{2.898\times10^{-8}}{5700} \approx 508\, nm$$

It is the wavelength at which maximum intensity is 508*nm*. Since the Sun's temperature is around 5700K, the spectrum of radiations emitted by Sun lie between 400 nm to 700 nm which is the visible part of the spectrum. It is shown in Figure 8.14

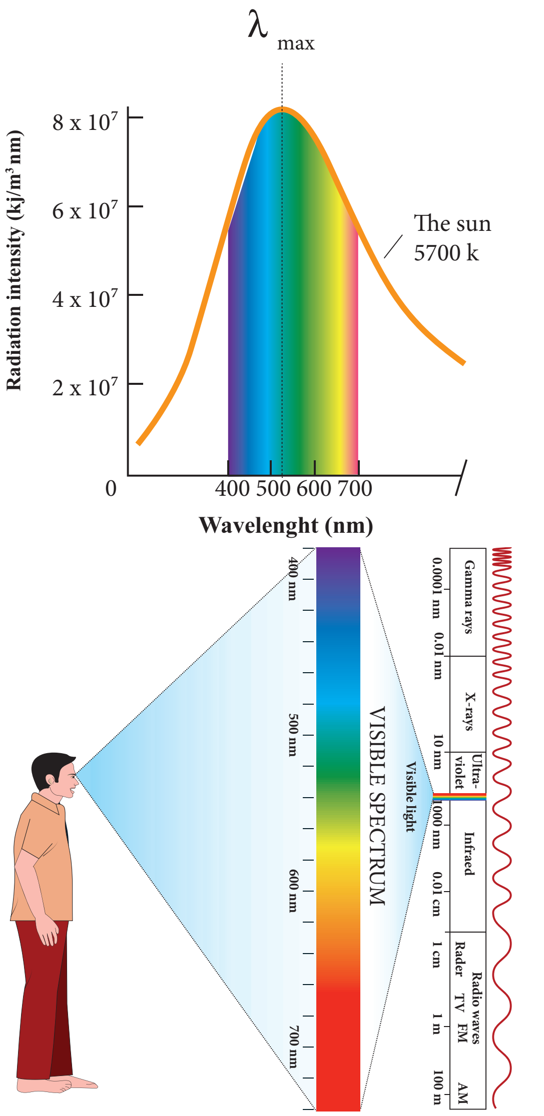


Figure 8.14 Wien's law and Human's vision

The humans evolved under the Sun by receiving its radiations. The human eye is sensitive only in the visible not in infrared or X-ray ranges in the spectrum.

Suppose if humans had evolved in a planet near the star Sirius (9940K), then they would have had the ability to see the Ultraviolet rays!

EXAMPLE 8.9

The power radiated by a black body A is E_A and the maximum energy radiated was at the wavelength λ_A. The power radiated by another black body B is $E_B = N\,E_A$ and the radiated energy was at the maximum wavelength, $\frac{1}{2}\lambda_A$. What is the value of N?

According to Wien's displacement law

$\lambda_{max}\,T$ = constant for both object A and B

$$\lambda_A T_A = \lambda_B T_B. \text{ Here } \lambda_B = \frac{1}{2}\lambda_A$$

$$\frac{T_B}{T_A} = \frac{\lambda_A}{\lambda_B} = \frac{1}{\frac{1}{2}} = 2$$

$$T_B = 2T_A$$

From Stefan-Boltzmann law

$$\frac{E_B}{E_A} = \left(\frac{T_B}{T_A}\right)^4 = (2)^4 = 16 = N$$

Object B has emitted at lower wavelength compared to A. So the object B would have emitted more energetic radiation than A.

8.4 THERMODYNAMICS:

8.4.1 Introduction

In the previous sections we have studied about the heat, temperature and thermal properties of matter. Thermodynamics is

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

a branch of physics which describes the laws governing the process of conversion of work into heat and conversion of heat into work. The laws of thermodynamics are formulated over three centuries of experimental works of Boyle, Charles, Bernoulli, Joule, Clausius, Kelvin, Carnot and Helmholtz. In our day to day life, the functioning of everything around us and even our body is governed by the laws of thermodynamics. Therefore thermodynamics is one of the essential branches of physics.

Thermodynamic system:

A thermodynamic system is a finite part of the universe. It is a collection of large number of particles (atoms and molecules) specified by certain parameters called pressure (P), Volume (V) and Temperature (T). The remaining part of the universe is called surrounding. Both are separated by a boundary. It is shown in Figure 8.15

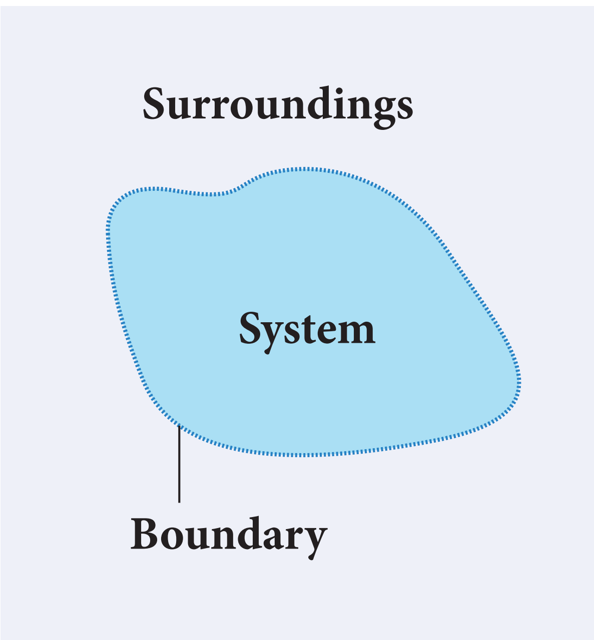


Figure 8.15 Thermodynamic system

Examples:

A thermodynamic system can be liquid, solid, gas and radiation.

Thermodynamic system	Surrounding
Bucket of water	Open atmosphere
Air molecules in the room	Outside air
Human body	Open atmosphere
Fish in the sea	Sea of water

We can classify thermodynamics system into three types: It is given in Figure 8.16

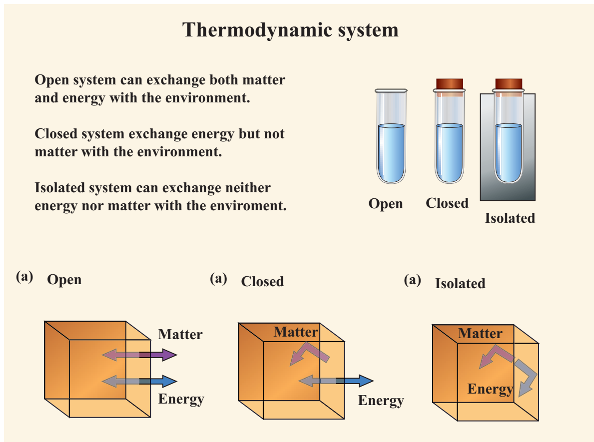


Figure 8.16 Different types of thermodynamic systems

8.4.2 Thermal equilibrium

When a hot cup of coffee is kept in the room, heat flows from coffee to the surrounding air. After some time the coffee reaches the same temperature as the surrounding air and there will be no heat flow from coffee to air or air to coffee. It implies that the coffee and surrounding air are in thermal equilibrium with each other.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which will not change with time.

Mechanical equilibrium:

Consider a gas container with piston as shown in Figure 8.17. When some mass is placed on the piston, it will move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position. When the downward gravitational force given by the piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermo dynamic system or on the surrounding by thermodynamic system.

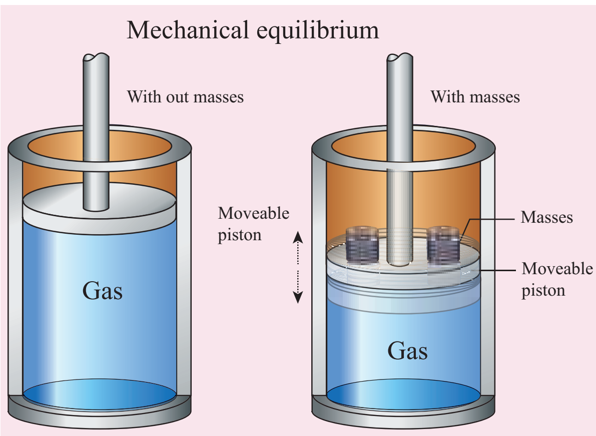


Figure 8.17 Mechanical equilibrium

Chemical equilibrium:

If there is no net chemical reaction between two thermodynamic systems in contact with each other then it is said to be in chemical equilibrium.

Thermodynamic equilibrium:

If two systems are set to be in thermodynamic equilibrium, then the systems are at thermal, mechanical and chemical equilibrium with each other. In a state of thermodynamic equilibrium the macroscopic variables such as pressure, volume and temperature will have fixed values and do not change with time.

8.4.3 Thermodynamic state variables

In mechanics velocity, momentum and acceleration are used to explain the state of any moving object (which you would have realized in Volume 1). In thermodynamics, the state of a thermodynamic system is represented by a set of variables called thermodynamic variables.

Examples: Pressure, temperature, volume and internal energy etc.

The values of these variables completely describe the equilibrium state of a thermodynamic system. Heat and work are not state variables rather they are process variables.

There are two types of thermodynamic variables: Extensive and Intensive

Extensive variable depends on the size or mass of the system.

Example: Volume, total mass, entropy, internal energy, heat capacity etc.

Intensive variables do not depend on the size or mass of the system.

Example: Temperature, pressure, specific heat capacity, density etc.

Equation of state:

The equation which connects the state variables in a specific manner is called equation of state. A thermodynamic equilibrium is completely specified by these state variables by the equation of state. If the system is not in thermodynamic equilibrium then these equations cannot specify the state of the system.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

An ideal gas obeys the equation PV = NkT at thermodynamic equilibrium. Since all four macroscopic variables (P,V,T and N) are connected by this equation, we cannot change one variable alone. For example, if we push the piston of a gas container, the volume of the gas will decrease but pressure will increase or if heat is supplied to the gas, its temperature will increase, pressure and volume of the gas may also increase.

There is another example of equation of state called van der Waals equation. Real gases obey this equation at thermodynamic equilibrium. The air molecules in the room truly obey van der Waals equation of state. But at room temperature with low density we can approximate it into an ideal gas.

8.5 ZEROTH LAW OF THERMODYNAMICS

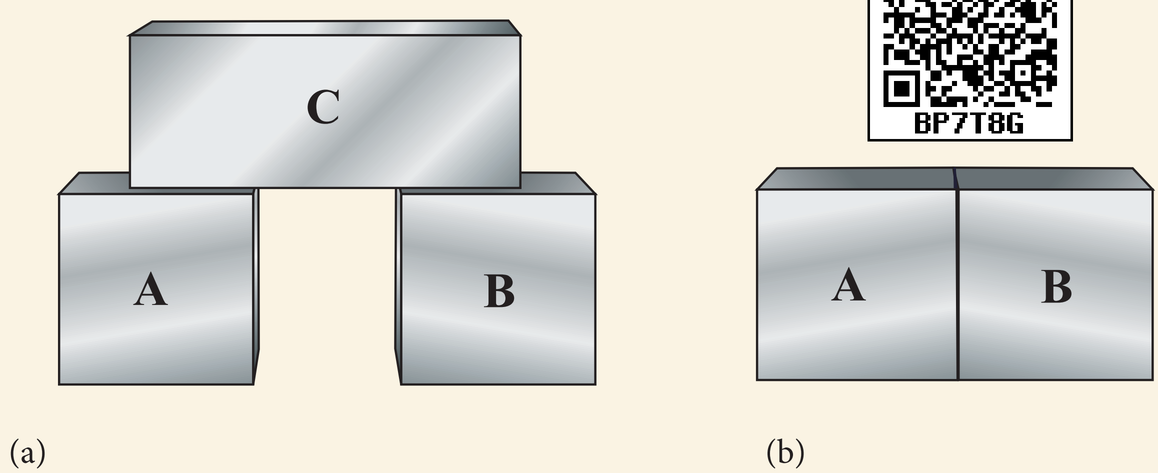
The zeroth law of thermodynamics states that if two systems, *A* and *B*, are in thermal equilibrium with a third system, *C*, then *A* and *B* are in thermal equilibrium with each other.

Consider three systems A, B and C which are initially at different temperatures. Assume that A and B are not in thermal contact with each other as shown in Figure 8.18 (a) but each of them is in thermal contact with a third system C. After a lapse of time, system A will be in thermal equilibrium with C and B also will be in thermal equilibrium with C. In this condition, if the systems A and B are kept in thermal contact as shown in the Figure 8.18 (b), there is no flow of heat between the systems A and B. It implies that the system A and B are also in thermal equilibrium with each other. Once the three systems are at thermal equilibrium, there will be no heat flow between them as they are at the same temperature. This can be mathematically expressed as

If $T_A = T_C$ and $T_B = T_C$, it implies that $T_A = T_B$,

where T_A, T_B and T_C are the temperatures of the systems A, B, and C respectively.

Temperature is the property which determines whether the system is in



BP7T8G

Figure 8.18 (a) Two systems A and B in thermal contact with object C separately (b) If systems A and B are in thermal contact, they are also in thermal equilibrium with each other.

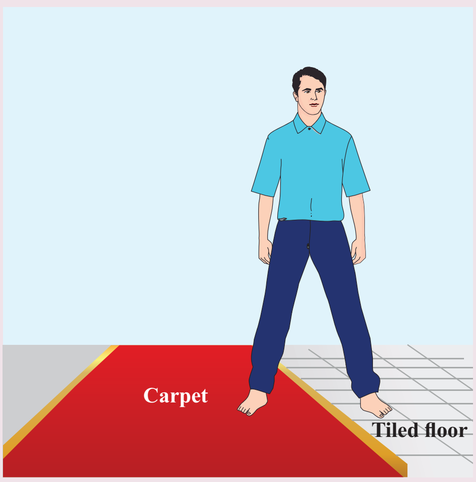
athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

thermal equilibrium with other systems or not. Zeroth law enables us to determine the temperature. For example, when a thermometer is kept in contact with a human body, it reaches thermal equilibrium with the body. At this condition, the temperature of the thermometer will be same as the human body. This principle is used in finding the body temperature.

ACTIVITY

We often associate the temperature as a measure of how hot or cold an object is while touching it. Can we use our sensory organs to determine the temperature of an object?

When you stand bare feet with one foot on the carpet and the other on the tiled floor, your foot on tiled floor feels cooler than the foot on the carpet even though both the tiled floor and carpet are at the same room temperature. It is because the tiled floor transfers the heat energy to your skin at higher rate than the carpet. So the skin is not measuring the actual temperature of the object; instead it measures the rate of heat energy transfer. But if we place a thermometer on the tiled floor or carpet it will show the same temperature.



8.6 INTERNAL ENERGY (U)

The internal energy of a thermodynamic system is the sum of kinetic and potential energies of all the molecules of the system with respect to the center of mass of the system.

The energy due to molecular motion including translational, rotational and vibrational motion is called internal kinetic energy (E_K)

The energy due to molecular interaction is called internal potential energy (E_P). Example: Bond energy.

$$U = E_K + E_P$$

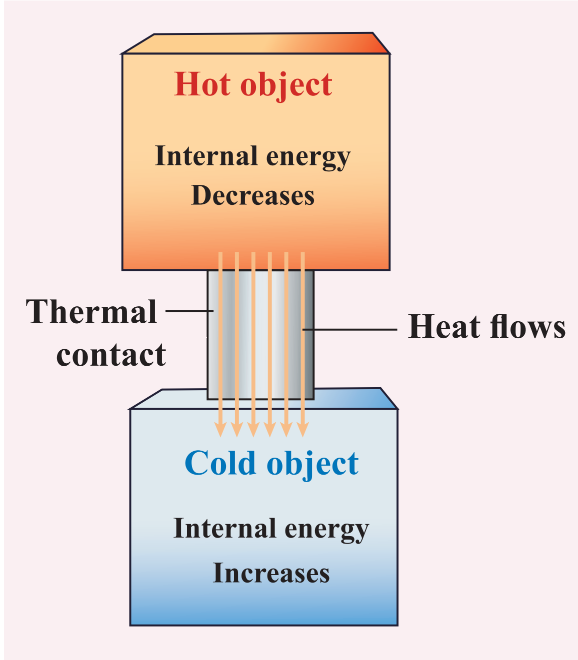
- Since ideal gas molecules are assumed to have no interaction with each other the internal energy consists of only kinetic energy part (E_K) which depends on the temperature, number of particles and is independent of volume. However this is not true for real gases like Van der Waals gases.
- Internal energy is a state variable. It depends only on the initial and final states of the thermodynamic system. For example, if the temperature of water is raised from 30°C to 40°C either by heating or by stirring, the final internal energy depends only on the final temperature 40°C and not the way it is arrived at.

Note It is very important to note that the internal energy of a thermodynamic system is associated with only the kinetic energy of the individual molecule due to its random motion and the potential energy of molecules which depends on their chemical nature. The bulk kinetic energy of the entire system or gravitational potential energy of the system should not be mistaken as a part of internal energy. For example

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Note (a) Consider two gas containers at the same temperature having the same internal energy, one is kept at rest on the ground and the other is kept in a moving train. Even though the gas container in the train is moving with the speed of the train, the internal energy of the gas in it will not increase.

(b) Consider two gas containers at the same temperature having the same internal energy, one is kept on the ground and the other is kept at some height h. Even though the container at height h is having higher gravitational potential energy, this has no influence on internal energy of the gas molecules.



EXAMPLE 8.10

When you mix a tumbler of hot water with one bucket of normal water, what will be the direction of heat flow? Justify.

The water in the tumbler is at a higher temperature than the bucket of normal water. But the bucket of normal water has larger internal energy than the hot water in the tumbler. This is because the internal energy is an extensive variable and it depends on the size or mass of the system.

Even though the bucket of normal water has larger internal energy than the tumbler of hot water, heat will flow from water in the tumbler to the water in the bucket. This is because heat flows from a body at higher temperature to the one at lower temperature and is independent of internal temperature and is independent of internal energy of the system.

Once the heat is transferred to an object it becomes internal energy of the object. The right way to say is 'object has certain amount of internal energy'. Heat is one of the ways to increase the internal energy of a system as shown in the Figure.

Note It is to be noted that heat does not always increases the internal energy. Later we shall see that in ideal gases during isothermal process the internal energy will not increase even though heat flows in to the system.

8.6.1 Joule's mechanical equivalent of heat

The temperature of an object can be increased by heating it or by doing some work on it. In the eighteenth century, James Prescott Joule showed that mechanical energy can be converted into internal energy and vice versa. In his experiment, two masses were attached with a rope and a paddle wheel as shown in Figure 8.19. When these masses fall through a distance h due to gravity, both the masses lose potential energy equal to 2mgh. When the masses fall, the paddle wheel turns. Due to the turning of wheel inside water, frictional force comes in between the water and the paddle wheel. This causes

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

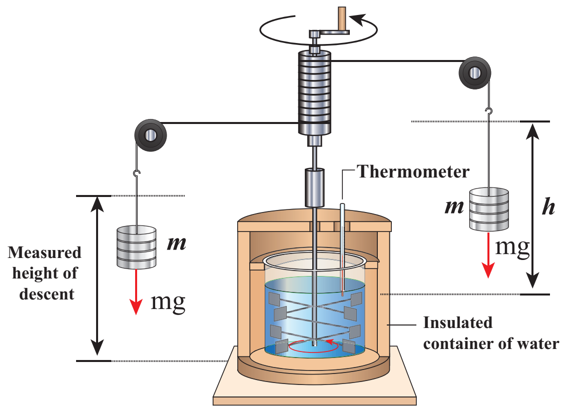


Figure.8.19 Joule's experiment for determining the mechanical equivalent of heat energy.

a rise in temperature of the water. This implies that gravitational potential energy is converted to internal energy of water. The temperature of water increases due to the work done by the masses. In fact, Joule was able to show that the mechanical work has the same effect as giving heat. He found that to raise 1 g of an object by 1°C , 4.186 J of energy is required. In earlier days the heat was measured in calorie.

$$1 \text{ cal} = 4.186 \text{ J}$$

This is called Joule's mechanical equivalent of heat.

Note Before James Prescott Joule, people thought that heat was a kind of fluid called caloric fluid which flows from an object at high temperature to that in low temperature. According to caloric fluid idea, the hot object contains more caloric fluid and the cold object contains less caloric fluid since heat was treated as a quantity. Now we understand that heat is not a quantity but it is an energy in transit. So the word 'mechanical equivalent of heat' is wrong terminology. Because mechanical energy is a quantity and any object can contain more or less mechanical energy, but this is not so with heat as it is not a quantity. However this terminology is retained for historical reasons. A correct name would be 'Joule's mechanical equivalence of internal energy'. Joule Essentially converted mechanical energy to internal energy. In his experiment potential energy is converted to rotational kinetic energy of paddle wheel and this rotational kinetic energy is converted to internal energy of water.

EXAMPLE 8.11

A student had a breakfast of 200 food calories. He thinks of burning this energy by drawing water from the well and watering the trees in his school. Depth of the well is about 25 m. The pot can hold 25L of water and each tree requires one pot of water. How many trees can he water? (Neglect the mass of the pot and the energy spent by walking. Take g =10 m s^{-2})

Solution

To draw 25 L of water from the well, the student has to do work against gravity by burning his energy.

Mass of the water = 25 L = 25 kg (1L=1kg)

The work required to draw 25 kg of water = gravitational potential energy gained by water.

$W = mgh = 25 \times 10 \times 25 = 6250\ J$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The total energy gained from the food = 200 food cal =200 kcal.

$= 200\times10^3 \times 4.186\ J = 8.37 \times 10^5\ J$

If we assume that by using this energy the student can drawn 'n' pots of water from the well, the total energy spent by him = $8.37 \times 10^5\ J = nmgh$

$$n = \frac{8.37\times10^5 J}{6250J} \approx 134.$$

This n is also equal to the number of trees that he can water.

Is it possible to draw 134 pots of water from the well just by having breakfast? No. Actually the human body does not convert entire food energy into work. It is only approximately 20% efficient. It implies that only 20% of 200 food calories is used to draw water from the well. So 20 % of the 134 is only 26 pots of water. It is quite meaningful. So he can water only 26 trees.

The remaining energy is used for blood circulation and other functions of the body. It is to be noted that some energy is always 'wasted'. Why is it that the body cannot have 100% efficiency? You will find the answer in section 8.9

8.6.2 First law of thermodynamics

The first law of thermodynamics is a statement of the law of conservation of energy. In Newtonian mechanics conservation of energy involves kinetic and potential energies of bulk objects. But the first law of thermodynamics includes heat also. This law states that '*Change in internal energy (ΔU) of the system is equal to heat supplied to the system (Q) minus the work done by the system (W) on the surroundings*'. Mathematically it is written as

$$\Delta U = Q - W \quad (8.13)$$

The internal energy of a thermodynamic system can be changed either by heating or by work as shown below.

Heat flows into the system	Internal energy increases
Heat flows out of the system	Internal energy decreases
Work is done on the system	Internal energy increases
Work is done by the system	Internal energy decreases

Based on the above table the sign convention is introduced to use first law of thermodynamics appropriately. It is shown in the following table and the Figure 8.20.

System gains heat	Q is positive
System loses heat	Q is negative
Work done on the system	W is negative
Work done by the system	W is positive

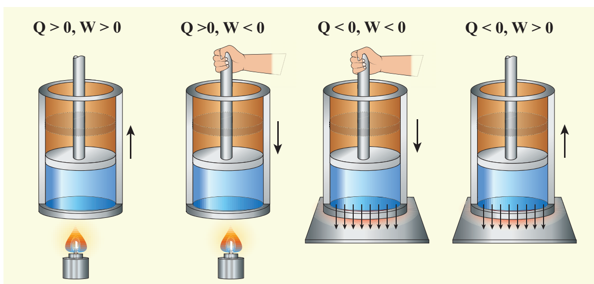


Figure 8.20 The Sign convention for heat and work

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Even though we often explain first law of thermodynamics using gases, this law is universal and applies to liquids and solids also.

> **Note** Some book presents the first law of thermodynamics as $\Delta U = Q + W$. Here the work done by the system is taken as negative and work done on the system is positive. However both the conventions are correct and we can follow any one of the convention.

EXAMPLE 8.12

A person does 30 kJ work on 2 kg of water by stirring using a paddle wheel. While stirring, around 5 kcal of heat is released from water through its container to the surface and surroundings by thermal conduction and radiation. What is the change in internal energy of the system?

Solution

Work done on the system (by the person while stirring), W = -30 kJ = -30,000J

Heat flowing out of the system, Q = -5 kcal = 5 × 4184 J =-20920 J

Using First law of thermodynamics

$$\Delta U = Q - W$$

$$\Delta U = -20{,}920\,J - (-30{,}000)\,J$$

$$\Delta U = -20{,}920\,J + 30{,}000\,J = 9080\,J$$

Here, the heat lost is less than the work done on the system, so the change in internal energy is positive.

EXAMPLE 8.13

Jogging every day is good for health. Assume that when you jog a work of 500 kJ is done and 230 kJ of heat is given off. What is the change in internal energy of your body?

Solution

Work done by the system (body),

W = +500 kJ

Heat released from the system (body),

Q = −230 kJ

The change in internal energy of a body

$= \Delta U = -\,230\,kJ - 500\,kJ = -\,730\,kJ$

8.6.3 Quasi-static process

Consider a system of an ideal gas kept in a cylinder of volume V at pressure P and temperature T. When the piston attached to the cylinder moves outward the volume of the gas will change. As a result the temperature and pressure will also change because all three variables P,T and V are related by the equation of state PV = NkT. If a block of some mass is kept on the piston, it will suddenly push the piston downward. The pressure near the piston will be larger than other parts of the system. It implies that the gas is in non-equilibrium state. We cannot determine pressure, temperature or internal energy of the system until it reaches another equilibrium state. But if the piston is pushed very slowly such that at every stage it is still in equilibrium with surroundings, we can use the equation of state to calculate the internal energy, pressure or temperature. This kind of process is called quasi-static process.

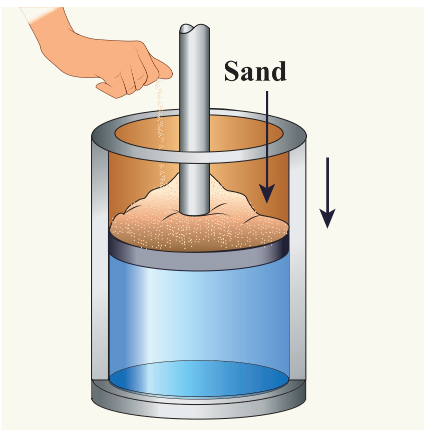
athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

A quasi-static process is an infinitely slow process in which the system changes its variables (P,V,T) so slowly such that it remains in thermal, mechanical and chemical equilibrium with its surroundings throughout. By this infinite slow variation, the system is always almost close to equilibrium state.

EXAMPLE 8.14

Give an example of a quasi-static process.

Consider a container of gas with volume V, pressure P and temperature T. If we add sand particles one by one slowly on the top of the piston, the piston will move inward very slowly. This can be taken as almost a quasi-static process. It is shown in the figure



Sand particles added slowly- quasi-static process

8.6.4 Work done in volume changes

Consider a gas contained in the cylinder fitted with a movable piston. Suppose the gas is expanded quasi-statically by pushing the piston by a small distance dx as shown in Figure 8.21. Since the expansion occurs quasi-statically the pressure, temperature and internal energy will have unique values at every instant.

The small work done by the gas on the piston

$$dW = Fdx \quad (8.14)$$

The force exerted by the gas on the piston F = PA. Here A is area of the piston and P is pressure exerted by the gas on the piston.

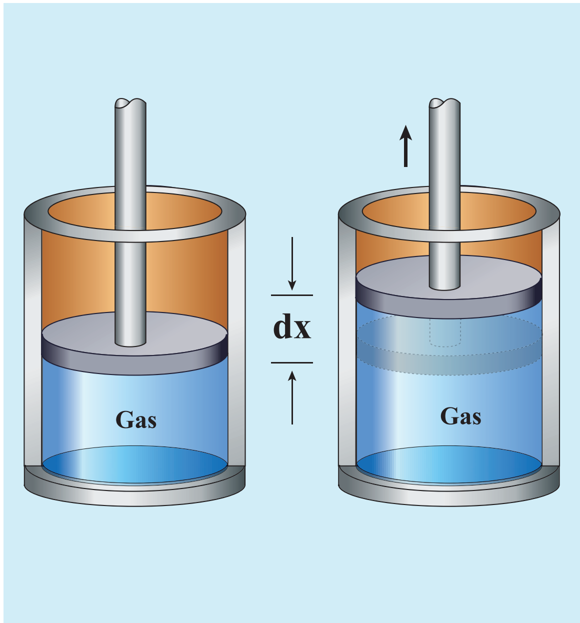


Figure 8.21 Work done by the gas

Equation (8.14) can be rewritten as

$$dW = PA\,dx \quad (8.15)$$

But Adx = dV= change in volume during this expansion process.

So the small work done by the gas during the expansion is given by

$$dW = PdV \quad (8.16)$$

Note here that is positive since the volume is increased. Here, is positive.

In general the work done by the gas by increasing the volume from V_i to V_f is given by

$$W = \int_{V_i}^{V_f} PdV \quad (8.17)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Suppose if the work is done on the system, then $V_i > V_f$. Then, W is negative.

Note here the pressure P is inside the integral in equation (8.17). It implies that while the system is doing work, the pressure need not be constant. To evaluate the integration we need to first express the pressure as a function of volume and temperature using the equation of state.

8.6.5 PV diagram

PV diagram is a graph between pressure P and volume V of the system. The P-V diagram is used to calculate the amount of work done by the gas during expansion or on the gas during compression. In Unit 2, we have seen that the area under the curve will give integration of the function from lower limit to upper limit. The area under the PV diagram will give the work done during expansion or compression which is shown in Figure 8.22

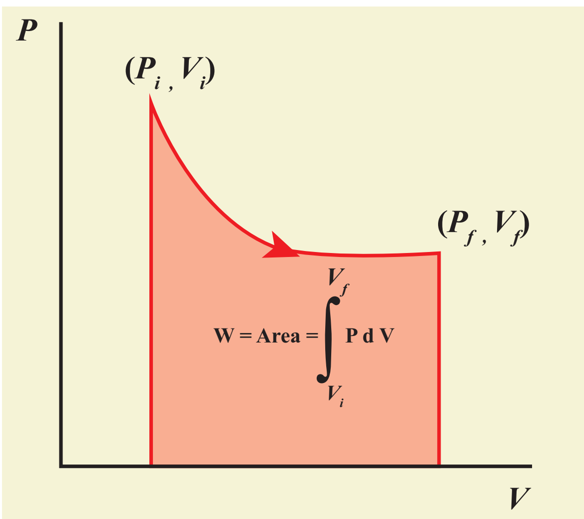


Figure 8.22 Work done by the gas during expansion

The shape of PV diagram depends on the nature of the thermodynamic process.

EXAMPLE 8.15

A gas expands from volume $1m^3$ to $2m^3$ at constant atmospheric pressure.

(a) Calculate the work done by the gas.

(b) Represent the work done in PV diagram.

Solution

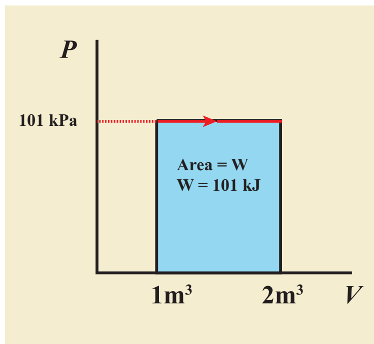
(a) The pressure P = 1 atm = 101 kPa, $V_f = 2\ m^3$ and $V_i = 1m^3$

From equation (8.17) $W = \int_{V_i}^{V_f} PdV = P \int_{V_i}^{V_f} dV$

Since P is constant. It is taken out of the integral.

$$W = P\,(V_f - V_i) = 101{\times}103 \times (2-1) = 101\ kJ$$

(b) Since the pressure is kept constant, PV diagram is straight line as shown in the figure. The area is equal to work done by the gas.



Note the arrow mark in the curve. Suppose the work is done on the system, then volume will decreases and the arrow will point in the opposite direction.

8.7 SPECIFIC HEAT CAPACITY OF A GAS

Specific heat capacity of a given system plays a very important role in determining

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

the structure and molecular nature of the system. Unlike solids and liquids, gases have two specific heats: specific heat capacity at constant pressure (s_p) and specific heat capacity at constant volume (s_v).

8.7.1 Specific heat capacity

Specific heat capacity at constant pressure (s_p):

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K or 1°C by keeping the pressure constant is called specific heat capacity of at constant pressure. When the heat energy is supplied to the gas, it expands to keep the pressure constant as shown in Figure 8.23

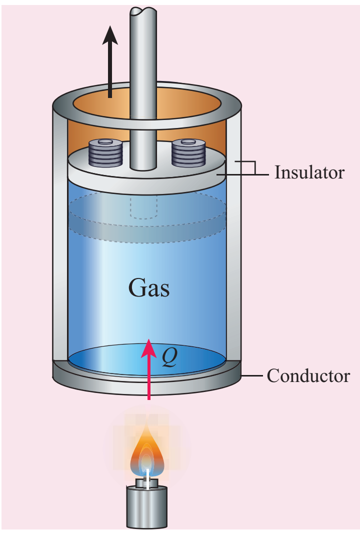


Figure 8.23 Specific heat capacity at constant pressure

In this process a part of the heat energy is used for doing work (expansion) and the remaining part is used to increase the internal energy of the gas.

Specific heat capacity at constant volume (s_v):

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K or 1°C by keeping the volume constant is called specific heat capacity at constant volume.

If the volume is kept constant, then the supplied heat is used to increase only the internal energy. No work is done by the gas as shown in Figure 8.24.

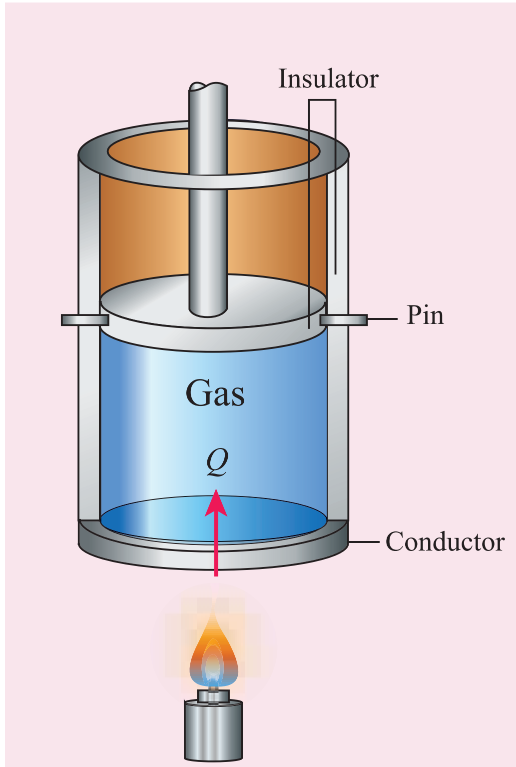


Figure 8.24 Specific heat capacity at constant volume

It implies that to increase the temperature of the gas at constant volume requires less heat than increasing the temperature of the gas at constant pressure. In other words s_p is always greater than s_v.

Molar Specific heat capacities

Sometimes it is useful to calculate the molar heat capacities C_p and C_v. The amount of heat required to raise the temperature of one mole of a substance by 1K or 1°C at constant volume is called molar specific heat capacity at constant volume (C_v). If pressure is kept constant, it is called molar specific heat capacity at constant pressure (C_p).

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

If Q is the heat supplied to mole of a gas at constant volume and if the temperature changes by an amount ΔT, we have

$$Q = \mu C_v \Delta T. \quad (8.18)$$

By applying the first law of thermodynamics for this constant volume process (W=0, since dV=0), we have

$$Q = \Delta U - 0 \quad (8.19)$$

By comparing the equations (8.18) and (8.19),

$$\Delta U = \mu C_v \Delta T \text{ or } C_v = \frac{1}{\mu}\frac{\Delta U}{\Delta T}$$

If the limit ΔT goes to zero, we can write

$$C_v = \frac{1}{\mu}\frac{dU}{dT} \quad (8.20)$$

Since the temperature and internal energy are state variables, the above relation holds true for any process.

8.7.2 Meyer's relation

Consider μ mole of an ideal gas in a container with volume V, pressure P and temperature T.

When the gas is heated at constant volume the temperature increases by dT. As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be dU.

If C_v is the molar specific heat capacity at constant volume, from equation (8.20)

$$dU = \mu C_v dT \quad (8.21)$$

Suppose the gas is heated at constant pressure so that the temperature increases by dT. If 'Q' is the heat supplied in this process and 'dV' the change in volume of the gas.

$$Q = \mu C_p dT \quad (8.22)$$

If W is the workdone by the gas in this process, then

$$W = PdV \quad (8.23)$$

But from the first law of thermodynamics,

$$Q = dU + W \quad (8.24)$$

Substituting equations (8.21), (8.22) and (8.23) in (8.24), we get,

$$\mu C_p dT = \mu C_v dT + PdV \quad (8.25)$$

For mole of ideal gas, the equation of state is given by

$$PV = \mu RT \Rightarrow PdV + VdP = \mu RdT \quad (8.26)$$

Since the pressure is constant, dP=0

$$\therefore C_p dT = C_v dT + RdT$$

$$\therefore C_p = C_v + R \quad \text{(or)} \quad C_p - C_v = R \quad (8.27)$$

This relation is called Meyer's relation

It implies that the molar specific heat capacity of an ideal gas at constant pressure is greater than molar specific heat capacity at constant volume.

The relation shows that specific heat at constant pressure (s_p) is always greater that specific heat at constant volume (s_v).

8.8 THERMODYNAMIC PROCESSES

8.8.1 Isothermal process

It is a process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change. The ideal gas equation is

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$PV = \mu RT$$

Here, T is constant for this process

So the equation of state for isothermal process is given by

$$PV = constant \quad (8.28)$$

This implies that if the gas goes from one equilibrium state (P_1, V_1) to another equilibrium state (P_2, V_2) the following relation holds for this process

$$P_1V_1 = P_2V_2 \quad (8.29)$$

Since PV = constant, P is inversely proportional to V ($P \propto \frac{1}{V}$). This implies that PV graph is a hyperbola. The pressure-volume graph for constant temperature is also called isotherm.

Figure 8.25 shows the PV diagram for quasi-static isothermal expansion and quasi-static isothermal compression.

We know that for an ideal gas the internal energy is a function of temperature only. For an isothermal process since temperature is constant, the internal energy is also constant. This implies that dU or $\Delta U = 0$.

For an isothermal process, the first law of thermodynamics can be written as follows,

$$Q = W \quad (8.30)$$

From equation (8.30), we infer that the heat supplied to a gas is used to do only external work. It is a common misconception that when there is flow of heat to the system, the temperature will increase. For isothermal process this is not true. The isothermal compression takes place when the piston of the cylinder is pushed. This will increase the internal energy which will flow out of the system through thermal contact. This is shown in Figure 8.26.

BPGPA4

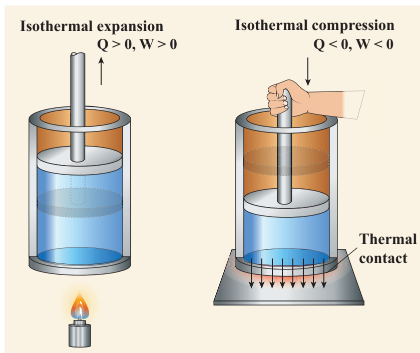


Figure 8.26 Isothermal expansion and isothermal compression

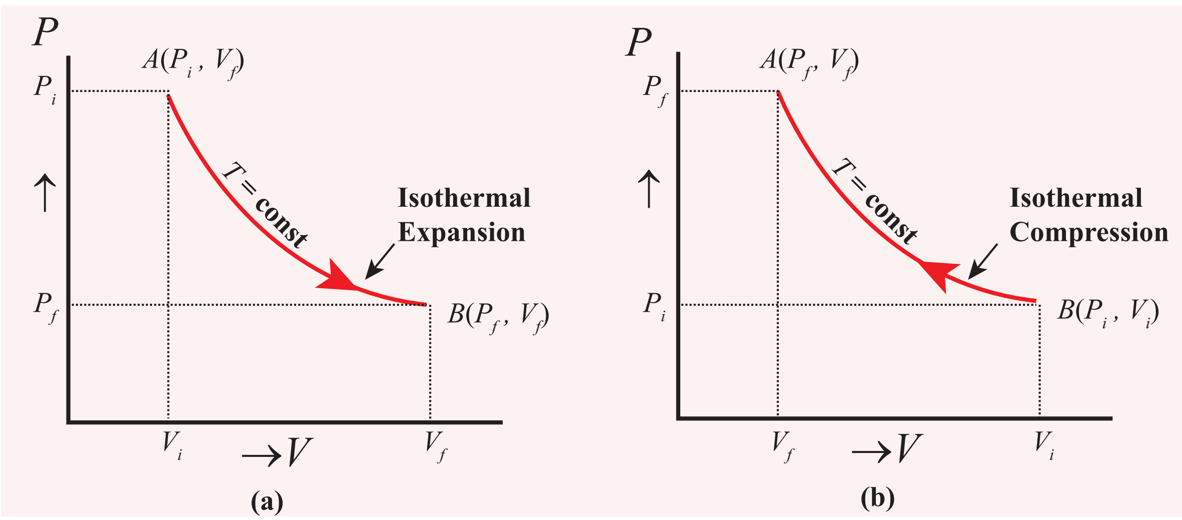


Figure 8.25 (a) Quasi-static isothermal expansion (b) Quasi-static isothermal compression

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Examples:

(i) When water is heated, at the boiling point, even when heat flows to water, the temperature will not increase unless the water completely evaporates. Similarly, at the freezing point, when the ice melts to water, the temperature of ice will not increase even when heat is supplied to ice.

(ii) All biological processes occur at constant body temperature (37°C).

Work done in an isothermal process:

Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state (P_i,V_i) to the final state (P_f, V_f). We can calculate the work done by the gas during this process. From equation (8.17) the work done by the gas,

$$W = \int_{V_i}^{V_f} PdV \qquad (8.31)$$

As the process occurs quasi-statically, at every stage the gas is at equilibrium with the surroundings. Since it is in equilibrium at every stage the ideal gas law is valid. Writing pressure in terms of volume and temperature,

$$P = \frac{\mu RT}{V} \qquad (8.32)$$

Substituting equation (8.32) in (8.31) we get

$$W = \int_{V_i}^{V_f} \frac{\mu RT}{V} dV$$

$$W = \mu RT \int_{V_i}^{V_f} \frac{dV}{V} \qquad (8.33)$$

In equation (8.33), we take μRT out of the integral, since it is constant throughout the isothermal process.

By performing the integration in equation (8.33), we get

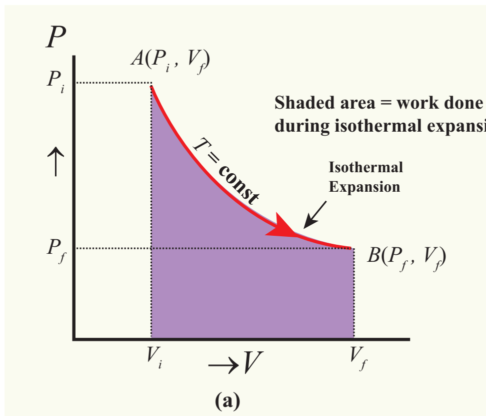
$$W = \mu RT \ln\left(\frac{V_f}{V_i}\right) \qquad (8.34)$$

Since we have an isothermal expansion, $\frac{V_f}{V_i} > 1$, so $ln\left(\frac{V_f}{V_i}\right) > 0$. As a result the work done by the gas during an isothermal expansion is positive.

The above result in equation (8.34) is true for isothermal compression also. But in an isothermal compression $\frac{V_f}{V_i} < 1$, so $ln\left(\frac{V_f}{V_i}\right) < 0$.

As a result the work done on the gas in an isothermal compression is negative.

In the PV diagram the work done during the isothermal expansion is equal to the area under the graph as shown in Figure 8.27



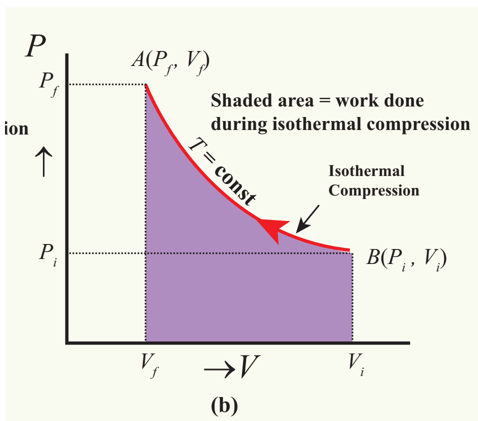


Figure 8.27 Work done in an isothermal process.

Similarly for an isothermal compression, the area under the PV graph is equal to the

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

work done on the gas which turns out to be the area with a negative sign.

Note To calculate the work done in an isothermal process, we assume that the process is quasi-static. If it is not quasi-static we cannot substitute $P = \frac{\mu RT}{V}$ in equation (8.31). It is because the ideal gas law is not valid for non equilibrium states. But equation (8.34) is valid even when the isothermal process is not quasi-static. This is because the state variables like pressure and volume depend on initial and final state alone of an ideal gas and do not depend on the way the final state is reached. The assumption of 'quasi-static' requires to do the integration.

EXAMPLE 8.16

A 0.5 mole of gas at temperature 300 K expands isothermally from an initial volume of 2 L to 6 L

(a) What is the work done by the gas?

(b) Estimate the heat added to the gas?

(c) What is the final pressure of the gas?

(The value of gas constant, R = 8.31 J mol^{-1} K^{-1})

Solution

(a) We know that work done by the gas in an isothermal expansion

Since μ = 0.5

$$W = 0.5mol \times \frac{8.31J}{mol.K} \times 300\ K \ In \left(\frac{6L}{2L}\right)$$

$$W = 1.369\ kJ$$

Note that W is positive since the work is done by the gas.

(b) From the First law of thermodynamics, in an isothermal process the heat supplied is spent to do work.

Therefore, $Q = W = 1.369\ kJ$. Thus Q is also positive which implies that heat flows in to the system.

(c) For an isothermal process

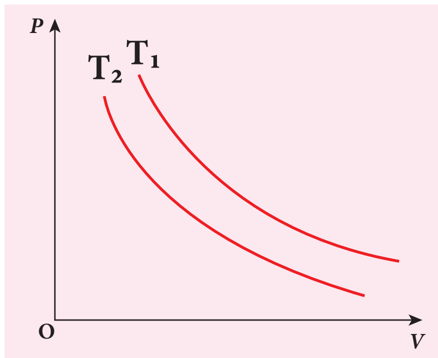
$$P_i V_i = P_f V_f = \mu RT$$

$$P_f = \frac{\mu RT}{V_f} = 0.5mol \times \frac{8.31J}{mol.K} \times \frac{300K}{6\times10^{-3}m^3}$$

$=207.75\ k\ Pa$

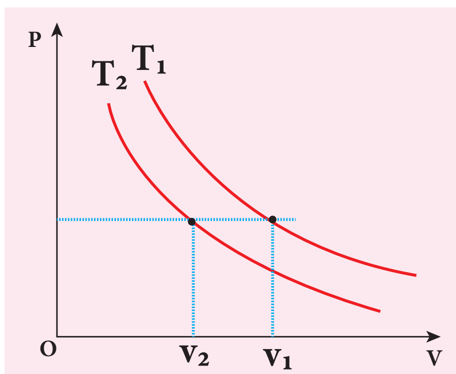
EXAMPLE 8.17

The following PV curve shows two isothermal processes for two different temperatures and. Identify the higher temperature of these two.



Solution

To determine the curve corresponding to higher temperature, draw a horizontal line parallel to x axis as shown in the figure. This is the constant pressure line. The volumes V_1 and V_2 belong to same pressure as the vertical lines from V_1 and V_2 meet the constant pressure line.



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

At constant pressure, higher the volume of the gas, higher will be the temperature. From the figure, as $V_1 > V_2$ we conclude $T_1 > T_2$. In general the isothermal curve closer to the origin, has lower temperature.

8.8.2 Adiabatic process

This is a process in which no heat flows into or out of the system (Q=0). But the gas can expand by spending its internal energy or gas can be compressed through some external work. So the pressure, volume and temperature of the system may change in an adiabatic process.

For an adiabatic process, the first law becomes $\Delta U = W$.

This implies that the work is done by the gas at the expense of internal energy or work is done on the system which increases its internal energy.

The adiabatic process can be achieved by the following methods

(i) Thermally insulating the system from surroundings so that no heat flows into or out of the system; for example, when thermally insulated cylinder of gas is compressed (adiabatic compression) or expanded (adiabatic expansion) as shown in the Figure 8.28

(ii) If the process occurs so quickly that there is no time to exchange heat with surroundings even though there is no thermal insulation. A few examples are shown in Figure 8.29.

Examples:

Figure 8.29 (a) When the tyre bursts the air expands so quickly that there is no time to exchange heat with the surroundings.

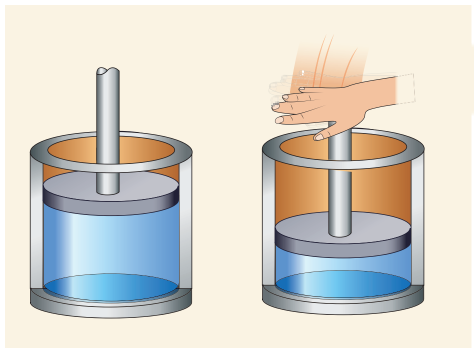


Figure 8.29 (b): When the gas is compressed or expanded so fast, the gas cannot exchange heat with surrounding even though there is no thermal insulation.

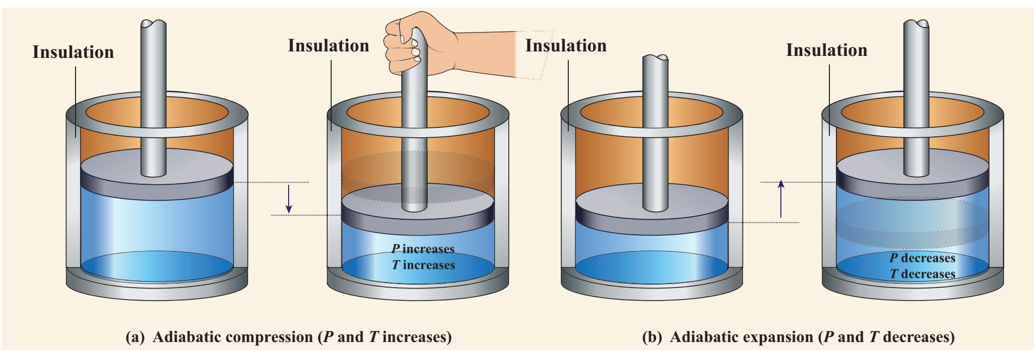


Figure 8.28 Adiabatic compression and expansion

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Figure 8.29 (c): When the warm air rises from the surface of the Earth, it adiabatically expands. As a result the water vapor cools and condenses into water droplets forming a cloud.

The equation of state for an adiabatic process is given by

$$PV^{\gamma} = constant \qquad (8.35)$$

Here γ is called adiabatic exponent ($\gamma = C_p/C_v$) which depends on the nature of the gas.
The equation (8.35) implies that if the gas goes from an equilibrium state (P_i, V_i) to another equilibrium state (P_f, V_f) adiabatically then it satisfies the relation

$$P_iV_i^{\gamma} = P_fV_f^{\gamma} \qquad (8.36)$$

The PV diagram of an adiabatic expansion and adiabatic compression process are shown in Figure 8.30. The PV diagram for an adiabatic process is also called *adiabat* Note that the PV diagram for isothermal (Figure 8.25) and adiabatic (Figure 8.30) processes look similar. But actually the adiabatic curve is steeper than isothermal curve.

We can also rewrite the equation (8.35) in terms of T and V. From ideal gas equation, the pressure $P = \frac{\mu RT}{V}$. Substituting this equation in the equation (8.35), we have

$$\frac{\mu RT}{V}V^{\gamma} = \text{constant (or) } \frac{T}{V}V^{\gamma} = \frac{constant}{\mu R}$$

Note here that is another constant. So it can be written as

$$TV^{\gamma-1} = constant. \qquad (8.37)$$

The equation (8.37) implies that if the gas goes from an initial equilibrium state (T_i, V_i) to final equilibrium state (T_f, V_f) adiabatically then it satisfies the relation

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1} \qquad (8.38)$$

The equation of state for adiabatic process can also be written in terms of T and P as

$$T^{\gamma}P^{1-\gamma} = constant. \qquad (8.39)$$

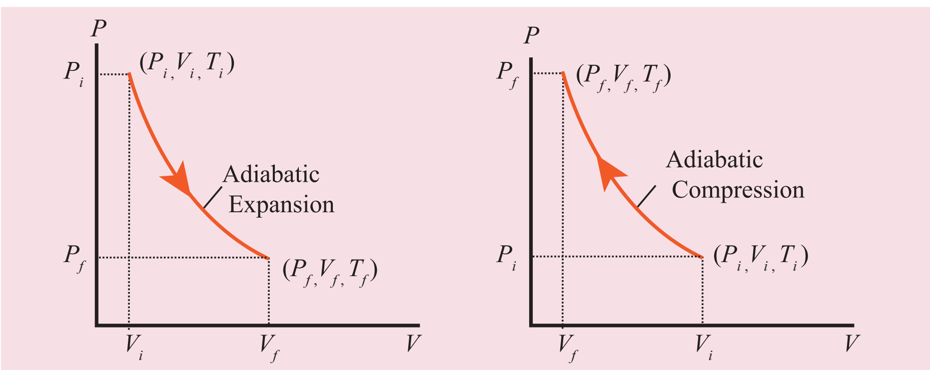


Figure 8.30 PV diagram for adiabatic expansion and adiabatic compression

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(The proof of equation (8.39) left as an exercise).

EXAMPLE 8.18

We often have the experience of pumping air into bicycle tyre using hand pump. Consider the air inside the pump as a thermodynamic system having volume V at atmospheric pressure and room temperature, 27°C. Assume that the nozzle of the tyre is blocked and you push the pump to a volume 1/4 of V. Calculate the final temperature of air in the pump? (For air , since the nozzle is blocked air will not flow into tyre and it can be treated as an adiabatic compression).

Solution

Here, the process is adiabatic compression. The volume is given and temperature is to be found. we can use the equation (8.38)

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}.$$

$$T_i = 300\,K \quad (273+27°C = 300\,K)$$

$$V_i = V \,\&\, V_f = \frac{V}{4}$$

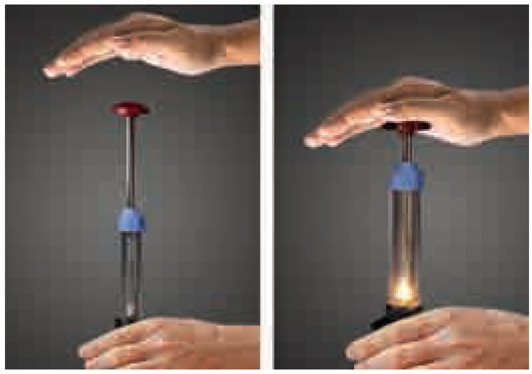
$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\,K \times 4^{1.4-1} = 300K \times 1.741$$

T2 ≈ 522 K or 249^{0}C

This temperature is higher than the boiling point of water. So it is very dangerous to touch the nozzle of blocked pump when you pump air.

DO YOU KNOW?

When the piston is compressed so quickly that there is no time to exchange heat to the surrounding, the temperature of the gas increases rapidly. This is shown in the figure. This principle is used in the diesel engine. The air-gasoline mixer is compressed so quickly (adiabatic compression) that the temperature increases enormously, which is enough to produce a spark.



Work done in an adiabatic process:

Consider μ moles of an ideal gas enclosed in a cylinder having perfectly non conducting walls and base. A frictionless and insulating piston of cross sectional area A is fitted in the cylinder as shown in Figure 8.31.

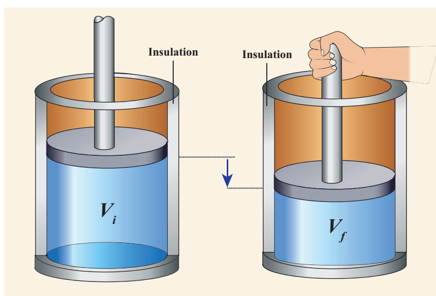


Figure 8.31 Work done in an adiabatic process

Let W be the work done when the system goes from the initial state (P_i, V_i, T_i) to the final state (P_f, V_f, T_f) adiabatically.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$W = \int_{V_i}^{V_f} PdV \qquad (8.40)$$

By assuming that the adiabatic process occurs quasi-statically, at every stage the ideal gas law is valid. Under this condition, the adiabatic equation of state is PV^{γ} = constant (or)

$P = \frac{\text{constant}}{V^{\gamma}}$ can be substituted in the equation (8.40), we get

$$\therefore W_{adia} = \int_{V_i}^{V_f} \frac{\text{constant}}{V^{\gamma}} dV$$

$$= \text{constant} \int_{V_i}^{V_f} V^{-\gamma} dV$$

$$= \text{constant} \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_i}^{V_f}$$

$$= \frac{\text{constant}}{1-\gamma} \left[\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right]$$

$$= \frac{1}{1-\gamma} \left[\frac{\text{constant}}{V_f^{\gamma-1}} - \frac{\text{constant}}{V_i^{\gamma-1}} \right]$$

But, $P_i V_i^{\gamma} = P_f V_f^{\gamma} = constant$.

$$\therefore W_{adia} = \frac{1}{1-\gamma} \left[\frac{P_f V_f^{\gamma}}{V_f^{\gamma-1}} - \frac{P_i V_i^{\gamma}}{V_i^{\gamma-1}} \right]$$

$$W_{adia} = \frac{1}{1-\gamma} [P_f V_f - P_i V_i] \qquad (8.41)$$

From ideal gas law,

$$P_f V_f = \mu R T_f \text{ and } P_i V_i = \mu R T_i$$

Substituting in equation (8.41), we get

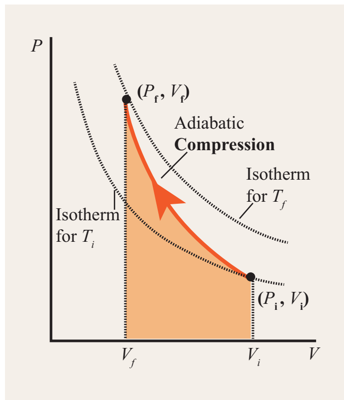
$$\therefore W_{adia} = \frac{\mu R}{\gamma - 1} [T_i - T_f] \qquad (8.42)$$

In adiabatic expansion, work is done by the gas. i.e., W_{adia} is positive. As $T_i > T_f$, the gas cools during adiabatic expansion.

In adiabatic compression, work is done on the gas. i.e., W_{adia} is negative. As $T_i < T_f$, the temperature of the gas increases during adiabatic compression.

Note Even though we have derived equations (8.41) and (8.42) by assuming that the adiabatic process is quasi-static, both the equations are valid even if the process is not quasi-static. This is because P and V are state variables and are independent of how the state is arrived.

In the adiabatic PV diagram shown in the Figure 8.32, the area under the adiabatic curve from initial state to final state will give the total work done in adiabatic process.



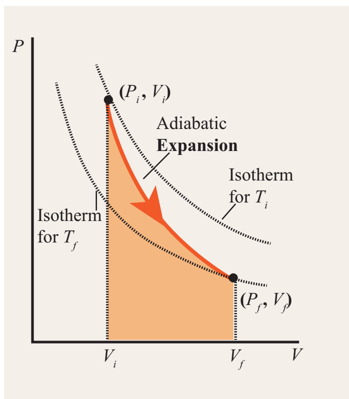


Figure 8.32 PV diagram -Work done in the adiabatic process

To differentiate between isothermal and adiabatic curves in (Figure 8.32) the adiabatic curve is drawn along with isothermal curve for T_f and T_i. Note that adiabatic curve is steeper than isothermal curve. This is because $\gamma > 1$ always.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

8.8.3 Isobaric process

This is a thermodynamic process that occurs at constant pressure. Even though pressure is constant in this process, temperature, volume and internal energy are not constant. From the ideal gas equation, we have

$$V = \left(\frac{\mu R}{P}\right) T \qquad (8.43)$$

$$\text{Here } \frac{\mu R}{P} = constant$$

In an isobaric process the temperature is directly proportional to volume.

$$V \propto T \quad \text{(Isobaric process)} \qquad (8.44)$$

This implies that for a isobaric process, the V-T graph is a straight line passing through the origin.

If a gas goes from a state (V_i, T_i) to (V_f, T_f) at constant pressure, then the system satisfies the following equation

$$\frac{T_f}{V_f} = \frac{T_i}{V_i} \qquad (8.45)$$

Examples for Isobaric process:

(i) When the gas is heated and pushes the piston so that it exerts a force equivalent to atmospheric pressure plus the force due to gravity then this process is isobaric. This is shown in Figure 8.33

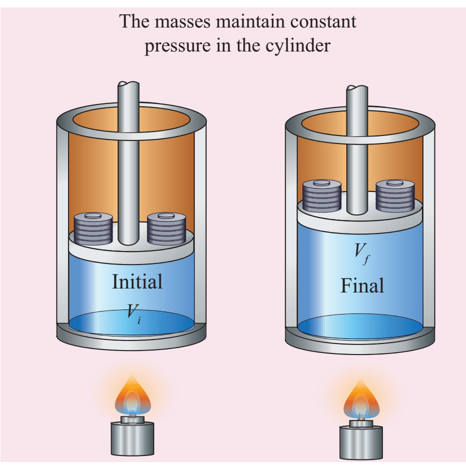


Figure 8.33 Isobaric process

(ii) Most of the cooking processes in our kitchen are isobaric processes. When the food is cooked in an open vessel, the pressure above the food is always at atmospheric pressure. This is shown in Figure 8.34

Figure 8.34 Isobaric process

The PV diagram for an isobaric process is a horizontal line parallel to volume axis as shown in Figure 8.35.

Figure 8.35 (a) represents isobaric process where volume decreases

Figure 8.35 (b) represents isobaric process where volume increases

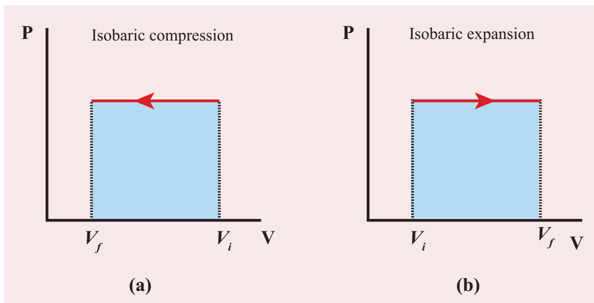


Figure 8.35 PV diagram for an isobaric process

The work done in an isobaric process:

Work done by the gas

$$W = \int_{V_i}^{V_f} P dV \qquad (8.46)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

In an isobaric process, the pressure is constant, so P comes out of the integral,

$$W = P\int_{V_i}^{V_f} dV \qquad (8.47)$$

$$W = P[V_f - V_i] = P\Delta V \qquad (8.48)$$

Where ΔV denotes change in the volume. If ΔV is negative, W is also negative. This implies that the work is done on the gas. If ΔV is positive, W is also positive, implying that work is done by the gas.

The equation (8.48) can also be rewritten using the ideal gas equation.

From ideal gas equation

$$PV = \mu RT \text{ and } V = \frac{\mu RT}{P}$$

Substituting this in equation (8.48) we get

$$W = \mu RT_f\left(1 - \frac{T_i}{T_f}\right) \qquad (8.49)$$

In the PV diagram, area under the isobaric curve is equal to the work done in isobaric process. The shaded area in the following Figure8.36 is equal to the work done by the gas.

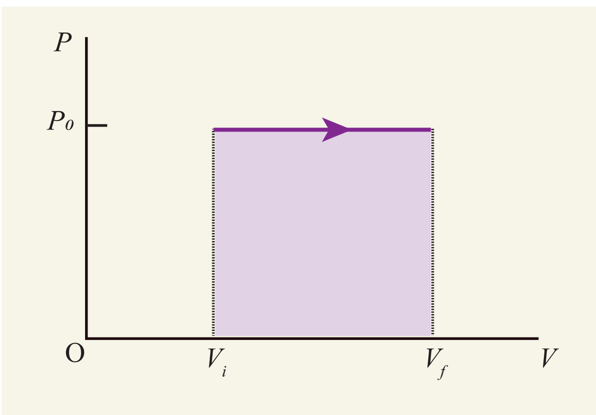


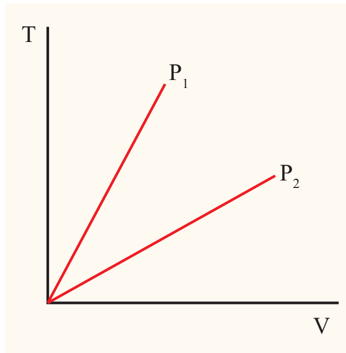
Figure 8.36 Work done in an isobaric process

The first law of thermodynamics for isobaric process is given by

$$\Delta U = Q - P\Delta V \qquad (8.50)$$

EXAMPLE 8.19

The following graph shows a V-T graph for isobaric processes at two different pressures. Identify which one occurs at higher pressure.



Solution

From the ideal gas equation, $V = \left(\frac{\mu R}{P}\right)T$

V-T graph is a straight line passing the origin.

The slope $= \frac{\mu R}{P}$

The slope of V-T graph is inversely proportional to the pressure. If the slope is greater, lower is the pressure.

Here P_1 has larger slope than P_2. So $P_2 > P_1$.

Suppose the graph is drawn between T and V (Temperature along the x-axis and Volume along the y-axis) then will we still have $P_2 > P_1$?

EXAMPLE 8.20

One mole of an ideal gas initially kept in a cylinder at pressure 1 MPa and temperature 27°C is made to expand until its volume is doubled.

(a) How much work is done if the expansion is (i) adiabatic (ii) isobaric (iii) isothermal?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(b) Identify the processes in which change in internal energy is least and is maximum.

(c) Show each process on a PV diagram.

(d) Name the processes in which the heat transfer is maximum and minimum.

(Take $\gamma = \frac{5}{3}$ and R=8.3 J $mol^{-1} K^{-1}$)

Solution

(a) (i) In an adiabatic process the work done by the system is

$$W_{adia} = \frac{\mu R}{\gamma - 1}[T_i - T_f]$$

To find the final temperature T_f, we can use adiabatic equation of state.

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300 \times \left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$= 0.63 \times 300K = 189.8K$$

$$W = 1 \times 8.3 \times \frac{3}{2}(300 - 189.8) = 1.37kJ$$

(ii) In an isobaric process the work done by the system

$$W = P\Delta V = P(V_f - V_i)$$

and $V_f = 2V_i$ so $W = 2PV_i$

To find V_i, we can use the ideal gas law for initial state. $P_i V_i = RT_i$

$$V_i = \frac{RT_i}{P_i} = 8.3 \times \frac{300}{1} \times 10^{-6} = 24.9\times10^{-4} m^3$$

The work done during isobaric process, $W = 2 \times 10^6 \times 24.9 \times 10^{-4} = 4.9\ kJ$

(iii) In an isothermal process the work done by the system,

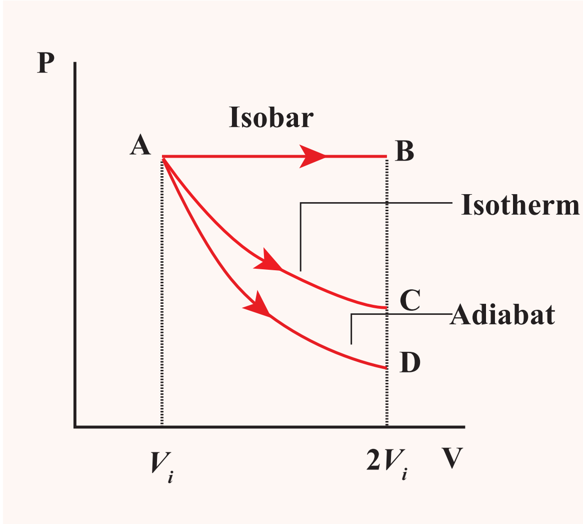
$$W = \mu RT\ ln\left(\frac{V_f}{V_i}\right)$$

In an isothermal process the initial room temperature is constant.

$$W = 1 \times 8.3 \times 300 \times ln(2) = 1.7kJ$$

(b) Comparing all three processes, we see that the work done in the isobaric process is the greatest, and work done in the adiabatic process is the least.

(c) The PV diagram is shown in the Figure.



The area under the curve AB = Work done during the isobaric process

The area under the curve AC = Work done during the isothermal process

The area under the curve AD= Work done during the adiabatic process

From the PV diagram the area under the curve AB is more, implying that the work done in isobaric process is highest and work done in adiabatic process is least.

(d) In an adiabatic process no heat enters into the system or leaves from the system. In an isobaric process the work done is more so heat supplied should be more compared to an isothermal process.

8.8.4 Isochoric process

This is a thermodynamic process in which the volume of the system is kept constant. But pressure, temperature and internal energy continue to be variables.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The pressure - volume graph for an isochoric process is a vertical line parallel to pressure axis as shown in Figure 8.37.

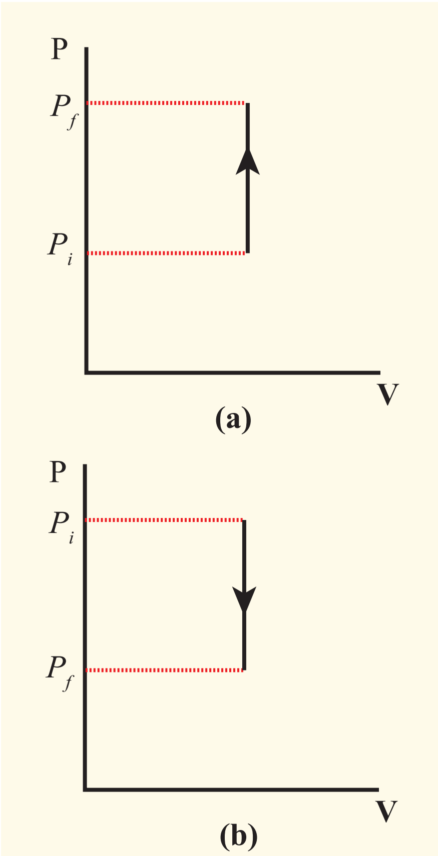


Figure 8.37 Isochoric process with (a) increased pressure and (b) decreased pressure

The equation of state for an isochoric process is given by

$$P = \left(\frac{\mu R}{V}\right) T \qquad (8.51)$$

where $\left(\frac{\mu R}{V}\right)$ = constant

We can infer that the pressure is directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin.

If a gas goes from state (P_i, T_i) to (P_f, T_f) at constant volume, then the system satisfies the following equation

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \qquad (8.52)$$

For an isochoric processes, ΔV=0 and W=0. Then the first law becomes

$$\Delta U = Q \qquad (8.53)$$

Implying that the heat supplied is used to increase only the internal energy. As a result the temperature increases and pressure also increases. This is shown in Figure 8.38

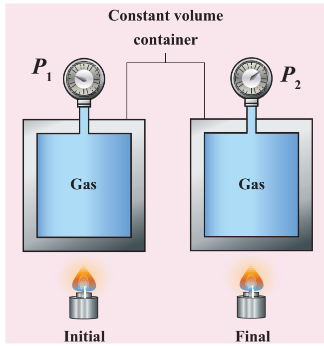


Figure 8.38 Isochoric process

Suppose a system loses heat to the surroundings through conducting walls by keeping the volume constant, then its internal energy decreases. As a result the temperature decreases; the pressure also decreases.

Examples:

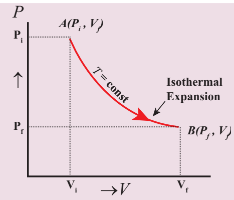
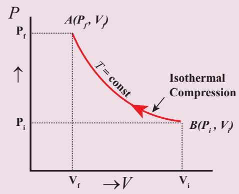
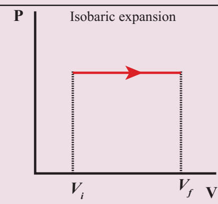
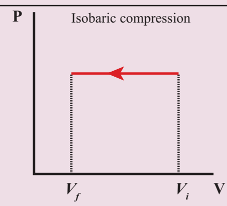
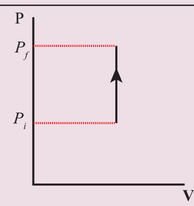
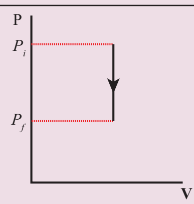
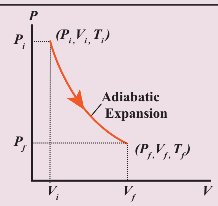
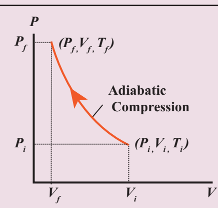
1. When food is cooked by closing with a lid as shown in figure.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Table 8.4: Summary of various thermodynamic processes

S. No.	Process		Heat Q	Temperature & internal energy	Pressure
1	Isothermal	Expansion	$Q > 0$	Constant	decreases
		Compression	$Q < 0$	Constant	Increases
2	Isobaric	Expansion	$Q > 0$	increases	Constant
		Compression	$Q < 0$	decreases	Constant
3	Isochoric		$Q > 0$	Increases	increases
			$Q < 0$	Decreases	decreases
4	Adiabatic	expansion	$Q = 0$	Decreases	Decreases
		Compression	$Q = 0$	Increases	increases

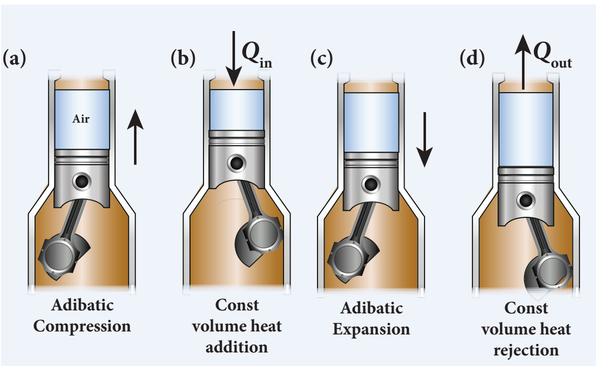
athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

	Volume	Equation of state	Work done (ideal gas)	Indicator diagram (PV diagram)
	increases	PV = Constant	$W=\mu RT \ln\left(\frac{V_f}{V_i}\right) > 0$	
	Decreases		$W=\mu RT \ln\left(\frac{V_f}{V_i}\right) < 0$	
	increases	$\frac{V}{T}$ = Constant	$W=P\,[V_f-V_i]=P\Delta V > 0$	
	decreases		$W=P\,[V_f-V_i]=P\Delta V < 0$	
	Constant	$\frac{P}{T}$ = Constant	Zero	
				
	increases	PV^{γ} = Constant	$W=\frac{\mu R}{\gamma-1}(T_i-T_f) > 0$	
	Decreases		$W=\frac{\mu R}{\gamma-1}(T_i-T_f) < 0$	

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

When food is being cooked in this closed position, after a certain time you can observe the lid is being pushed upwards by the water steam. This is because when the lid is closed, the volume is kept constant. As the heat continuously supplied, the pressure increases and water steam tries to push the lid upwards.

2. In automobiles the petrol engine undergoes four processes. First the piston is adiabatically compressed to some volume as shown in the Figure (a). In the second process (Figure (b)), the volume of the air-fuel mixture is kept constant and heat is being added. As a result the temperature and pressure are increased. This is an isochoric process. For a third stroke (Figure (c)) there will be an adiabatic expansion, and fourth stroke again isochoric process by keeping the piston immoveable (Figure (d)).



The summary of various thermodynamic processes is give in the Table 8.4.

EXAMPLE 8.21

500 g of water is heated from 30°C to 60°C. Ignoring the slight expansion of water, calculate the change in internal energy of the water? (specific heat of water 4184 J/kg.K)

Solution

When the water is heated from 30°C to 60°C,there is only a slight change in its volume. So we can treat this process as isochoric. In an isochoric process the work done by the system is zero. The given heat supplied is used to increase only the internal energy.

$$\Delta U = Q = ms_v \Delta T$$

The mass of water = 500 g =0.5 kg

The change in temperature = 30K

The heat Q = 0.5×4184×30 = 62.76 *kJ*

8.8.5 Cyclic processes

This is a thermodynamic process in which the thermodynamic system returns to its initial state after undergoing a series of changes. Since the system comes back to the initial state, the change in the internal energy is zero. In cyclic process, heat can flow in to system and heat flow out of the system. From the first law of thermodynamics, the net heat transferred to the system is equal to work done by the gas.

$$Q_{net} = Q_{in} - Q_{out} = W \quad \text{(for a cyclic process)} \tag{8.54}$$

8.8.6 PV diagram for a cyclic process

In the PV diagram the cyclic process is represented by a closed curve.

Let the gas undergo a cyclic process in which it returns to the initial stage after an expansion and compression as shown in Figure 8.39

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

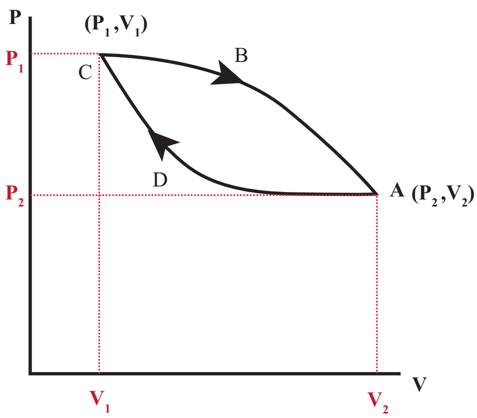


Figure 8.39 PV diagram for cyclic process

Let W_1 be the work done by the gas during expansion from volume V_1 to volume V_2. It is equal to area under the graph CBA as shown in Figure 8.40 (a) .

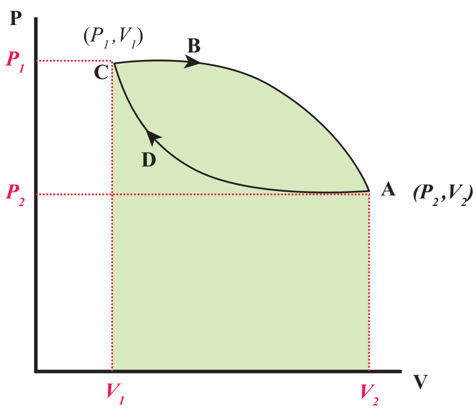


Figure 8.40 (a) W for path CBA

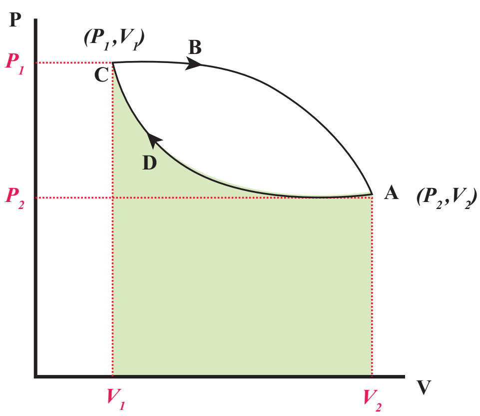


Figure 8.40 (b) W for path ADC

Let W_2 be the work done on the gas during compression from volume V_2 to volume V_1. It is equal to the area under the graph ADC as shown in Figure 8.40 (b)

The total work done in this cyclic process $= W_1 - W_2 =$ Green shaded area inside the loop, as shown in Figure 8.41.

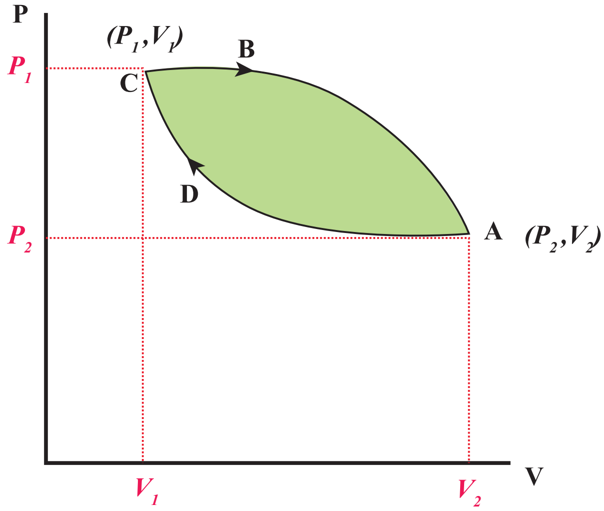


Figure 8.41 Net work done in a cyclic process

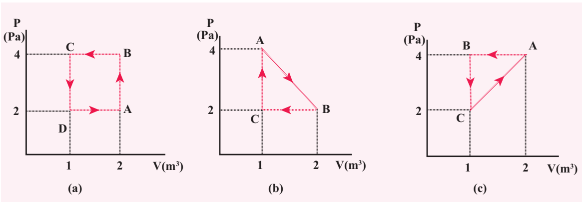
Thus the net work done during the cyclic process shown above is not zero. In general the net work done can be positive or negative. If the net work done is positive, then work done by the system is greater than the work done on the system. If the net work done is negative then the work done by the system is less than the work done on the system.

Note Further, in a cyclic process the net work done is positive if the process goes clockwise and net work done is negative if the process goes anti-clockwise. In Figure 8.41the process goes clockwise.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 8.22

The PV diagrams for a thermodynamical system is given in the figure below. Calculate the total work done in each of the cyclic processes shown.



Solution

In the case (a) the closed curve is anticlockwise. So the net work done is negative, implying that the work done on the system is greater than the work done by the system. The area under the curve BC will give work done on the gas (isobaric compression) and area under the curve DA (work done by the system) will give the total work done by the system.

Area under the curve BC = Area of rectangle BC12 = $1 \times 4 = -4J$

Area under the curve DA = $1 \times 2 = +2J$

Net work done in cyclic process $= -4 + 2 = -2$ J

In the case (b) the closed curve is clockwise. So the net work done is positive, implying that the work done on the system is less than the work done by the system. Area under the curve BC will give work done on the gas (isobaric compression) and area under the curve AB will give the total work done by the system.

Area under the curve AB = rectangle area+ triangle area = $(1\times2) + \frac{1}{2} \times 1\times2 = +3J$

Area under the curve BC = rectangle area $= 1 \times 2 = -2J$

Network done in the cyclic process = 1 J, which is positive.

In the case (c) the closed curve is anticlockwise. So the net work done is negative, implying that the work done on the system is greater than work done by the system. The area under the curve AB will give the work done on the gas (isobaric compression) and area under the curve CA (work done by the system) will give the total work done by the system.

The area under the curve AB =Rectangle of area = $4 \times 1 = -4J$

The area under the curve CA = Rectangle area + triangle area = $(1\times2) + \frac{1}{2} \times 1\times2 = +3J$

The total work in the cyclic process = -1 J. It is negative

8.8.7 Limitations of first law of thermodynamics

The first law of thermodynamics explains well the inter convertibility of heat and work. But it does not indicate the direction of change.

For example,

a. When a hot object is in contact with a cold object, heat always flows from the hot object to cold object but not in the reverse direction. According to first law, it is possible for the energy to flow from hot object to cold object or from cold object to hot object. But in nature the direction of heat flow is always from higher temperature to lower temperature.
b. When brakes are applied, a car stops due to friction and the work done against friction is converted into heat. But this

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

heat is not reconverted to the kinetic energy of the car.

So the first law is not sufficient to explain many of natural phenomena.

8.8.8 Reversible process

A thermodynamic process can be considered reversible only if it possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states as in the initial,direct process.

Example: A quasi–static isothermal expansion of gas, slow compression and expansion of a spring.

Conditions for reversible process:

1. The process should proceed at an extremely slow rate.
2. The system should remain in mechanical, thermal and chemical equilibrium state at all the times with the surroundings, during the process.
3. No dissipative forces such as friction, viscosity, electrical resistance should be present.

Note All reversible processes are quasi-static but all quasi-static processes need not be reversible. For example when we push the piston very slowly, if there is friction between cylinder wall and piston some amount of energy is lost to surroundings, which cannot be retrieved back.

Irreversible process:

All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure, temperature at every stage of the process.

The first law of thermodynamics is the statement about conservation of energy in a thermodynamic process. For example, if a hotter object is placed on a colder object, heat flows from hotter to colder object. Why does heat not flow from the colder object to hotter object? Even if energy flows from colder object to hotter object, the first law of thermodynamics is not violated. For example, if 5 J of heat flows form hotter to colder or from colder to hotter objects the total internal energy of this combined system remains the same. But 5 J of heat never flows from the colder object to hotter object. In nature all such process occur only in one direction but not in the reverse direction, even if the energy is conserved in both the processes. Thus the first law of thermodynamics has no explanation for this irreversibility. When the scientists of the eighteenth century tried to explain this irreversibility, they discovered a new law of nature. This is called the second law of thermodynamics. According to second law of thermodynamics

"Heat always flows from hotter object to colder object spontaneously". This is known as the Clausius form of second law of thermodynamics.

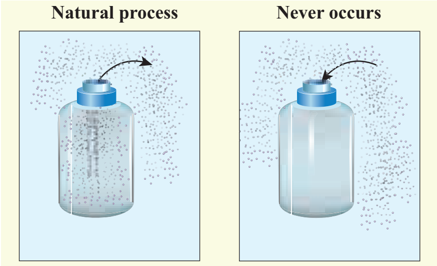
EXAMPLE 8.23

Give some examples of irreversible processes.

All naturally occuring processes are irreversible. Here we give some interesting examples.

(a) When we open a gas bottle, the gas molecules slowly spread into the entire room. These gas molecules can never get back in to the bottle.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181



(b) Suppose one drop of an ink is dropped in water, the ink droplet slowly spreads in the water. It is impossible to get the ink droplet back.

(c) When an object falls from some height, as soon as it hits the earth it comes to rest. All the kinetic energy of the object is converted to kinetic energy of molecules of the earth surface, molecules of the object and small amount goes as sound energy. The spreaded kinetic energy to the molecules never collected back and object never goes up by itself.

Note that according to first law of thermodynamics all the above processes are possible in both directions. But second law of thermodynamics forbids The processes to occur in the reverse direction. The second law of thermodynamics is one of the very important laws of nature. It controls the way the natural processes occur.

8.9 HEAT ENGINE

In the modern technological world, the role of automobile engines plays a vital role in for transportation. In motor bikes and cars there are engines which take in petrol or diesel as input, and do work by rotating wheels. Most of these automobile engines have efficiency not greater than 40%. The second law of thermodynamics puts a fundamental restriction on efficiency of engines. Therefore understanding heat engines is very important.

Reservoir:

It is defined as a thermodynamic system which has very large heat capacity. By taking in heat from reservoir or giving heat to reservoir, the reservoir's temperature does not change.

Example: Pouring a tumbler of hot water in to lake will not increase the temperature of the lake. Here the lake can be treated as a reservoir.

When a hot cup of coffee attains equilibrium with the open atmosphere, the temperature of the atmosphere will not appreciably change. The atmosphere can be taken as a reservoir.

We can define heat engine as follows.

Heat engine is a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.

A heat engine has three parts:

(a) Hot reservoir
(b) Working substance
(c) Cold reservoir

A Schematic diagram for heat engine is given below in the figure 8.42.

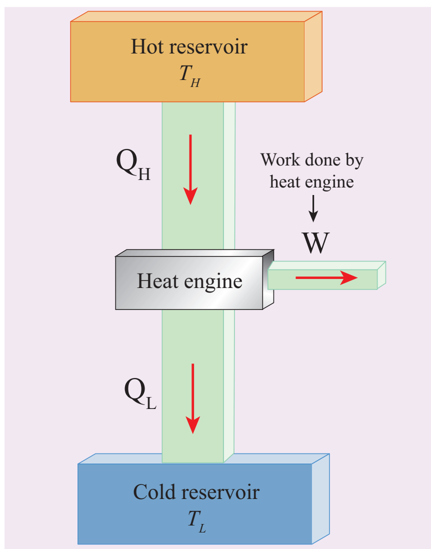


Figure 8.42 Heat Engine

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

1. Hot reservoir (or) Source: It supplies heat to the engine. It is always maintained at a high temperature T_H
2. Working substance: It is a substance like gas or water, which converts the heat supplied into work.

   A simple example of a heat engine is a steam engine. In olden days steam engines were used to drive trains. The working substance in these is water which absorbs heat from the burning of coal. The heat converts the water into steam. This steam is does work by rotating the wheels of the train, thus making the train move.
3. Cold reservoir (or) Sink: The heat engine ejects some amount of heat (Q_L) in to cold reservoir after it doing work. It is always maintained at a low temperature T_L.

   For example, in the automobile engine, the cold reservoir is the surroundings at room temperature. The automobile ejects heat to these surroundings through a silencer.

The heat engine works in a cyclic process. After a cyclic process it returns to the same state. Since the heat engine returns to the same state after it ejects heat, the change in the internal energy of the heat engine is zero.

The efficiency of the heat engine is defined as the ratio of the work done (out put) to the heat absorbed (input) in one cylic process.

Let the working substance absorb heat Q_H units from the source and reject Q_L units to the sink after doing work W units, as shown in the Figure 8.43.

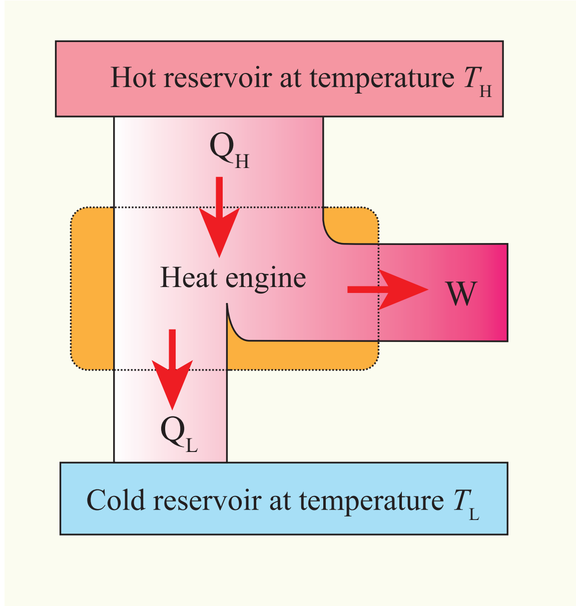


Figure 8.43 Heat engine

We can write

Input heat = Work done + ejected heat

$$Q_H = W + Q_L$$

$$W = Q_H - Q_L$$

Then the efficiency of heat engine

$$\eta = \frac{output}{input} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$\eta = \frac{output}{input} = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H} \qquad (8.55)$$

Note here that Q_H, Q_L and W all are taken as positive, a sign convention followed in this expression.

Since $Q_L < Q_H$, the efficiency (η) always less than 1. This implies that heat absorbed is not completely converted into work. The second law of thermodynamics placed fundamental restrictions on converting heat completely into work.

We can state the heat engine statement of second law of thermodynamics. This is also called Kelvin-Planck's statement.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Kelvin-Planck statement:

It is impossible to construct a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work. This implies that no heat engine in the universe can have 100% efficiency.

Note According to first law of thermodynamics, in an isothermal process the given heat is completely converted into work ($Q = W$). Is it a violation of the second law of thermodynamics? No. For non-cyclic process like an isothermal expansion, the heat can be completely converted into work. But Second law of thermodynamics implies that 'In a cyclic process only a portion of the heat absorbed is converted into work'. All heat engines operate in a cyclic process.

EXAMPLE 8.24

During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine?

Solution

The efficiency of heat engine is given by

$$\eta = 1 - \frac{Q_L}{Q_H}$$

$$\eta = 1 - \frac{300}{500} = 1 - \frac{3}{5}$$

$$\eta = 1 - 0.6 = 0.4$$

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

8.9.1 Carnot's ideal heat engine

In the previous section we have seen that the heat engine cannot have 100% efficiency. What is the maximum possible efficiency can a heat engine have?. In the year 1824 a young French engineer Sadi Carnot proved that a certain reversible engine operated in cycle between hot and cold reservoir can have maximum efficiency. This engine is called Carnot engine.

A reversible heat engine operating in a cycle between two temperatures in a particular way is called a Carnot Engine.

The carnot engine has four parts which are given below.

i Source: It is the source of heat maintained at constant high temperature T_H. Any amount of heat can be extracted from it, without changing its temperature.

ii Sink: It is a cold body maintained at a constant low temperature T_L. It can absorb any amount of heat.

iii Insulating stand: It is made of perfectly non-conducting material. Heat is not conducted through this stand.

iv Working substance: It is an ideal gas enclosed in a cylinder with perfectly non-conducting walls and perfectly conducting bottom. A non-conducting and frictionless piston is fitted in it.

The four parts are shown in the following Figure 8.44

BPQKBR

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

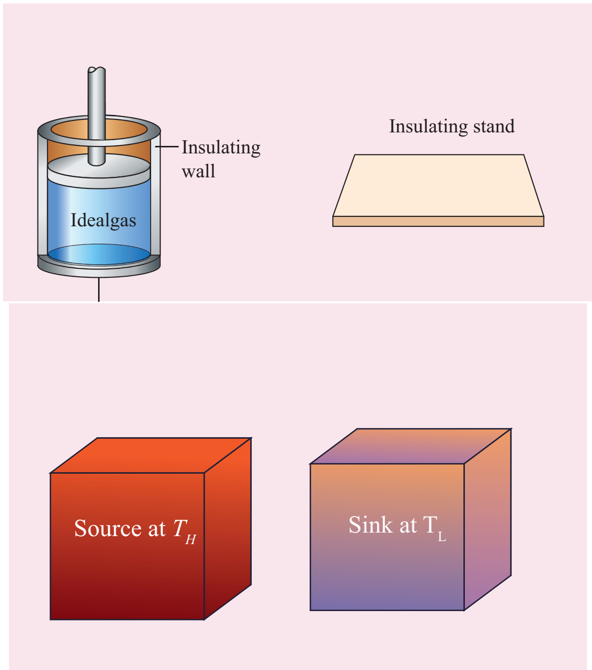


Figure 8.44 Carnot engine

Carnot's cycle:

The working substance is subjected to four successive reversible processes forming what is called Carnot's cycle.

Let the initial pressure, volume of the working substance be P_1,V_1.

Step A to B: Quasi-static isothermal expansion from (P_1,V_1,T_H) to (P_2,V_2,T_H):

The cylinder is placed on the source. The heat (Q_H) flows from source to the working substance (ideal gas) through the bottom of the cylinder. Since the process is isothermal, the internal energy of the working substance will not change. The input heat increases the volume of the gas. The piston is allowed to move out very slowly(quasi-statically). It is shown in the figure 8.47(a).

W_1 is the work done by the gas in expanding from volume V_1 to volume V_2 with a decrease of pressure from P_1 to P_2. This is represented by the P-V diagram along the path AB as shown in the Figure 8.45.

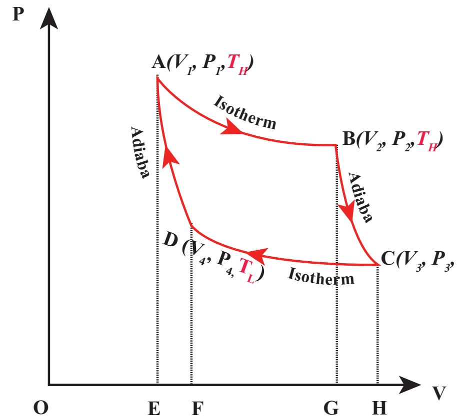


Figure 8.45 PV diagram for Carnot cycle

Then the work done by the gas (working substance) is given by

$$\therefore Q_H = W_{A\to B} = \int_{V_1}^{V_2} PdV$$

Since the process occurs quasi-statically, the gas is in equilibrium with the source till it reaches the final state. The work done in the isothermal expansion is given by the equation (8.34)

$$W_{A\to B} = \mu RT_H \ln\left(\frac{V_2}{V_1}\right) = \text{Area under the curve AB} \quad (8.56)$$

This is shown in Figure 8.46 (a)

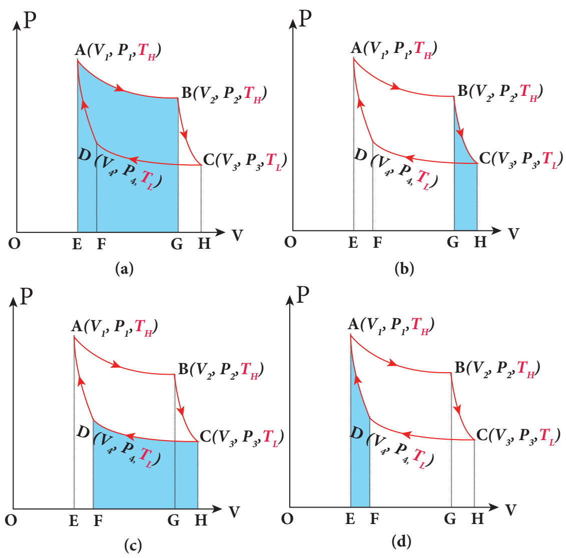


Figure 8.46 Work done in Carnot cycle

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

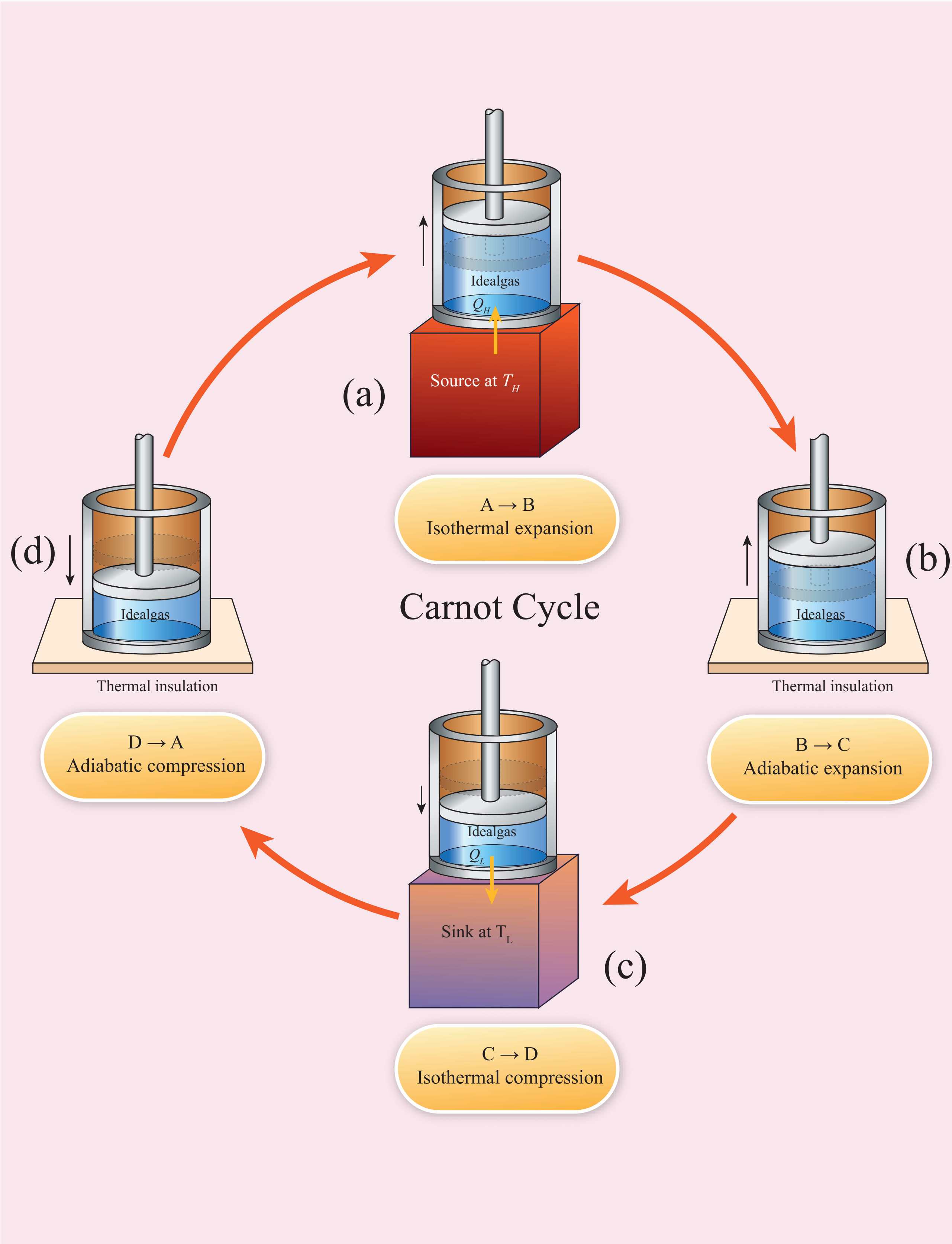


Figure 8.47 Carnot cycle

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Step B to C: Quasi-static adiabatic expansion from (P_2,V_2,T_H) to (P_3,V_3,T_L)

The cylinder is placed on the insulating stand and the piston is allowed to move out. As the gas expands adiabatically from volume V_2 to volume V_3 the pressure falls from P_2 to P_3. The temperature falls to T_L. This adiabatic expansion is represented by curve *BC* in the P-V diagram. This adiabatic process also occurs quasi-statically and implying that this process is reversible and the ideal gas is in equilibrium throughout the process. It is shown in the figure 8.47(b).

From the equation (8.42)

The work done by the gas in an adiabatic expansion is given by,

$$W_{B\to C} = \int_{V_2}^{V_3} PdV = \frac{\mu R}{\gamma - 1}[T_H - T_L] =$$

Area under the curve BC (8.57)

This is shown in Figure 8.46 (b)

Step C→D: Quasi-static isothermal compression from (P_3,V_3,T_L) to (P_4,V_4,T_L): It is shown in the figure 8.47(c)

The cylinder is placed on the sink and the gas is isothermally compressed until the pressure and volume become P_4 and V_4 respectively. This is represented by the curve *CD* in the PV diagram as shown in Figure 8.45. Let $W_{C\to D}$ be the work done on the gas. According to first law of thermodynamics

$$\therefore W_{C\to D} = \int_{V_3}^{V_4} PdV = \mu R T_L \ln\left(\frac{V_4}{V_3}\right) = -\mu R T_L \ln\left(\frac{V_3}{V_4}\right)$$

. = - Area under the curve CD (8.58)

This is shown in Figure 8.46 (c)

Here V_3 is greater than V_4. So the work done is negative, implying work is done on the gas.

Step D→A: Quasi-static adiabatic compression from (P_4,V_4,T_L) to (P_1,V_1,T_H): It is shown in the figure 8.47(d)

The cylinder is placed on the insulating stand again and the gas is compressed adiabatically till it attains the initial pressure P_1, volume V_1 and temperature T_H. This is shown by the curve *DA* in the P-V diagram.

$$\therefore W_{D\to A} = \int_{V_4}^{V_1} PdV = \frac{\mu R}{\gamma - 1}(T_L - T_H) = -\text{ Area}$$

under the curve DA (8.59)

In the adiabatic compression also work is done on the gas so it is negative, as is shown in Figure 8.46 (d)

Let 'W' be the net work done by the working substance in one cycle

∴ W=Work done by the gas – work done on the gas

$$= W_{A\to B} + W_{B\to C} - W_{C\to D} - W_{D\to A}$$

$$\text{since } W_{B\to C} = W_{D\to A}$$

$$= W_{A\to B} - W_{C\to D}$$

The net work done by the Carnot engine in one cycle $W = W_{A\to B} - W_{C\to D}$ (8.60)

Equation (8.60) shows that the net work done by the working substance in one cycle is equal to the area (enclosed by ABCD) of the P-V diagram (Figure 8.48)

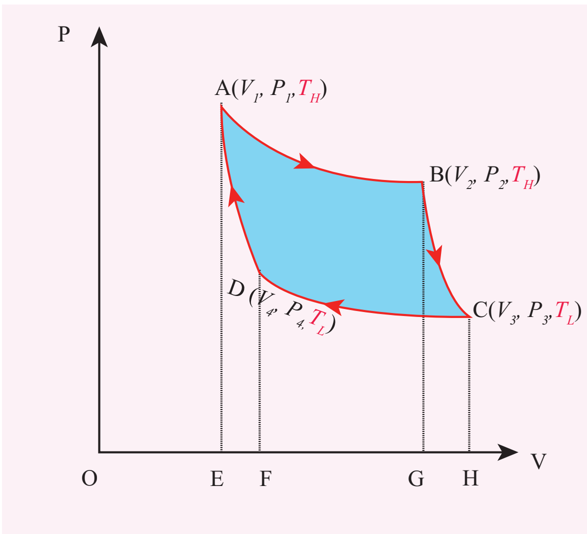


Figure 8.48 Net work done in Carnot cycle

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

It is very important to note that after one cycle the working substance returns to the initial temperature T_H. This implies that the change in internal energy of the working substance after one cycle is zero.

8.9.2 Efficiency of a Carnot engine

Efficiency is defined as the ratio of work done by the working substance in one cycle to the amount of heat extracted from the source.

$$\eta = \frac{work\,done}{Heat\,extracted} = \frac{W}{Q_H} \qquad (8.61)$$

From the first law of thermodynamics, $W = Q_H - Q_L$

$$\therefore \eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \qquad (8.62)$$

Applying isothermal conditions, we get,

$$Q_H = \mu R T_H \ln\left(\frac{V_2}{V_1}\right)$$
$$Q_L = \mu R T_L \ln\left(\frac{V_3}{V_4}\right) \qquad (8.63)$$

Here we omit the negative sign. Since we are interested in only the amount of heat (Q_L) ejected into the sink, we have

$$\therefore \frac{Q_L}{Q_H} = \frac{T_L \ln\left(\frac{V_3}{V_4}\right)}{T_H \ln\left(\frac{V_2}{V_1}\right)} \qquad (8.64)$$

By applying adiabatic conditions, we get,

$$T_H V_2^{\gamma-1} = T_L V_3^{\gamma-1}$$
$$T_H V_1^{\gamma-1} = T_L V_4^{\gamma-1}$$

By dividing the above two equations, we get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

Which implies that $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ (8.65)

Substituting equation (8.65) in (8.64), we get

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad (8.66)$$

$$\therefore \text{The efficiency } \eta = 1 - \frac{T_L}{T_H} \qquad (8.67)$$

Note : T_L and T_H should be expressed in Kelvin scale.

Important results:

1. η is always less than 1 because T_L is less than T_H. This implies the efficiency cannot be 100%. It can be 1 or 100% only when T_L = *0K* (absolute zero of temperature) which is impossible to attain practically.
2. The efficiency of the Carnot's engine is independent of the working substance. It depends only on the temperatures of the source and the sink. The greater the difference between the two temperatures, higher the efficiency.
3. When $T_H = T_L$ the efficiency η =0. No engine can work having source and sink at the same temperature.
4. The entire process is reversible in the Carnot engine cycle. So Carnot engine is itself a reversible engine and has maximum efficiency. But all practical heat engines like diesel engine, petrol engine and steam engine have cycles which are not perfectly reversible. So their efficiency is always less than the Carnot efficiency. This can be stated in the form of the Carnot theorem. It is stated as follows '*Between two constant temperatures reservoirs, only Carnot engine can have maximum efficiency. All real heat engines will have efficiency less than the Carnot engine*'

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 8.25

(a) A steam engine boiler is maintained at 250°C and water is converted into steam. This steam is used to do work and heat is ejected to the surrounding air at temperature 300K. Calculate the maximum efficiency it can have?

Solution

The steam engine is not a Carnot engine, because all the process involved in the steam engine are not perfectly reversible. But we can calculate the maximum possible efficiency of the steam engine by considering it as a Carnot engine.

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300\,K}{523\,K} = 0.43$$

The steam engine can have maximum possible 43% of efficiency, implying this steam engine can convert 43% of input heat into useful work and remaining 57% is ejected as heat. In practice the efficiency is even less than 43%.

EXAMPLE 8.26

There are two Carnot engines A and B operating in two different temperature regions. For Engine A the temperatures of the two reservoirs are 150°C and 100°C. For engine B the temperatures of the reservoirs are 350°C and 300°C. Which engine has lesser efficiency?

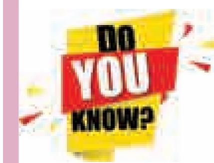
Solution

The efficiency for engine A = $1 - \frac{373}{423}$ = 0.11. Engine A has 11% efficiency

The efficiency for engine B = $1 - \frac{573}{623}$ = 0.08

Engine B has only 8% efficiency.

Even though the differences between the temperature of hot and cold reservoirs in both engines is same, the efficiency is not same. The efficiency depends on the ratio of the two temperature and not on the difference in the temperature. The engine which operates in lower temperature has highest efficiency.



Diesel engines used in cars and petrol engines used in our motor bikes are all real heat engines. The efficiency of diesel engines has maximum up to 44% and the efficiency of petrol engines are maximum up to 30%. Since these engines are not ideal heat engines (Carnot engine), their efficiency is limited by the second law of thermodynamics. Now a days typical bikes give a mileage of 50 km per Liter of petrol. This implies only 30% of 1 Liter of petrol is converted into mechanical work and the remaining 70% goes out as wasted heat and ejected into the surrounding atmosphere!

8.9.3 Entropy and second law of thermodynamics

We have seen in the equation (8.66) that the quantity $\frac{Q_H}{T_H}$ is equal to $\frac{Q_L}{T_L}$. The quantity $\frac{Q}{T}$ is called entropy. It is a very important thermodynamic property of a system. It is also a state variable. $\frac{Q_H}{T_H}$ is the entropy received by the Carnot engine from hot reservoir and $\frac{Q_L}{T_L}$ is entropy given out by the Carnot engine to the cold reservoir. For reversible engines (Carnot Engine) both entropies should be same, so that the change

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

in entropy of the Carnot engine in one cycle is zero. This is proved in equation (8.66). But for all practical engines like diesel and petrol engines which are not reversible engines, they satisfy the relation $\frac{Q_L}{T_L} > \frac{Q_H}{T_H}$. In fact we can reformulate the second law of thermodynamics as follows

"For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.

We now come back to the question: Why does heat always flows from a state of higher temperature to one of lower temperature and not in the opposite direction? Because entropy increases when heat flows from hot object to cold object. If heat were to flow from a cold to a hot object, entropy will decrease leading to violation of second law thermodynamics.

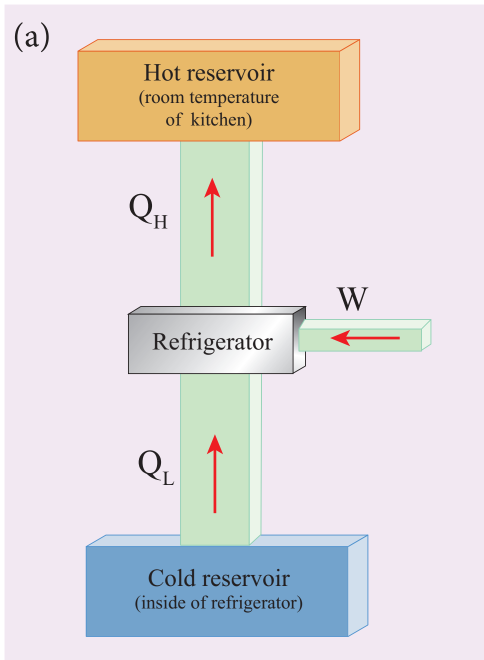
Entropy is also called 'measure of disorder'. All natural process occur such that the disorder should always increases.

Consider a bottle with a gas inside. When the gas molecules are inside the bottle it has less disorder. Once it spreads into the entire room it leads to more disorder. In other words when the gas is inside the bottle the entropy is less and once the gas spreads into entire room, the entropy increases. From the second law of thermodynamics, entropy always increases. If the air molecules go back in to the bottle, the entropy should decrease, which is not allowed by the second law of thermodynamics. The same explanation applies to a drop of ink diffusing into water. Once the drop of ink spreads, its entropy is increased. The diffused ink can never become a drop again. So the natural processes occur in such a way that entropy should increase for all irreversible process.

8.10 REFRIGERATOR

A refrigerator is a Carnot's engine working in the reverse order. It is shown in the figure 8.49.

Working Principle:



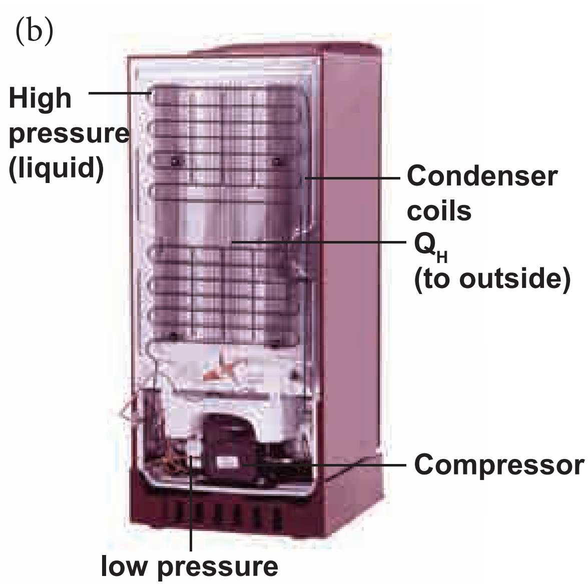


Fig 8.49 (a) Schematic diagram of a refrigerator (b) Actual refrigerator

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The working substance (gas) absorbs a quantity of heat Q_L from the cold body (sink) at a lower temperature T_L. A certain amount of work W is done on the working substance by the compressor and a quantity of heat Q_H is rejected to the hot body (source) ie, the atmosphere at T_H. When you stand beneath of refrigerator, you can feel warmth air.

From the first law of thermodynamics , we have

$$Q_L + W = Q_H \qquad (8.68)$$

As a result the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.

Coefficient of performance (COP) (β):

COP is a measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor W.

$$\text{COP} = \beta = \frac{Q_L}{W} \qquad (8.69)$$

From the equation (8.68)

$$\beta = \frac{Q_L}{Q_H - Q_L}$$

$$\beta = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad (8.70)$$

But we know that $\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$

BPZGDE

Substituting this equation into equation (8.70) we get

$$\beta = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}$$

Inferences:

1. The greater the COP, the better is the condition of the refrigerator. A typical refrigerator has COP around 5 to 6.
2. Lesser the difference in the temperatures of the cooling chamber and the atmosphere, higher is the COP of a refrigerator.
3. In the refrigerator the heat is taken from cold object to hot object by doing external work. Without external work heat cannot flow from cold object to hot object. It is not a violation of second law of thermodynamics, because the heat is ejected to surrounding air and total entropy of (refrigerator + surrounding) is always increased.

EXAMPLE 8.27

A refrigerator has COP of 3. How much work must be supplied to the refrigerator in order to remove 200 J of heat from its interion?

$$\text{COP} = \beta = \frac{Q_L}{W}$$

$$W = \frac{Qc}{COP} = \frac{200}{3} = 66.67 J$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Green House Effect

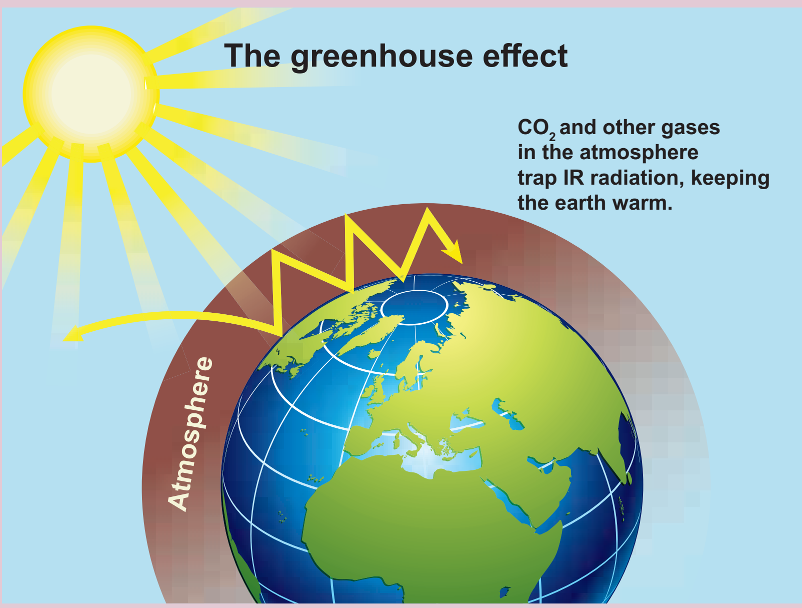
The presence of atmosphere in the earth plays very important role in human lives. Top of the atmosphere is at -19°C and bottom of the atmosphere is at +14°C. The increase in 33°C from top to bottom is due to some gases present in the atmosphere. These gases are called Greenhouse gases and this effect is called Greenhouse effect.

The greenhouse gases are mainly CO_2, water vapour, Ne, He, NO_2, CH_4, Xe, Kr, ozone and NH_3. Except CO_2 and water vapor, all others are present only in very small amount in the atmosphere. The radiation from the Sun is mainly in the visible region of the spectrum. The earth absorbs these radiations and reradiate in the infrared region. Carbon dioxide and water Vapour are good absorbers of infrared radiation since they have more vibrational degree of freedom compared to nitrogen and oxygen (you will learn in unit 9) which keeps earth warmer as shown in Figure.

The amount of CO_2 present in the atmosphere is increased from 20% to 40% due to human activities since 1900s. The major emission of CO_2 comes from burning of fossil fuels. The increase in automobile usage worldwide causes this damage. Due to this increase in the CO_2 content in the atmosphere, the average temperature of the earth increases by 1°C. This effect is called global warming. It has serious influence and alarming effect on ice glaciers on Arctic and Antarctic regions. In addition, the CO_2 content is also increasing in ocean which is very dangerous to species in the oceans.

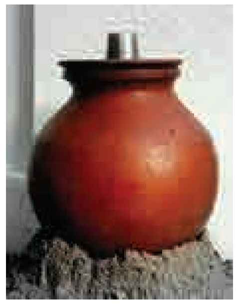
In addition to CO_2, another very important greenhouse gas is Chloro flouro carbon(CFC) which is used as coolant in refrigerators worldwide. In the human made greenhouse gases CO_2 is 55%, CFCs are 24%. Nitrogen oxide is 6% and methane is 15%. CFCs also has made huge damage to ozone layer.

Lot of efforts are taken internationally to reduce the emission of CO_2 and CFCs in various countries. Nowadays a lot of research is going to replace non fossil fuels to replace the fossil-fuels in automobile industry. The major emission of CO_2 comes from developed countries like USA and European countries. Various treaties are formed between countries to reduce the emission of CO_2 to considerable level before 2020s. But still global warming is not taken seriously in various countries.



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

In hot summer, we use earthern pots to drink cold water. The pot reduces the temperature of water inside it. Does the earthern pot act as a refrigerator? No. cyclic process is the basic necessity for heat engine or refrigerator. In earthern pot, the cooling process is not due to any cyclic process. The cooling occurs due to evaporation of water molecules which oozes out through pores of the pot. Once the water molecules evaporate, they never come back to the pot. Even though the heat flows from cold water to open atmosphere, it is not a violation of second law of thermodynamics. The water inside the pot is an open thermodynamic system, so the entropy of water + surrounding always increases.



SUMMARY

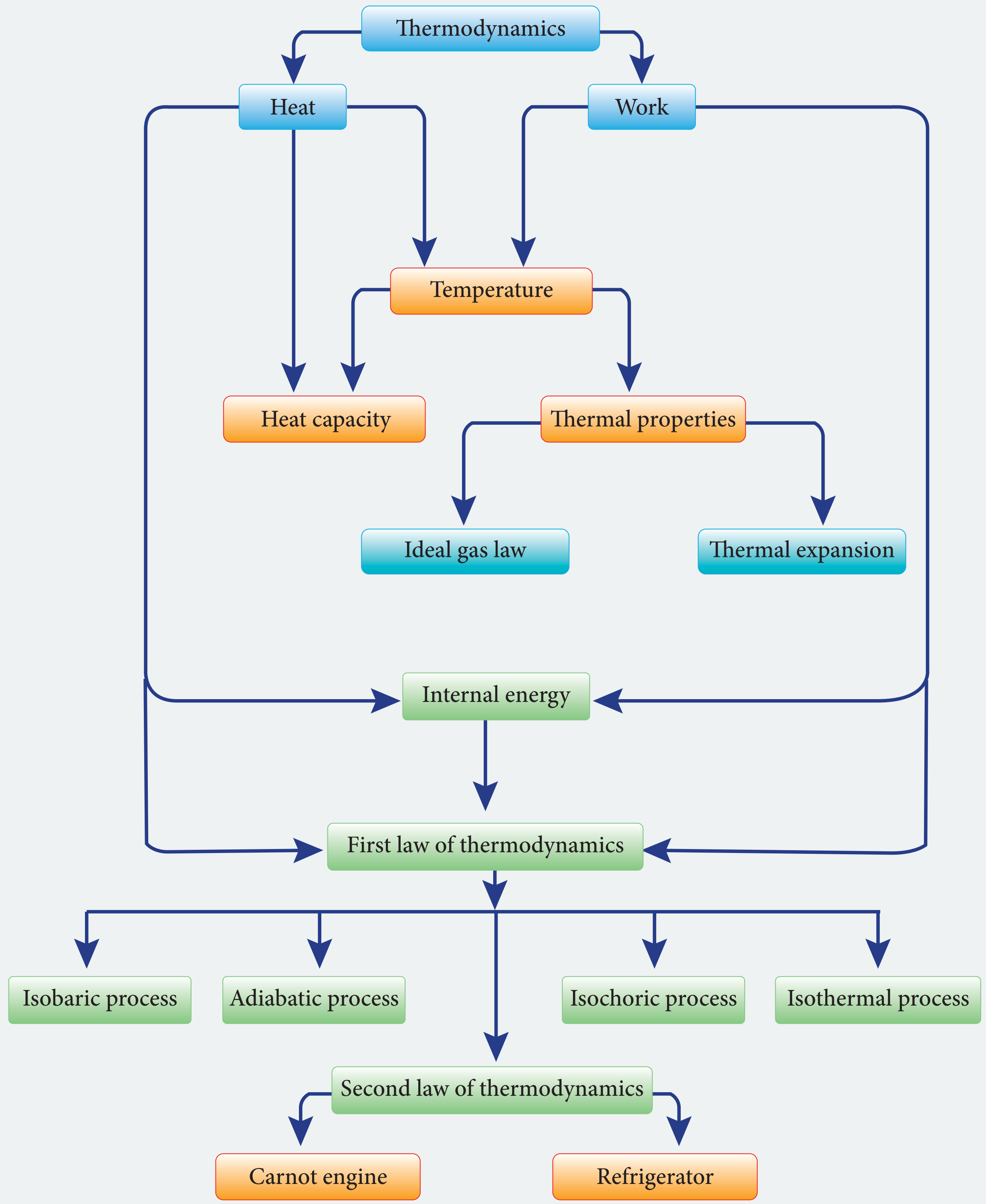
- Heat is energy in transit which flows from hot object to cold object. However it is not a quantity.
- Work is a process to transfer energy from one object to another object.
- Temperature is a measure of hotness of the object. It determines the direction of the flow of heat.
- The ideal gas law is $PV = NkT$ or $PV = \mu RT$. The Ideal gas law holds for only at thermodynamic equilibrium. For non-equilibrium process, it is not valid.
- Heat capacity is the amount of heat energy required to increase the object's temperature by 1°C or 1K. It is denoted by S.
- Specific heat capacity is the amount of heat energy required to increase the 1 kg of object's temperature by 1°C or 1K. It is denoted by s.
- Molar specific heat capacity is the amount of heat energy requires to increase the 1 mole of substance's temperature by 1°C or 1K. It is denoted by C.
- Thermal expansion is a tendency of an object to change its shape, area, and volume due to change in temperature.
- Water has an anomalous behavior of expansion.
- Latent heat capacity is the amount of heat energy required to change the phase of the substance.
- Calorimetry is the measurement of the amount of heat energy released or absorbed by a thermodynamic system during the heating process.
- Heat transfers in three different modes: conduction, convection and radiation
- Stefan-Boltmann law: $E = \sigma T^4$ and Wien's law: $\lambda_{max} T = b$
- Thermodynamic equilibrium: thermal, mechanical and chemical equilibrium
- Thermodynamic variables : Pressure, temperature, volume, internal energy and entropy

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- Zeroth law of thermodynamics: If two objects are separately in thermal equilibrium with the third object, then these two are in thermal equilibrium. Temperature is a property which is the same for both the systems.
- Internal energy is the sum of kinetic and potential energies of molecules in the thermodynamic system.
- Joule converted mechanical energy to internal energy of the thermodynamic system
- First law of thermodynamics is a statement of conservation of energy. It included heat energy of the thermodynamic system.
- A quasi-static process is an infinitely slow process in which the system is always at equilibrium with the surrounding.
- When the volume of the system changes, the work done $W = \int P\, dV$
- The area under the PV diagram gives the work done by the system or work done on the system.
- Specific heat capacity at constant volume is always less than specific heat capacity at constant pressure.
- Isothermal process: T = constant, Isobaric process: P=constant, Isochoric process: V= constant, Adiabatic process Q = 0
- Work done in the isobaric process is most and work done in the adiabatic process is least
- In a cyclic process, change in internal energy is zero.
- The total work done in the cyclic process is given by a closed area in PV diagram
- A reversible process is an ideal process.
- All natural processes are irreversible.
- Heat engine takes input from the hot reservoir, performs work and rejects some amount of heat energy into sink.
- Carnot engine is a reversible engine. It has the highest efficiency. No real heat engine can have the efficiency of that of a Carnot engine.
- A refrigerator is reverse of a Carnot engine. COP (coefficient of performance) of the practical refrigerator is always less than ideal refrigerator.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CONCEPT MAP



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

I. Multiple choice questions:

1. In hot summer after a bath, the body's
   a) internal energy decreases
   b) internal energy increases
   c) heat decreases
   d) no change in internal energy and heat

2. The graph between volume and temperature in Charles' law is
   a) an ellipse   b) a circle
   c) a straight line   d) a parabola

3. When a cycle tyre suddenly bursts, the air inside the tyre expands. This process is
   a) isothermal   b) adiabatic
   c) isobaric   d) isochoric

4. An ideal gas passes from one equilibrium state (P_1, V_1, T_1, N) to another equilibrium state ($2P_1$, $3V_1$, T_2, N). Then
   a) $T_1 = T_2$ b) $T_1 = \frac{T_2}{6}$
   c) $T_1 = 6T_2$ d) $T_1 = 3T_2$

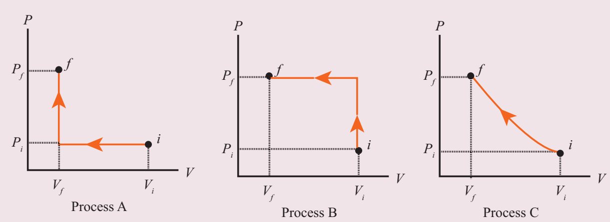
5. When a uniform rod is heated, which of the following quantity of the rod will increase
   a) mass
   b) weight
   c) center of mass
   d) moment of inertia

6. When food is cooked in a vessel by keeping the lid closed, after some time the steam pushes the lid outward. By considering the steam as a thermodynamic system, then in the cooking process
   a) $Q > 0$, $W > 0$,
   b) $Q < 0$, $W > 0$,
   c) $Q > 0$, $W < 0$,
   d) $Q < 0$, $W < 0$,

7. When you exercise in the morning, by considering your body as thermodynamic system, which of the following is true?
   a) $\Delta U > 0$, $W > 0$,
   b) $\Delta U < 0$, $W > 0$,
   c) $\Delta U < 0$, $W < 0$,
   d) $\Delta U = 0$, $W > 0$,

8. A hot cup of coffee is kept on the table. After some time it attains a thermal equilibrium with the surroundings. By considering the air molecules in the room as a thermodynamic system, which of the following is true
   a) $\Delta U > 0$, $Q = 0$
   b) $\Delta U > 0$, $W < 0$
   c) $\Delta U > 0$, $Q > 0$
   d) $\Delta U = 0$, $Q > 0$

9. An ideal gas is taken from (P_i,V_i) to (P_f,V_f) in three different ways. Identify the process in which the work done on the gas the most.



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

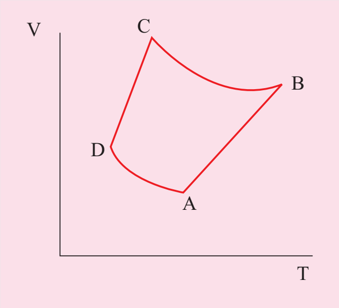
a) Process A

b)Process B

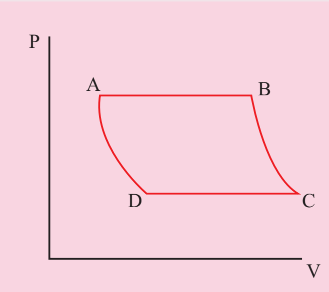
c) Process C

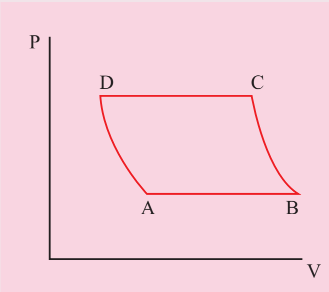
d) Equal work is done in Process A,B &C

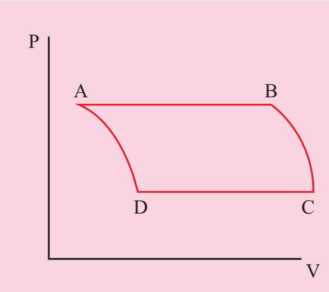
10. The V-T diagram of an ideal gas which goes through a reversible cycle $A \to B \to C \to D$ is shown below. (Processes $D \to A$ and $B \to C$ are adiabatic)

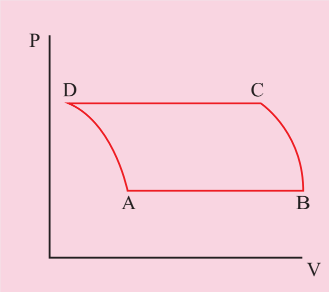


The corresponding PV diagram for the process is (all figures are schematic)

a) 

b) 

c) 

d) 

11. A distant star emits radiation with maximum intensity at 350 nm. The temperature of the star is

a) 8280 K b) 5000K

c) 7260 K d) 9044 K

12. Identify the state variables given here?

a) *Q, T, W* b) *P, T, U*

c) *Q, W* d) *P, T, Q*

13. In an isochoric process, we have

a) W = 0 b) Q = 0

c) $\Delta U = 0$ d) $\Delta T = 0$

14. The efficiency of a heat engine working between the freezing point and boiling point of water is

(NEET 2018)

a) 6.25% b)20%

c) 26.8% d)12.5%

15. An ideal refrigerator has a freezer at temperature −12°C. The coefficient of performance of the engine is 5. The temperature of the air (to which the heat ejected) is

a) 50°C (b)45.2°C

c) 40.2°C (d)37.5°C

Answers:

1) a	2) c	3) b	4) b
5) d	6) a	7) b	8) c
9) b	10) b	11) a	12) b
13) a	14) b	15) c	

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

II. Short answer questions:

1. 'An object contains more heat'- is it a right statement? If not why?
2. Obtain an ideal gas law from Boyle's and Charles' law.
3. Define one mole.
4. Define specific heat capacity and give its unit.
5. Define molar specific heat capacity.
6. What is a thermal expansion?
7. Give the expressions for linear, area and volume thermal expansions.
8. Define latent heat capacity. Give its unit.
9. State Stefan-Boltzmann law.
10. What is Wien's law?
11. Define thermal conductivity. Give its unit.
12. What is a black body?
13. What is a thermodynamic system? Give examples.
14. What are the different types of thermodynamic systems?
15. What is meant by 'thermal equilibrium'?
16. What is mean by state variable? Give example.
17. What are intensive and extensive variables? Give examples.
18. What is an equation of state? Give an example.
19. State Zeroth law of thermodynamics.
20. Define the internal energy of the system.
21. Are internal energy and heat energy the same? Explain.
22. Define one calorie.
23. Did joule converted mechanical energy to heat energy? Explain.
24. State the first law of thermodynamics.
25. Can we measure the temperature of the object by touching it?
26. Give the sign convention for Q and W.
27. Define the quasi-static process.
28. Give the expression for work done by the gas.
29. What is PV diagram?
30. Explain why the specific heat capacity at constant pressure is greater than the specific heat capacity at constant volume.
31. Give the equation of state for an isothermal process.
32. Give an expression for work done in an isothermal process.
33. Express the change in internal energy in terms of molar specific heat capacity.
34. Apply first law for (a) an isothermal (b) adiabatic (c) isobaric processes.
35. Give the equation of state for an adiabatic process.
36. Give an equation state for an isochoric process.
37. If the piston of a container is pushed fast inward. Will the ideal gas equation be valid in the intermediate stage? If not, why?
38. Draw the PV diagram for
    a. Isothermal process
    b. Adiabatic process
    c. isobaric process
    d. Isochoric process
39. What is a cyclic process?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

40. What is meant by a reversible and irreversible processes?
41. State Clausius form of the second law of thermodynamics
42. State Kelvin-Planck statement of second law of thermodynamics.
43. Define heat engine.
44. What are processes involves in a Carnot engine?
45. Can the given heat energy be completely converted to work in a cyclic process? If not, when can the heat can completely converted to work?
46. State the second law of thermodynamics in terms of entropy.
47. Why does heat flow from a hot object to a cold object?
48. Define the coefficient of performance.

III. Long answer Questions:

1. Explain the meaning of heat and work with suitable examples.
2. Discuss the ideal gas laws.
3. Explain in detail the thermal expansion.
4. Describe the anomalous expansion of water. How is it helpful in our lives?
5. Explain Calorimetry and derive an expression for final temperature when two thermodynamic systems are mixed.
6. Discuss various modes of heat transfer.
7. Explain in detail Newton's law of cooling.
8. Explain Wien's law and why our eyes are sensitive only to visible rays?
9. Discuss the
   a. thermal equilibrium
   b. mechanical equilibrium
   c. Chemical equilibrium
   d. thermodynamic equilibrium.
10. Explain Joule's Experiment of the mechanical equivalent of heat.
11. Derive the expression for the work done in a volume change in a thermodynamic system.
12. Derive Mayer's relation for an ideal gas.
13. Explain in detail the isothermal process.
14. Derive the work done in an isothermal process
15. Explain in detail an adiabatic process.
16. Derive the work done in an adiabatic process
17. Explain the isobaric process and derive the work done in this process
18. Explain in detail the isochoric process.
19. What are the limitations of the first law of thermodynamics?
20. Explain the heat engine and obtain its efficiency.
21. Explain in detail Carnot heat engine.
22. Derive the expression for Carnot engine efficiency.
23. Explain the second law of thermodynamics in terms of entropy.
24. Explain in detail the working of a refrigerator.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

IV. Numerical Problems

1. Calculate the number of moles of air is in the inflated balloon at room temperature as shown in the figure.

The radius of the balloon is 10 cm, and pressure inside the balloon is 180 kPa.

Answer: $\mu \cong 0.3\ mol$

2. In the planet Mars, the average temperature is around -53°C and atmospheric pressure is 0.9 kPa. Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

Answer: $\mu_{Mars} = 0.49\ mol$

$\mu_{Earth} \cong 40\ mol$

3. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gases, calculate the final equilibrium temperature of the container.

Answer: $T = \dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$

4. The temperature of a uniform rod of length L having a coefficient of linear expansion α_L is changed by ΔT. Calculate the new moment of inertia of the uniform rod about axis passing through its center and perpendicular to an axis of the rod.

Answer: $I' = I\,(1 + \alpha_L\,\Delta T)^2$

5. Draw the TP diagram (P-x axis, T-y axis), VT(T-x axis, V-y axis) diagram for
   a. Isochoric process
   b. Isothermal process
   c. isobaric process

6. A man starts bicycling in the morning at a temperature around 25°C, he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

Answer: $T = 36.9°C$

7. Normal human body of the temperature is 98.6°F. During high fever if the temperature increases to 104°F, what is the change in peak wavelength that emitted by our body? (Assume human body is a black body)

Answer: (a) $\lambda_{max} \approx 9348$ nm at 98.6°F

(b) $\lambda_{max} \approx 9258$ nm at 104°F

8. In an adiabatic expansion of the air, the volume is increased by 4%, what is percentage change in pressure?

(For air $\gamma = 1.4$)

Answer: 5.6%

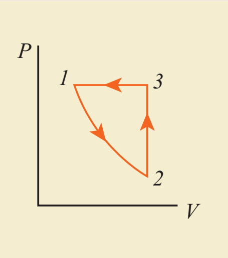
athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

9. In a petrol engine, (internal combustion engine) air at atmospheric pressure and temperature of 20°C is compressed in the cylinder by the piston to 1/8 of its original volume. Calculate the temperature of the compressed air.

(For air $\gamma = 1.4$)

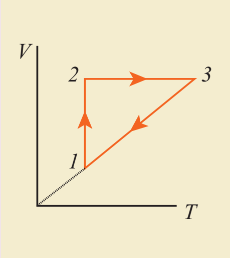
Answer: T ≅ 400°C

10. Consider the following cyclic process consist of isotherm, isochoric and isobar which is given in the figure.



Draw the same cyclic process qualitatively in the V-T diagram where T is taken along x direction and V is taken along y-direction. Analyze the nature of heat exchange in each process.

Answer: *T*= 36.9°C



Process 1 to 2 = increase in volume. So heat must be added.

Process 2 to 3 = Volume remains constant. Increase in temperature. The given heat is used to increase the internal energy.

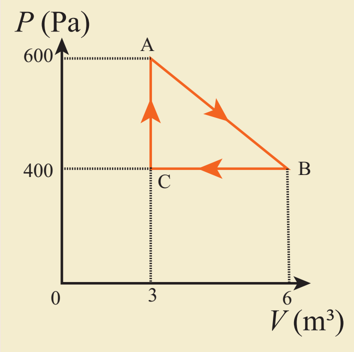
Process 3 to 1 : Pressure remains constant. Volume and Temperature are reduced. Heat flows out of the system. It is an isobaric compression where the work is done on the system.

11. An ideal gas is taken in a cyclic process as shown in the figure. Calculate

(a) work done by the gas.

(b) work done on the gas

(c) Net work done in the process



Answer: (a) $W = +1.5$kJ

(b) $W = -1.2$kJ

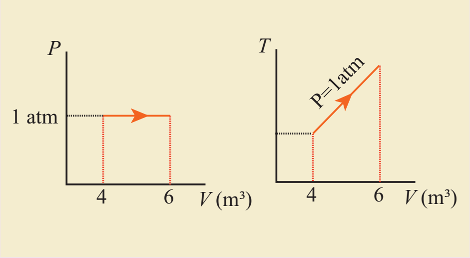
(c) $W = +300$J.

12. For a given ideal gas $6 \times 10^5 J$ heat energy is supplied and the volume of gas is increased from 4 m^3 to 6 m^3 at atmospheric pressure. Calculate (a) the work done by the gas (b) change in internal energy of the gas (c) graph this process in PV and TV diagram.

Answer: (a) $W = +202.6\ kJ$

(b) $dU = 397.4\ kJ$

(c)



13. Suppose a person wants to increase the efficiency of the reversible heat engine that is operating between 100°C and 300°C. He had two ways to increase the efficiency. (a) By decreasing the cold reservoir temperature from 100°C to 50°C and keeping the hot reservoir temperature constant (b) by increasing

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

the temperature of the hot reservoir from 300°C to 350°C by keeping the cold reservoir temperature constant. Which is the suitable method?

Answer: Initial efficiency = 44.5%

Efficiency in method (a) =52 %

Efficiency in method (b) =48 %

Method (a) is more efficient.

14. A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of 327°C. To have an engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

Answer: 552°C

15. An ideal refrigerator keeps its content at 0°C while the room temperature is 27°C. Calculate its coefficient of performance.

Answer: β=10.11

BOOKS FOR REFERENCE

1. Serway and Jewett, Physics for scientist and Engineers with modern physics, Brook/Coole publishers, Eighth edition

2. Paul Tipler and Gene Mosca, Physics for scientist and engineers with modern physics,
Sixth edition, W.H.Freeman and Company

3. James Walker, Physics, Addison-Wesley publishers, 4th Edition

4. Douglas C Giancoli, Physics for scientist & Engineers with modern physics, Pearson Prentice Hall, 4th edition.

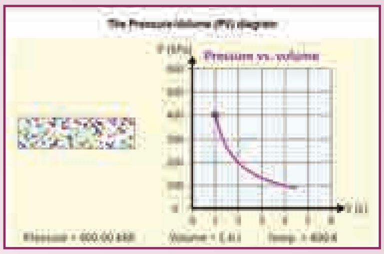
5. H.C.Verma, Concepts of physics volume 1 and Volume 2, Bharati Bhawan Publishers

6. Tarasov and Tarasova, Question and problems in school physics, Mir Publishers

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ICT CORNER

Heat and Thermodynamics

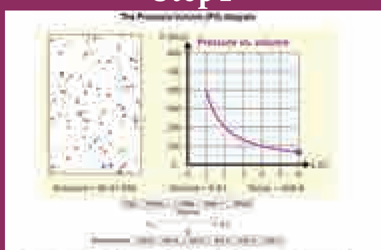


Through this activity you will be able to learn the PV diagrams for various thermodynamic process.

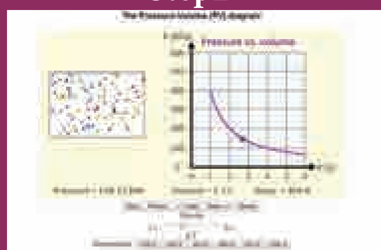
STEPS:

- Use the URL or scan the QR code to open interactive simulation on 'Pressure and Volume Diagram ".
- At selected temperature, change the "Volume" given below the graph and click play button.
- Now select a different temperature, change the 'Volume' again and find the change in the pressure both in the left image and graph.
- Repeat the same with different values and try drawing the graph accordingly. This also helps to understand isothermal process.

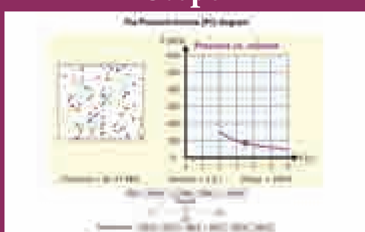
Step1



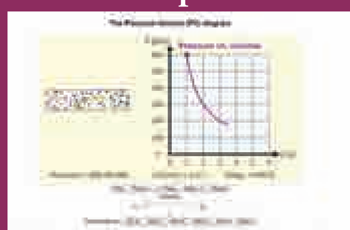
Step2



Step3



Step4



URL:

http://physics.bu.edu/~duffy/HTML5/PV_diagram.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

UNIT 9

KINETIC THEORY OF GASES

"With thermodynamics one can calculate almost everything crudely; with kinetic theory, one can calculate fewer things, but more accurately." - Eugene Wigner

Learning Objectives

BQI8GP

In this unit, the student is exposed to

- necessity of kinetic theory of gases
- the microscopic origin of pressure and temperature
- correlate the internal energy of the gas and translational kinetic energy of gas molecules
- meaning of degrees of freedom
- calculate the total degrees of freedom for mono atomic, diatomic and triatomic molecules
- law of equipartition of energy
- calculation of the ratio of C_P and C_V
- mean free path and its dependence with pressure, temperature and number density
- Brownian motion and its microscopic origin

9.1 KINETIC THEORY

9.1.1 Introduction

Thermodynamics is basically a macroscopic science. We discussed macroscopic parameters like pressure, temperature and volume of thermodynamical systems in unit 8. In this unit we discuss the microscopic origin of pressure and temperature by considering a thermodynamic system as collection of particles or molecules. Kinetic theory relates pressure and temperature to molecular motion of sample of a gas and it is a bridge between Newtonian mechanics and thermodynamics. The present chapter introduces the kinetic nature of gas molecules.

9.1.2 Postulates of kinetic theory of gases

Kinetic theory is based on certain assumptions which makes the mathematical treatment simple. None of these assumptions are strictly true yet the model based on these assumptions can be applied to all gases.

1. All the molecules of a gas are identical, elastic spheres.
2. The molecules of different gases are different.
3. The number of molecules in a gas is very large and the average separation between them is larger than size of the gas molecules.
4. The molecules of a gas are in a state of continuous random motion.
5. The molecules collide with one another and also with the walls of the container.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

6. These collisions are perfectly elastic so that there is no loss of kinetic energy during collisions.
7. Between two successive collisions, a molecule moves with uniform velocity.
8. The molecules do not exert any force of attraction or repulsion on each other except during collision. The molecules do not possess any potential energy and the energy is wholly kinetic.
9. The collisions are instantaneous. The time spent by a molecule in each collision is very small compared to the time elapsed between two consecutive collisions.
10. These molecules obey Newton's laws of motion even though they move randomly.

9.2 PRESSURE EXERTED BY A GAS

9.2.1 Expression for pressure exerted by a gas

Consider a monatomic gas of N molecules each having a mass m inside a cubical container of side l as shown in the Figure 9.1 (a).

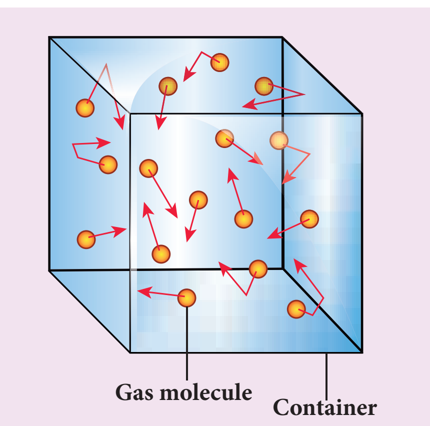


Figure 9.1 (a) Container of gas molecules

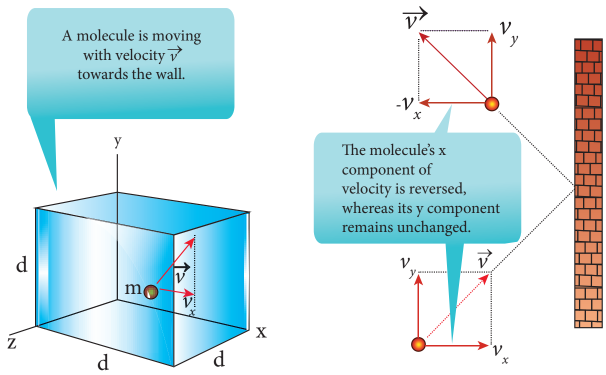


Figure 9.1 (b) Collision of a molecule with the wall

The molecules of the gas are in random motion. They collide with each other and also with the walls of the container. As the collisions are elastic in nature, there is no loss of energy, but a change in momentum occurs.

The molecules of the gas exert pressure on the walls of the container due to collision on it. During each collision, the molecules impart certain momentum to the wall. Due to transfer of momentum, the walls experience a continuous force. The force experienced per unit area of the walls of the container determines the pressure exerted by the gas. It is essential to determine the total momentum transferred by the molecules in a short interval of time.

A molecule of mass m moving with a velocity $\vec{v}$ having components (v_x, v_y, v_z) hits the right side wall. Since we have assumed that the collision is elastic, the particle rebounds with same speed and its x-component is reversed. This is shown in the Figure 9.1 (b). The components of velocity of the molecule after collision are $(-v_x, v_y, v_z)$.

The x-component of momentum of the molecule before collision $= mv_x$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The x-component of momentum of the molecule after collision = $-mv_x$

The change in momentum of the molecule in x direction

=Final momentum – initial momentum = $-mv_x - mv_x = -2mv_x$

According to law of conservation of linear momentum, the change in momentum of the wall = $2mv_x$

Note In x direction, the total momentum of the system before collision is equal to momentum of the molecule (mv_x) since the momentum of the wall is zero. According to the law of conservation of momentum the total momentum of system after the collision must be equal to total momentum of system before collision. The momentum of the molecule (in x direction) after the collision is $-mv_x$ and the momentum of the wall after the collision is $2mv_x$. So total momentum of the system after the collision is $(2mv_x - mv_x) = mv_x$ which is same as the total momentum of the system before collision.

The number of molecules hitting the right side wall in a small interval of time Δt is calculated as follows.

The molecules within the distance of $v_x\Delta t$ from the right side wall and moving towards the right will hit the wall in the time interval Δt. This is shown in the Figure 9.2. The number of molecules that will hit the right side wall in a time interval Δt is equal to the product of volume ($Av_x\Delta t$) and number density of the molecules (n). Here A is area of the wall and n is number of molecules per unit volume $\left(\frac{N}{V}\right)$. We have assumed that the number density is the same throughout the cube.

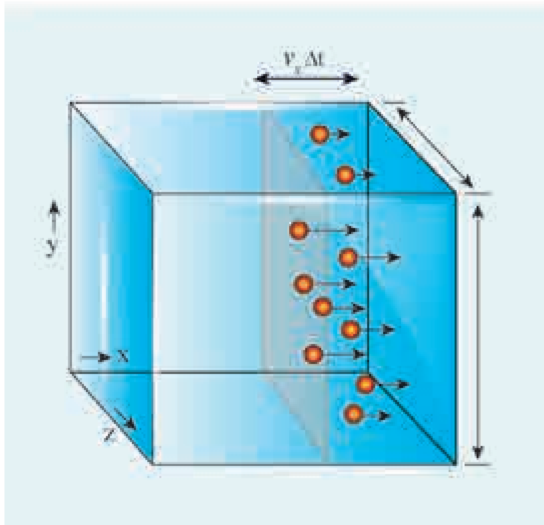


Figure 9.2 Number of molecules hitting the wall

Not all the n molecules will move to the right, therefore on an average only half of the n molecules move to the right and the other half moves towards left side.

The number of molecules that hit the right side wall in a time interval

$$\Delta t = \frac{n}{2} A v_x \Delta t \qquad (9.1)$$

In the same interval of time Δt, the total momentum transferred by the molecules

$$\Delta p = \frac{n}{2} A v_x \Delta t \times 2mv_x = Av_x^2 mn\Delta t \qquad (9.2)$$

From Newton's second law, the change in momentum in a small interval of time gives rise to force.

The force exerted by the molecules on the wall (in magnitude)

$$F = \frac{\Delta p}{\Delta t} = nmAv_x^2 \qquad (9.3)$$

Pressure, P = force divided by the area of the wall

$$P = \frac{F}{A} = nmv_x^2 \qquad (9.4)$$

Since all the molecules are moving completely in random manner, they do not have same speed. So we can replace the term v_x^2 by the average $\overline{v_x^2}$ in equation (9.4)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$P = nm\overline{v_x^2} \quad (9.5)$$

Since the gas is assumed to move in random direction, it has no preferred direction of motion (the effect of gravity on the molecules is neglected). It implies that the molecule has same average speed in all the three direction. So, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$. The mean square speed is written as

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}$$

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2}$$

Using this in equation (9.5), we get

$$P = \frac{1}{3}nm\overline{v^2} \text{ or } P = \frac{1}{3}\frac{N}{V}m\overline{v^2} \quad (9.6)$$

as $\left[n = \frac{N}{V}\right]$

The following inference can be made from the above equation. The pressure exerted by the molecules depends on

(i) **Number density** $n = \frac{N}{V}$. It implies that if the number density increases then pressure will increase. For example when we pump air inside the cycle tyre or car tyre essentially the number density increases and as a result the pressure increases.

(ii) **Mass of the molecule** Since the pressure arises due to momentum transfer to the wall, larger mass will have larger momentum for a fixed speed. As a result the pressure will increase.

(iii) **Mean square speed** For a fixed mass if we increase the speed, the average speed will also increase. As a result the pressure will increase.

For simplicity the cubical container is taken into consideration. The above result is true for any shape of the container as the area A does not appear in the final expression (9.6). Hence the pressure exerted by gas molecules on the wall is independent of area of the wall.

9.2.2 Kinetic interpretation of temperature

To understand the microscopic origin of temperature in the same way,

Rewrite the equation (9.6)

$$P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} \quad (9.7)$$

Comparing the equation (9.7) with ideal gas equation $PV=NkT$,

$$NkT = \frac{1}{3}Nm\overline{v^2}$$

$$kT = \frac{1}{3}m\overline{v^2} \quad (9.8)$$

Multiply the above equation by 3/2 on both sides,

$$\frac{3}{2}kT = \frac{1}{2}m\overline{v^2} \quad (9.9)$$

R.H.S of the equation (9.9) is called average kinetic energy of a single molecule ($\overline{KE}$).

The average kinetic energy per molecule

$$\overline{KE} = \epsilon = \frac{3}{2}kT \quad (9.10)$$

Equation (9.9) implies that the temperature of a gas is a measure of the average translational kinetic energy per molecule of the gas.

Note Compare this with the definition of temperature studied in lower classes: Temperature is the degree of hotness or coldness!

Equation 9.10 is a very important result from kinetic theory of gas. We can infer the following from this equation.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(i) The average kinetic energy of the molecule is directly proportional to absolute temperature of the gas. The equation (9.9) gives the connection between the macroscopic world (temperature) to microscopic world (motion of molecules).

(ii) The average kinetic energy of each molecule depends only on temperature of the gas not on mass of the molecule. In other words, if the temperature of an ideal gas is measured using thermometer, the average kinetic energy of each molecule can be calculated without seeing the molecule through naked eye.

By multiplying the total number of gas molecules with average kinetic energy of each molecule, the internal energy of the gas is obtained.

Internal energy of ideal gas $U = N\left(\frac{1}{2}m\overline{v^2}\right)$

By using equation (9.9)

$$U = \frac{3}{2}NkT \qquad (9.11)$$

From equation (9.11), we understand that the internal energy of an ideal gas depends only on absolute temperature and is independent of pressure and volume.

EXAMPLE 9.1

A football at 27°C has 0.5 mole of air molecules. Calculate the internal energy of air in the ball.

Solution

The internal energy of ideal gas = $\frac{3}{2}$ NkT.

The number of air molecules is given in terms of number of moles so, rewrite the expression as follows

$$U = \frac{3}{2}\mu RT$$

Since $Nk = \mu R$. Here μ is number of moles.

Gas constant $R = 8.31 \frac{J}{mol\,k}$

Temperature T =273+27=300K

$U = \frac{3}{2} \times 0.5 \times 8.31 \times 300 = 1869.75J$

This is approximately equivalent to the kinetic energy of a man of 57 kg running with a speed of 8 m s^{-1}.

9.2.3 Relation between pressure and mean kinetic energy

From earlier section, the internal energy of the gas is given by

$$U = \frac{3}{2}NkT$$

The above equation can also be written as

$$U = \frac{3}{2}PV$$

since $PV = NkT$

$$P = \frac{2}{3}\frac{U}{V} = \frac{2}{3}u \qquad (9.12)$$

From the equation (9.12), we can state that the pressure of the gas is equal to two thirds of internal energy per unit volume or internal energy density ($u = \frac{U}{V}$).

Writing pressure in terms of mean kinetic energy density using equation (9.6)

$$P = \frac{1}{3}nm\overline{v^2} = \frac{1}{3}\rho\overline{v^2} \qquad (9.13)$$

where $\rho = nm = mass\ density$ (Note n is number density)

Multiply and divide R.H.S of equation (9.13) by 2, we get

$$P = \frac{2}{3}\left(\frac{\rho}{2}\overline{v^2}\right)$$

$$P = \frac{2}{3}\overline{KE} \qquad (9.14)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

From the equation (9.14), pressure is equal to 2/3 of mean kinetic energy per unit volume.

9.2.4 Some elementary deductions from kinetic theory of gases

Boyle's law:

From equation (9.12), we know that $PV = \frac{2}{3}U$

But the internal energy of an ideal gas is equal to N times the average kinetic energy ($\in$) of each molecule.

$$U = N\in$$

For a fixed temperature, the average translational kinetic energy $\in$ will remain constant. It implies that

$$PV = \frac{2}{3}N\in \qquad \text{Thus} \quad PV = \text{constant}$$

Therefore, *pressure of a given gas is inversely proportional to its volume provided the temperature remains constant. This is Boyle's law.*

Charles' law:

From the equation (9.12), we get $PV = \frac{2}{3}U$

For a fixed pressure, the volume of the gas is proportional to internal energy of the gas or average kinetic energy of the gas and the average kinetic energy is directly proportional to absolute temperature. It implies that

$$V \propto T \text{ or } \frac{V}{T} = \text{constant}$$

This is Charles' law.

Avogadro's law:

This law states that at constant temperature and pressure, equal volumes of all gases contain the same number of molecules. For two different gases at the same temperature and pressure, according to kinetic theory of gases,

From equation (9.6)

$$P = \frac{1}{3}\frac{N_1}{V}m_1\overline{v_1^2} = \frac{1}{3}\frac{N_2}{V}m_2\overline{v_2^2} \qquad (9.15)$$

where $\overline{v_1^2}$ and $\overline{v_2^2}$ are the mean square speed for two gases and N_1 and N_2 are the number of gas molecules in two different gases.

At the same temperature, average kinetic energy per molecule is the same for two gases.

$$\frac{1}{2}m_1\overline{v_1^2} = \frac{1}{2}m_2\overline{v_2^2} \qquad (9.16)$$

Dividing the equation (9.15) by (9.16) we get $N_1 = N_2$

This is Avogadro's law. It is sometimes referred to as Avogadro's hypothesis or Avogadro's Principle.

9.2.5 Root mean square speed (v_{rms})

Root mean square speed (v_{rms}) is defined as the square root of the mean of the square of speeds of all molecules. It is denoted by $v_{rms} = \sqrt{\overline{v^2}}$

Equation (9.8) can be re-written as,

$$\text{mean square speed } \overline{v^2} = \frac{3kT}{m} \qquad (9.17)$$

root mean square speed,

$$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}} \qquad (9.18)$$

From the equation (9.18) we infer the following

(i) rms speed is directly proportional to square root of the temperature and inversely proportional to square root of mass of the molecule. At a given temperature the molecules of lighter mass move faster on an average than the molecules with heavier masses.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Example: Lighter molecules like hydrogen and helium have high 'v_{rms}' than heavier molecules such as oxygen and nitrogen at the same temperature.

(ii) Increasing the temperature will increase the r.m.s speed of molecules

We can also write the v_{rms} in terms of gas constant R. Equation (9.18) can be rewritten as follows

$v_{rms} = \sqrt{\dfrac{3N_A kT}{N_A m}}$ Where N_A is Avogadro number.

Since $N_A k = R$ and $N_A m = M$ (molar mass)

The root mean square speed or r.m.s speed

$$v_{rms} = \sqrt{\frac{3RT}{M}} \qquad (9.19)$$

The equation (9.6) can also be written in terms of rms speed $P = \frac{1}{3} n m v^2_{rms}$

since $v^2_{rms} = \overline{v^2}$

Note Root mean square speed is not the same as average speed. Average speed is 0.92 times of r.m.s speed.

Impact of v_{rms} in nature:

1. **Moon has no atmosphere.**

   The escape speed of gases on the surface of Moon is much less than the root mean square speeds of gases due to low gravity. Due to this all the gases escape from the surface of the Moon.

2. **No hydrogen in Earth's atmosphere**.

   As the root mean square speed of hydrogen is much less than that of nitrogen, it easily escapes from the earth's atmosphere.

   In fact, the presence of nonreactive nitrogen instead of highly combustible hydrogen deters many disastrous consequences.

EXAMPLE 9.2

A room contains oxygen and hydrogen molecules in the ratio 3:1. The temperature of the room is 27°C. The molar mass of 0_2 is 32 g mol^{-1} and for H_2 2 g mol^{-1}. The value of gas constant R is 8.32 J mol^{-1} K^{-1}

Calculate

(a) rms speed of oxygen and hydrogen molecule

(b) Average kinetic energy per oxygen molecule and per hydrogen molecule

(c) Ratio of average kinetic energy of oxygen molecules and hydrogen molecules

Solution

(a) Absolute Temperature

T=27°C =27+273=300 K.

Gas constant R=8.32 J mol^{-1} k^{-1}

For Oxygen molecule: Molar mass M=32 gm=32 x 10^{-3} kg mol^{-1}

rms speed $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3\times 8.32\times 300}{32\times 10^{-3}}}$

$$= 483.73\ m\ s^{-1} \approx 484\ m\ s^{-1}$$

For Hydrogen molecule:

Molar mass M = 2 × 10^{-3} kg mol^{-1}

rms speed $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3\times 8.32\times 300}{2\times 10^{-3}}}$

$$= 1934\ m s^{-1} = 1.93\ km\ s^{-1}$$

Note that the rms speed is inversely proportional to $\sqrt{M}$ and the molar mass of oxygen is 16 times higher than molar mass of hydrogen. It implies that the rms speed of hydrogen is 4 times greater than rms speed of oxygen at the same temperature. $\frac{1934}{484} \approx 4$.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(b) The average kinetic energy per molecule is $\frac{3}{2}kT$. It depends only on absolute temperature of the gas and is independent of the nature of molecules. Since both the gas molecules are at the same temperature, they have the same average kinetic energy per molecule. k is Boltzmaan constant.

$$\frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times300 = 6.21\times10^{-21}J$$

(c) Average kinetic energy of total oxygen molecules = $\frac{3}{2}N_O kT$ where N_o - number of oxygen molecules in the room

Average kinetic energy of total hydrogen molecules = $\frac{3}{2}N_H kT$ where N_H - number of hydrogen molecules in the room.

It is given that the number of oxygen molecules is 3 times more than number of hydrogen molecules in the room. So the ratio of average kinetic energy of oxygen molecules with average kinetic energy of hydrogen molecules is 3:1

9.2.6 Mean (or) average speed ($\bar{v}$)

It is defined as the mean (or) average of all the speeds of molecules

If v_1, v_2, v_3.....v_N are the individual speeds of molecules then

$$\bar{v} = \frac{v_1+v_2+v_3........+v_n}{N} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8kT}{\pi m}} \quad (9.20)$$

Here M- Molar Mass and m – mass of the molecule.

$$\bar{v} = 1.60\sqrt{\frac{kT}{m}} \quad (9.21)$$

9.2.7. Most probable speed (V_{mp})

It is defined as the speed acquired by most of the molecules of the gas.

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}} \quad (9.22)$$

$$v_{mp} = 1.41\sqrt{\frac{kT}{m}} \quad (9.23)$$

The derivation of equations (9.20), (9.22) is beyond the scope of the book

Comparison of v_{rms}, $\bar{v}$ and v_{mp}

Among the speeds v_{rms} is the largest and v_{mp} is the least

$$v_{rms} > \bar{v} > v_{mp}$$

Ratio-wise,

$$v_{rms} : \bar{v} : v_{mp} = \sqrt{3} : \sqrt{8/\pi} : \sqrt{2} = 1.732 : 1.6 : 1.414$$

EXAMPLE 9.3

Ten particles are moving at the speed of 2, 3, 4, 5, 5, 5, 6, 6, 7 and 9 m s^{-1}. Calculate rms speed, average speed and most probable speed.

Solution

The average speed

$$\bar{v} = \frac{2+3+4+5+5+5+6+6+7+9}{10} = 5.2\, m\, s^{-1}$$

To find the rms speed, first calculate the mean square speed $\overline{v^2}$

$$\overline{v^2} = \frac{2^2+3^2+4^2+5^2+5^2+5^2+6^2+6^2+7^2+9^2}{10}$$

$$= 30.6\, m^2\, s^{-2}$$

The rms speed

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{30.6} = 5.53\, m\, s^{-1}$$

The most probable speed is 5 m s^{-1} because three of the particles have that speed.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 9.4

Calculate the rms speed, average speed and the most probable speed of 1 mole of hydrogen molecules at 300 K. Neglect the mass of electron.

Solution

The hydrogen atom has one proton and one electron. The mass of electron is negligible compared to the mass of proton.

Mass of one proton = $1.67 \times 10^{-27} kg$.

One hydrogen molecule = 2 hydrogen atoms = $2 \times 1.67 \times 10^{-27} kg$.

The average speed

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 1.60\sqrt{\frac{kT}{m}} =$$

$$= 1.60\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.78\times10^{3} m s^{-1}$$

(Boltzmann Constant $k = 1.38 \times 10^{-23} J K^{-1}$)

The rms speed $$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$$

$$= 1.73\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.93\times10^{3} m s^{-1}$$

Most probable speed $$v_{mp} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$$

$$= 1.41\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.57\times10^{3} m s^{-1}$$

Note that $v_{rms} > \bar{v} > v_{mp}$

9.2.8 Maxwell-Boltzmann speed distribution function

In a classroom, the air molecules are moving in random directions. The speed of each molecule is not the same even though macroscopic parameters like temperature and pressure are fixed. Each molecule collides with every other molecule and they exchange their speed. In the previous section we calculated the rms speed of each molecule and not the speed of each molecule which is rather difficult. In this scenario we can find the number of gas molecules that move with the speed of 5 m s^{-1} to 10 m s^{-1} or 10 m $s^{-1}$to 15 m s^{-1} etc. In general our interest is to find how many gas molecules have the range of speed from v to $v + dv$. This is given by Maxwell's speed distribution function.

$$N_v = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} \qquad (9.24)$$

The above expression is graphically shown as follows

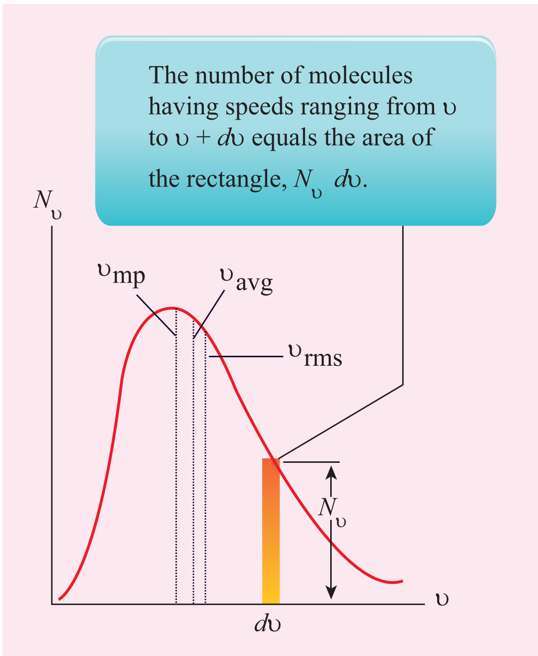


Figure 9.3 Maxwell's molecular speed distribution

From the Figure 9.3, it is clear that, for a given temperature the number of molecules having lower speed increases parabolically but decreases exponentially after reaching most probable speed. The rms speed, average speed and most probable speed are indicated in the Figure 9.3. It can be seen that the rms speed is greatest among the three.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

To know the number of molecules in the range of speed between 50 m s^{-1} and 60 m s^{-1}, we need to integrate $\int_{50}^{60} 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv =$ $= N(50 \; to \; 60 \; ms^{-1})$. In general the number of molecules within the range of speed v and v+dv is given by

$$\int_{v}^{v+dv} 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv = N(v \; to \; v+dv).$$

The exact integration is beyond the scope of the book. But we can infer the behavior of gas molecules from the graph.

(i) The area under the graph will give the total number of gas molecules in the system

(ii) Figure 9.4 shows the speed distribution graph for two different temperatures. As temperature increases, the peak of the curve is shifted to the right. It implies that the average speed of each molecule will increase. But the area under each graph is same since it represents the total number of gas molecules.

Note

Interestingly once the gas molecule attains equilibrium, the number of molecules in the given range of speeds are fixed. For example if a molecule initially moving with speed 12 m s^{-1}, collides with some other molecule and changes its speed to 9 m s^{-1}, then the other molecule initially moving with different speed reaches the speed 12 m s^{-1} due to another collision. So in general once the gas molecules attain equilibrium, the number of molecules that lie in the range of v to v+dv is always fixed.

9.3 DEGREES OF FREEDOM

9.3.1 Definition

The minimum number of independent coordinates needed to specify the position and configuration of a thermo-dynamical system in space is called the degree of freedom of the system.

BR1ZJZ

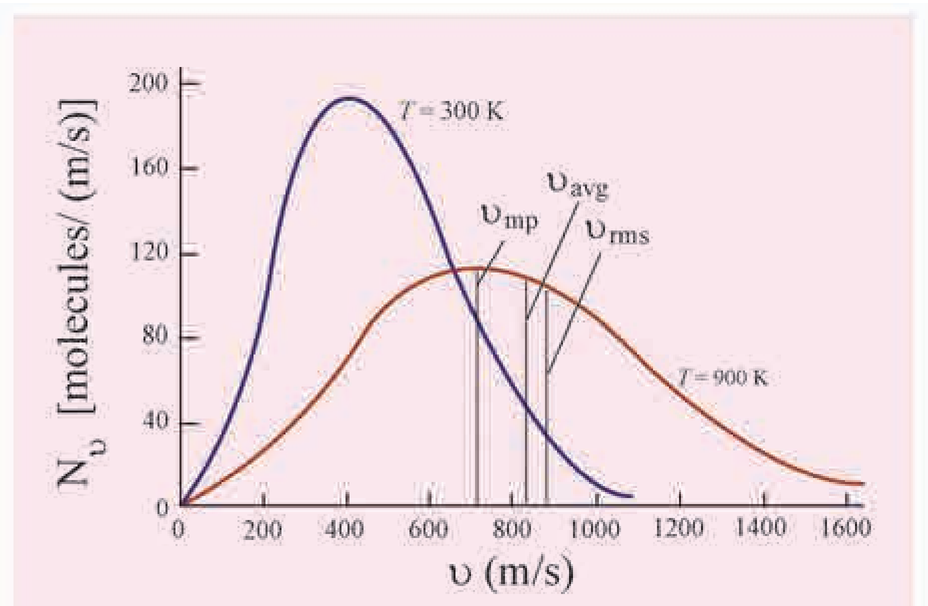


Figure 9.4 Maxwell distribution graph for two different temperatures

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Example:

1. A free particle moving along x-axis needs only one coordinate to specify it completely. So its degree of freedom is one.
2. Similarly a particle moving over a plane has two degrees of freedom.
3. A particle moving in space has three degrees of freedom.

Suppose if we have N number of gas molecules in the container, then the total number of degrees of freedom is f = 3N.

But, if the system has *q* number of constraints (restrictions in motion) then the degrees of freedom decreases and it is equal to $f = 3N-q$ where N is the number of particles.

9.3.2 Monoatomic molecule

A monoatomic molecule by virtue of its nature has only three translational degrees of freedom.

Therefore $f = 3$

Example: Helium, Neon, Argon

9.3.3 Diatomic molecule

There are two cases.

1. At Normal temperature

A molecule of a diatomic gas consists of two atoms bound to each other by a force of attraction. Physically the molecule can be regarded as a system of two point masses fixed at the ends of a massless elastic spring.

The center of mass lies in the center of the diatomic molecule. So, the motion of the center of mass requires three translational degrees of freedom (figure 9.5 a). In addition, the diatomic molecule can rotate

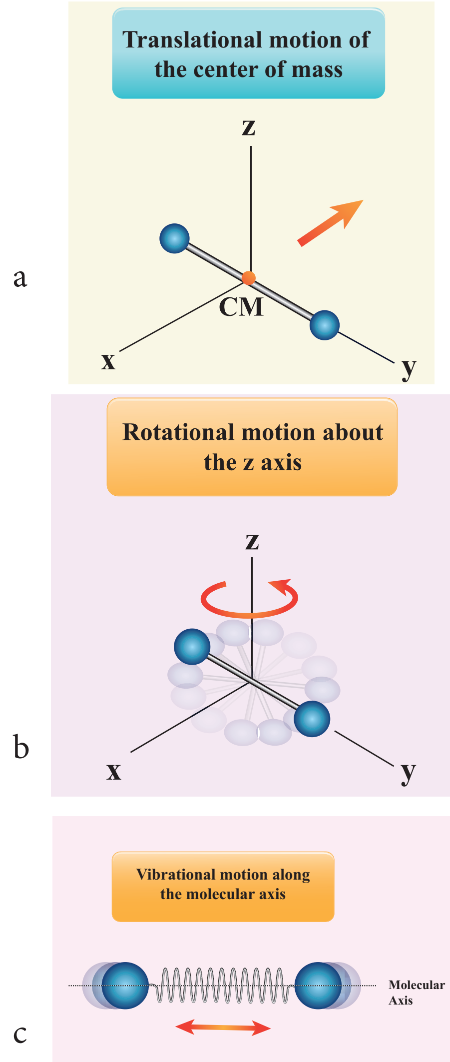


Figure 9.5 Degree of freedom of diatomic molecule

about three mutually perpendicular axes (figure 9.5 b). But the moment of inertia about its own axis of rotation is negligible (about y axis in the figure 9.5). Therefore, it has only two rotational degrees of freedom (one rotation is about Z axis and another rotation is about Y axis). Therefore totally there are five degrees of freedom.

$$f = 5$$

2. At High Temperature

At a very high temperature such as 5000 K, the diatomic molecules possess additional

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

two degrees of freedom due to vibrational motion[one due to kinetic energy of vibration and the other is due to potential energy] (Figure 9.5c). So totally there are seven degrees of freedom.

$$f = 7$$

Examples: Hydrogen, Nitrogen, Oxygen.

9.3.4 Triatomic molecules

There are two cases.

Linear triatomic molecule

In this type, two atoms lie on either side of the central atom as shown in the Figure 9.6

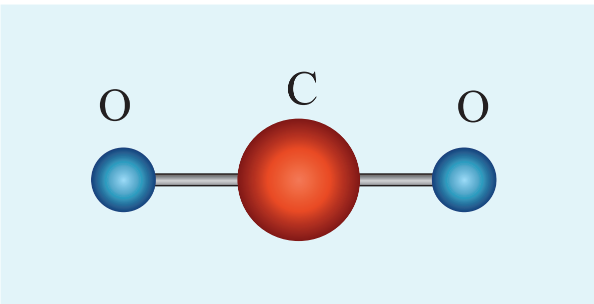


Figure 9.6 A linear triatomic molecule.

Linear triatomic molecule has three translational degrees of freedom. It has two rotational degrees of freedom because it is similar to diatomic molecule except there is an additional atom at the center. At normal temperature, linear triatomic molecule will have five degrees of freedom. At high temperature it has two additional vibrational degrees of freedom.

So a linear triatomic molecule has seven degrees of freedom.

Example: Carbon dioxide.

Non-linear triatomic molecule

In this case, the three atoms lie at the vertices of a triangle as shown in the Figure 9.7

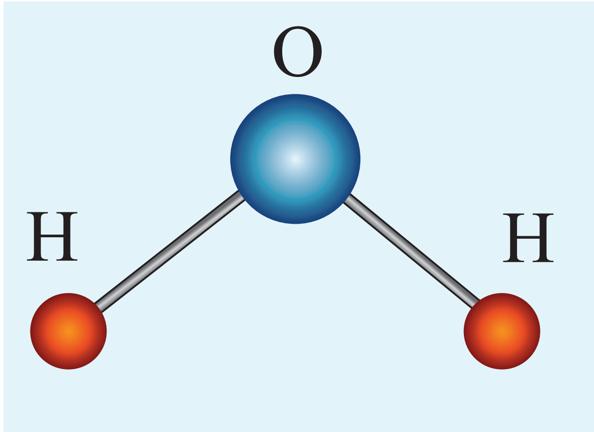


Figure 9.7 A non-linear triatomic molecule

It has three translational degrees of freedom and three rotational degrees of freedom about three mutually orthogonal axes. The total degrees of freedom, f = 6

Example: Water, Sulphurdioxide.

9.4 LAW OF EQUIPARTITION OF ENERGY

We have seen in Section 9.2.1 that the average kinetic energy of a molecule moving in x direction is $\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}kT$.

Similarly, when the motion is in y direction, $\frac{1}{2}m\overline{v_y^2} = \frac{1}{2}kT$ and

For the motion z direction, $\frac{1}{2}m\overline{v_z^2} = \frac{1}{2}kT$.

According to kinetic theory, the average kinetic energy of system of molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom (x or y or

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

z directions of motion) so that each degree of freedom will get $\frac{1}{2}kT$ of energy. This is called law of equipartition of energy.

Average kinetic energy of a monatomic molecule (with f=3) = $3\times\frac{1}{2}kT=\frac{3}{2}kT$

Average kinetic energy of diatomic molecule at low temperature (with f = 5) $= 5\times\frac{1}{2}kT=\frac{5}{2}kT$

Average kinetic energy of a diatomic molecule at high temperature (with f =7) $=7\times\frac{1}{2}kT=\frac{7}{2}kT$

Average kinetic energy of linear triatomic molecule (with f = 7) = $7\times\frac{1}{2}kT=\frac{7}{2}kT$

Average kinetic energy of non linear triatomic molecule (with f = 6) = $6\times\frac{1}{2}kT=3kT$

9.4.1 Application of law of equipartition energy in specific heat of a gas

Meyer's relation $C_P - C_V = R$ connects the two specific heats for one mole of an ideal gas.

Equipartition law of energy is used to calculate the value of $C_P - C_V$ and the ratio between them $\gamma = \frac{C_P}{C_V}$. Here γ is called adiabatic exponent.

i) Monatomic molecule

Average kinetic energy of a molecule

$$=\left[\frac{3}{2}kT\right]$$

Total energy of a mole of gas

$$=\frac{3}{2}kT\times N_A=\frac{3}{2}RT$$

For one mole, the molar specific heat at constant volume $C_V=\frac{dU}{dT}=\frac{d}{dT}\left[\frac{3}{2}RT\right]$

$$C_V=\left[\frac{3}{2}R\right]$$

$$C_P=C_V+R=\frac{3}{2}R+R=\frac{5}{2}R$$

The ratio of specific heats,

$$\gamma=\frac{C_P}{C_V}=\frac{\frac{5}{2}R}{\frac{3}{2}R}=\frac{5}{3}=1.67$$

ii) Diatomic molecule

Average kinetic energy of a diatomic molecule at low temperature = $\frac{5}{2}kT$

Total energy of one mole of gas

$$=\frac{5}{2}kT\times N_A=\frac{5}{2}RT$$

(Here, the total energy is purely kinetic)

For one mole Specific heat at constant volume

$$C_V=\frac{dU}{dT}=\left[\frac{5}{2}RT\right]=\frac{5}{2}R$$

But $C_P=C_V+R=\frac{5}{2}R+R=\frac{7}{2}R$

$$\therefore\gamma=\frac{C_P}{C_V}=\frac{\frac{7}{2}R}{\frac{5}{2}R}=7/5=1.40$$

Energy of a diatomic molecule at high temperature is equal to $\frac{7}{2}RT$

$$C_V=\frac{dU}{dT}=\left[\frac{7}{2}RT\right]=\frac{7}{2}R$$

$$\therefore C_P=C_V+R=7/2\,R+R$$

$$C_P=9/2\,R$$

Note that the C_V and C_P are higher for diatomic molecules than the mono atomic molecules. It implies that to increase the temperature of

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

diatomic gas molecules by 1°C it require more heat energy than monoatomic molecules.

$$\therefore \gamma = \frac{C_P}{C_V} = \frac{{}^9\!/\!_2 R}{{}^7\!/\!_2 R} = {}^9\!/\!_7 = 1.28$$

iii) Triatomic molecule

a) Linear molecule

Energy of one mole$= \frac{7}{2}kT \times N_A = \frac{7}{2}RT$

$$C_V = \frac{dU}{dT} = \frac{d}{dT}\left[\frac{7}{2}RT\right]$$

$$C_V = \frac{7}{2}R$$

$$C_P = C_V + R = \frac{7}{2}R + R = \frac{9R}{2}$$

$$\therefore \gamma = \frac{C_P}{C_V} = \frac{\frac{9}{2}R}{\frac{7}{2}R} = \frac{9}{7} = 1.28$$

b) Non-linear molecule

Energy of a mole $= \frac{6}{2}kT \times N_A = \frac{6}{2}RT = 3RT$

$$C_V = \frac{dU}{dT} = 3R$$

$$C_P = C_V + R = 3R + R = 4R$$

$$\therefore \gamma = \frac{C_P}{C_V} = \frac{4R}{3R} = \frac{4}{3} = 1.33$$

Note that according to kinetic theory model of gases the specific heat capacity at constant volume and constant pressure are independent of temperature. But in reality it is not sure. The specific heat capacity varies with the temperature.

EXAMPLE 9.5

Find the adiabatic exponent γ for mixture of μ_1 moles of monoatomic gas and μ_2 moles of a diatomic gas at normal temperature.

Solution

The specific heat of one mole of a monoatomic gas $C_V = \frac{3}{2}R$

For μ_1 mole, $C_V = \frac{3}{2}\mu_1 R \quad C_P = \frac{5}{2}\mu_1 R$

The specific heat of one mole of a diatomic gas

$$C_V = \frac{5}{2}R$$

For μ_2 mole, $C_V = \frac{5}{2}\mu_2 R \quad C_P = \frac{7}{2}\mu_2 R$

The specific heat of the mixture at constant volume $C_V = \frac{3}{2}\mu_1 R + \frac{5}{2}\mu_2 R$

The specific heat of the mixture at constant pressure $C_P = \frac{5}{2}\mu_1 R + \frac{7}{2}\mu_2 R$

The adiabatic exponent $\gamma = \frac{C_P}{C_V} = \frac{5\mu_1 + 7\mu_2}{3\mu_1 + 5\mu_2}$

9.5 MEAN FREE PATH

Usually the average speed of gas molecules is several hundred meters per second even at room temperature. Odor from an open perfume bottle takes some time to reach us even if we are closer to the room. The time delay is because the odor of the molecules cannot travel straight to us as it undergoes a lot of collisions with the nearby air molecules and moves in a zigzag path. This *average distance travelled by the molecule between collisions is called mean free path (λ). We can calculate the mean free path based on kinetic theory.*

Expression for mean free path

We know from postulates of kinetic theory that the molecules of a gas are in random motion and they collide with each other. Between two successive collisions, a molecule

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

moves along a straight path with uniform velocity. This path is called mean free path.

Consider a system of molecules each with diameter d. Let n be the number of molecules per unit volume.

Assume that only one molecule is in motion and all others are at rest as shown in the Figure 9.8

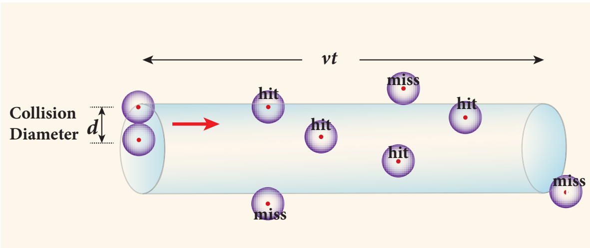


Figure 9.8 Mean free path

If a molecule moves with average speed v in a time t, the distance travelled is vt. In this time t, consider the molecule to move in an imaginary cylinder of volume $\pi d^2 vt$. It collides with any molecule whose center is within this cylinder. Therefore, the number of collisions is equal to the number of molecules in the volume of the imaginary cylinder. It is equal to $\pi d^2 vtn$. The total path length divided by the number of collisions in time t is the mean free path.

Mean free path, $\lambda = \dfrac{distance\ travelled}{Number\ of\ collisions}$

$$\lambda = \frac{vt}{n\pi d^2 vt} = \frac{1}{n\pi d^2} \qquad (9.25)$$

Though we have assumed that only one molecule is moving at a time and other molecules are at rest, in actual practice all the molecules are in random motion. So the average relative speed of one molecule with respect to other molecules has to be taken into account. After some detailed calculations (you will learn in higher classes) the correct expression for mean free path

$$\therefore \lambda = \frac{1}{\sqrt{2} n\pi d^2} \qquad (9.26)$$

The equation (9.26) implies that the mean free path is inversely proportional to number density. When the number density increases the molecular collisions increases so it decreases the distance travelled by the molecule before collisions.

Case1: Rearranging the equation (9.26) using 'm' (mass of the molecule)

$$\therefore \lambda = \frac{m}{\sqrt{2}\pi d^2 mn}$$

But mn=mass per unit volume = ρ (density of the gas)

$$\therefore \lambda = \frac{m}{\sqrt{2}\,\pi d^2 \rho} \qquad (9.27)$$

Also we know that PV = NkT

$$P = \frac{N}{V}kT = nkT$$

$$\therefore n = \frac{P}{kT}$$

Substituting n= $\dfrac{P}{kT}$ in equation (9.26), we get

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} \qquad (9.28)$$

The equation (9.28) implies the following

1. Mean free path increases with increasing temperature. As the temperature increases, the average speed of each molecule will increase. It is the reason why the smell of hot sizzling food reaches several meter away than smell of cold food.
2. Mean free path increases with decreasing pressure of the gas and diameter of the gas molecules.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 9.6

An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is $1.2 \times 10^{-10} m$. Calculate the mean free path of oxygen molecule.

Solution

From (9.26) $\quad \lambda = \frac{1}{\sqrt{2}\pi n d^2}$

We have to find the number density n

By using ideal gas law

$$n = \frac{N}{V} = \frac{P}{kT} = \frac{101.3 \times 10^3}{1.381 \times 10^{-23} \times 300}$$

$$= 2.449 \times 10^{25}\ molecues/m^3$$

$$\lambda = \frac{1}{\sqrt{2} \times \pi \times 2.449 \times 10^{25} \times (1.2 \times 10^{-10})^2}$$

$$= \frac{1}{15.65 \times 10^5}$$

$$\lambda = 0.63 \times 10^{-6} m$$

BRAVLH

9.6 BROWNIAN MOTION

In 1827, Robert Brown, a botanist reported that grains of pollen suspended in a liquid moves randomly from one place to other. The random (Zig - Zag path) motion of pollen suspended in a liquid is called Brownian motion. In fact we can observe the dust particle in water moving in random directions. This discovery puzzled scientists for long time. There were a lot of explanations for pollen or dust to move in random directions but none of these explanations were found adequate. After a systematic study, Wiener and Gouy proposed that Brownian motion is due to the bombardment of suspended particles by molecules of the surrounding fluid. But during 19th century people did not accept that every matter is made up of small atoms or molecules. In the year 1905, Einstein gave systematic theory of Brownian motion based on kinetic theory and he deduced the average size of molecules.

According to kinetic theory, any particle suspended in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random and zig–zag manner as shown in Figure 9.9. But when we put our hand in water it causes no random motion because the mass of our hand is so large that the momentum transferred by the molecular collision is not enough to move our hand.

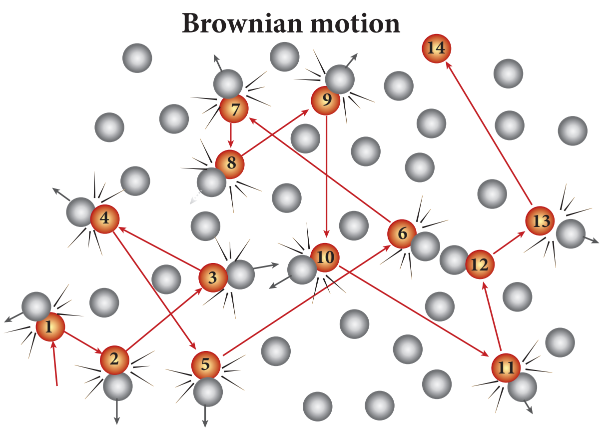


Figure 9.9 Particles in Brownian motion

Factors affecting Brownian Motion

1. Brownian motion increases with increasing temperature.
2. Brownian motion decreases with bigger particle size, high viscosity and density of the liquid (or) gas.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Note

The experimental verification on Einstein's theoretical explanation of Brownian motion was done by Jean Perrin in the year 1908. The Einstein's explanation on Brownian motion and Perrin experiment was of great importance in physics because it provided direct evidence of reality of atoms and molecules.

SUMMARY

- Kinetic theory explains the microscopic origin of macroscopic parameters like temperature, pressure.
- The pressure exerted on the walls of gas container is due to the momentum imparted by the gas molecules on the walls.
- The pressure $P = \frac{1}{3}n m \overline{v^2}$. The pressure is directly proportional to the number density, mass of molecule and mean square speed.
- The temperature of a gas is a measure of the average translational kinetic energy per molecule of the gas. The average kinetic energy per molecule is directly proportional to absolute temperature of gas and independent of nature of molecules.
- The pressure is also equal to 2/3 of internal energy per unit volume.
- The rms speed of gas molecules = $v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$
- The average speed of gas molecules $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 1.60\sqrt{\frac{kT}{m}}$
- The most probable speed of gas molecules $v_{mp} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$
- Among the speeds v_{rms} is the largest and v_{mp} is the least

$$v_{rms} > \bar{v} > v_{mp}$$

- The number of gas molecules in the range of speed v to v+dv is given by Maxwell-Boltzmann distribution

$$N_v dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv$$

- The minimum number of independent coordinates needed to specify the position and configuration of a thermodynamical system in space is called the degrees of freedom of the system. If a sample of gas has N molecules, then the total degrees of freedom f = 3N. If there are q number of constraints then total degrees of freedom f = 3N-q.
- For a monoatomic molecule, f = 3

  For a diatomic molecule (at normal temperature), f = 5

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

For a diatomic molecule (at high temperature), f = 7

For a triatomic molecule (linear type), f = 7

For a triatomic molecule (non-linear type), f = 6

- The average kinetic energy of sample of gas is equally distributed to all the degrees of freedom. It is called law of equipartition of energy. Each degree of freedom will get $\frac{1}{2}kT$ energy.
- The ratio of molar specific heat at constant pressure and constant volume of a gas $\left[\gamma = \frac{C_p}{C_v}\right]$

For

Monoatomic molecule: 1.67

Diatomic molecule (Normal temperature) : 1.40

Diatomic molecule (High temperature): 1.28

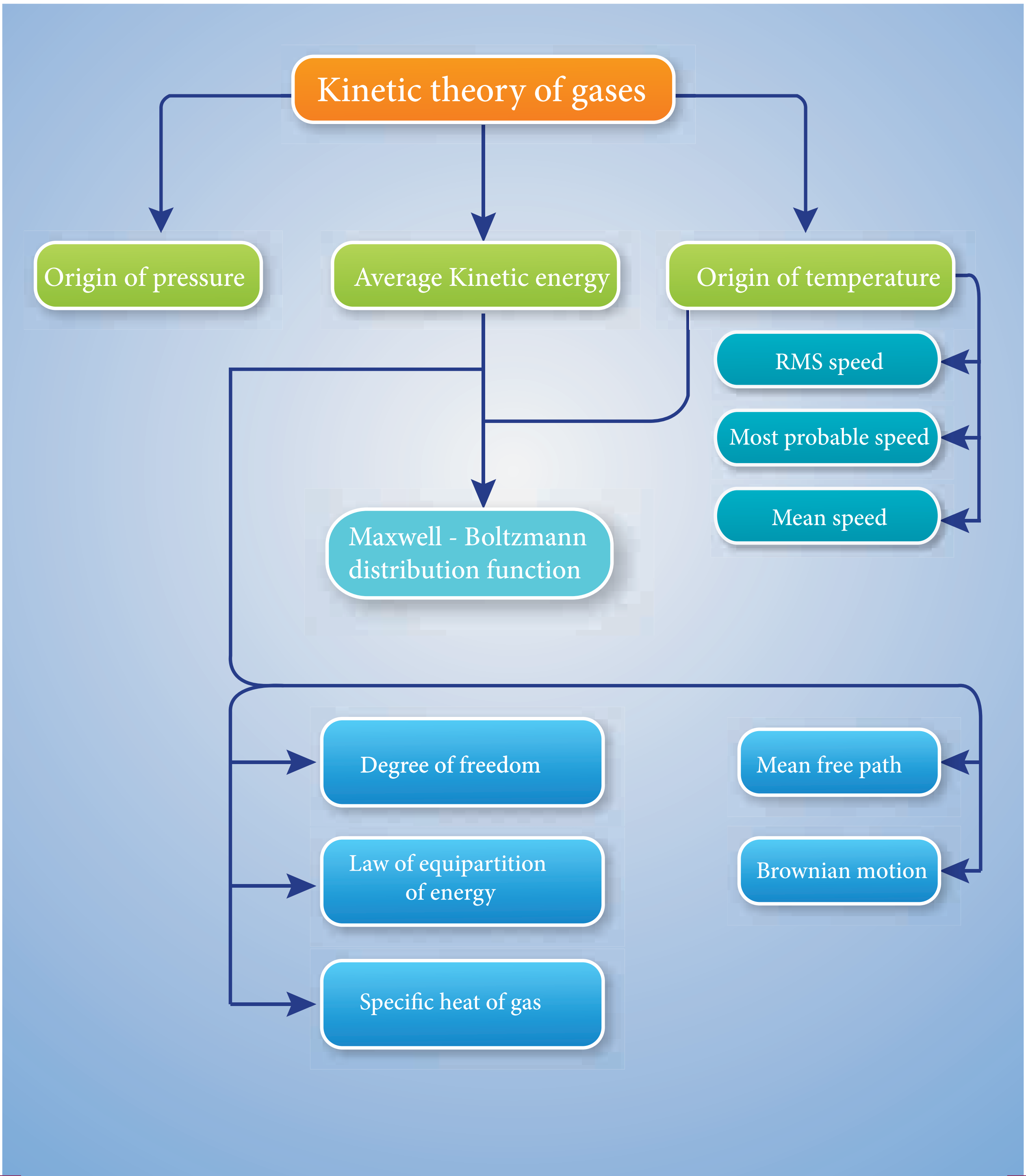
Triatomic molecule (Linear type): 1.28.

Triatomic molecule (Non-linear type): 1.33

- The mean free path $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$. The mean free path is directly proportional to temperature and inversely proportional to size of the molecule and pressure of the molecule
- The Brownian motion explained by Albert Einstein is based on kinetic theory. It proves the reality of atoms and molecules.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

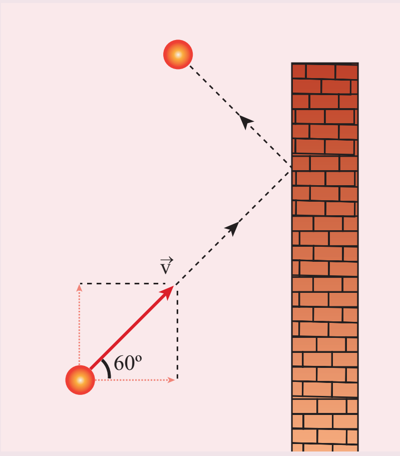
CONCEPT MAP



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

I. Multiple choice questions

1. A particle of mass m is moving with speed u in a direction which makes 60° with respect to x axis. It undergoes elastic collision with the wall. What is the change in momentum in x and y direction?



(a) $\Delta p_x = -mu, \Delta p_y = 0$

(b) $\Delta p_x = -2mu, \Delta p_y = 0$

(c) $\Delta p_x = 0, \Delta p_y = mu$

(d) $\Delta p_x = mu, \Delta p_y = 0$

2. A sample of ideal gas is at equilibrium. Which of the following quantity is zero?

(a) rms speed

(b) average speed

(c) average velocity

(d) most probable speed

3. An ideal gas is maintained at constant pressure. If the temperature of an ideal gas increases from 100K to 1000K then the rms speed of the gas molecules

(a) increases by 5 times

(b) increases by 10 times

(c) remains same

(d) increases by 7 times

4. Two identically sized rooms A and B are connected by an open door. If the room A is air conditioned such that its temperature is 4° lesser than room B, which room has more air in it?

(a) Room A

(b) Room B

(c) Both room has same air

(d) Cannot be determined

5. The average translational kinetic energy of gas molecules depends on

(a) number of moles and T

(b) only on T

(c) P and T

(d) P only

6. If the internal energy of an ideal gas U and volume V are doubled then the pressure

(a) doubles

(b) remains same

(c) halves

(d) quadruples

7. The ratio $\gamma = \frac{C_p}{C_v}$ for a gas mixture consisting of 8 g of helium and 16 g of oxygen is (Physics Olympiad -2005)

(a) 23/15

(b) 15/23

(c) 27/11

(d) 17/27

BRJRNA

8. A container has one mole of monoatomic ideal gas. Each molecule has f degrees of freedom. What is the ratio of $\gamma = \frac{C_p}{C_v}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(a) f (b) $\frac{f}{2}$

(c) $\frac{f}{f+2}$ (d) $\frac{f+2}{f}$

9. If the temperature and pressure of a gas is doubled the mean free path of the gas molecules

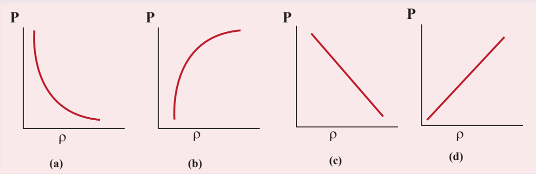
(a) remains same

(b) doubled

(c) tripled

(d) quadrapoled

10. Which of the following shows the correct relationship between the pressure and density of an ideal gas at constant temperature?



11. A sample of gas consists of μ_1 moles of monoatomic molecules, μ_2 moles of diatomic molecules and μ_3 moles of linear triatomic molecules. The gas is kept at high temperature. What is the total number of degrees of freedom?

(a) $[3\mu_1 + 7(\mu_2 + \mu_3)]\ N_A$

(b) $[3\mu_1 + 7\mu_2 + 6\mu_3]\ N_A$

(c) $[7\mu_1 + 3(\mu_2 + \mu_3)]\ N_A$

(d) $[3\mu_1 + 6(\mu_2 + \mu_3)]\ N_A$

12. If s_P and s_V denote the specific heats of nitrogen gas per unit mass at constant pressure and constant volume respectively, then (JEE 2007)

(a) $s_P - s_V = 28R$

(b) $s_P - s_V = R/28$

(c) $s_P - s_V = R/14$

(d) $s_P - s_V = R$

13. Which of the following gases will have least rms speed at a given temperature?

(a) Hydrogen

(b) Nitrogen

(c) Oxygen

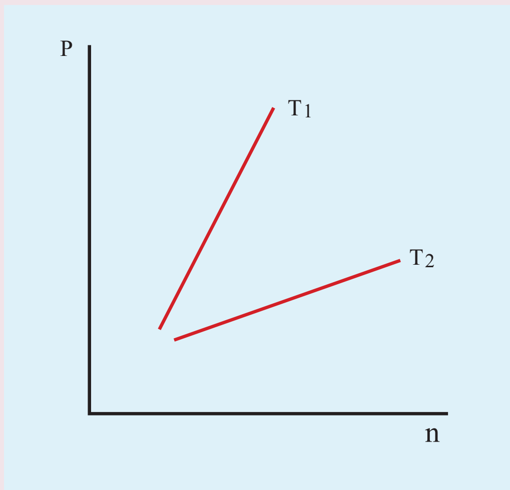
(d) Carbon dioxide

14. For a given gas molecule at a fixed temperature, the area under the Maxwell-Boltzmann distribution curve is equal to

(a) $\frac{PV}{kT}$ (b) $\frac{kT}{PV}$

(c) $\frac{P}{NkT}$ (d) PV

15. The following graph represents the pressure versus number density for ideal gas at two different temperatures T_1 and T_2. The graph implies



(a) $T_1 = T_2$

(b) $T_1 > T_2$

(c) $T_1 < T_2$

d) Cannot be determined

Answers:

1) a	2) c	3) b	4) a
5) a	6) b	7) c	8) d
9) a	10) d	11) a	12) b
13) d	14) a	15) b	

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

II. Short answer questions

1. What is the microscopic origin of pressure?
2. What is the microscopic origin of temperature?
3. Why moon has no atmosphere?
4. Write the expression for rms speed, average speed and most probable speed of a gas molecule.
5. What is the relation between the average kinetic energy and pressure?
6. Define the term degrees of freedom.
7. State the law of equipartition of energy.
8. Define mean free path and write down its expression.
9. Deduce Charles' law based on kinetic theory.
10. Deduce Boyle's law based on kinetic theory.
11. Deduce Avogadro's law based on kinetic theory.
12. List the factors affecting the mean free path.
13. What is the reason for Brownian motion?

III. Long answer questions

1. Write down the postulates of kinetic theory of gases.
2. Derive the expression of pressure exerted by the gas on the walls of the container.
3. Explain in detail the kinetic interpretation of temperature.
4. Describe the total degrees of freedom for monoatomic molecule, diatomic molecule and triatomic molecule.
5. Derive the ratio of two specific heat capacities of monoatomic, diatomic and triatomic molecules
6. Explain in detail the Maxwell Boltzmann distribution function.
7. Derive the expression for mean free path of the gas.
8. Describe the Brownian motion.

IV Numerical Problems

1. A fresh air is composed of nitrogen N_2(78%) and oxygen O_2(21%). Find the rms speed of N_2 and O_2 at 20°C.

   Ans: For $v_{rms} = 511\ m\ s^{-1}$
   For $O_2 v_{rms} = 478\ m\ s^{-1}$

2. If the rms speed of methane gas in the Jupiter's atmosphere is 471.8 m s^{-1}, show that the surface temperature of Jupiter is sub-zero.

   Ans: -130°C

3. Calculate the temperature at which the rms velocity of a gas triples its value at S.T.P.

   Ans: $T_1 = 273$ K, $T_2 = 2457$ K

4. A gas is at temperature 80°C and pressure $5 \times 10^{-10} N\,m^{-2}$. What is the number of molecules per m^3 if Boltzmann's constant is $1.38 \times 10^{-23}\ J\,K^{-1}$

   Ans: 1.02×10^{11}

5. From kinetic theory of gases, show that Moon cannot have an atmosphere (Assume $k = 1.38 \times 10^{-23}\ J\,K^{-1}$ Temperature T=0°C=273K).

   Ans: $v_{escape} = v_{rms} = 1.86\ km\ s^{-1}$

6. If 10^{20} oxygen molecules per second strike 4 cm^2 of wall at an angle of 30° with the normal when moving at a speed of 2×10^3 m s^{-1}, find the pressure

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

exerted on the wall. (mass of 1 atom = 1.67×10^{-27} kg)

Ans: 92.4 N m^{-2}

7. During an adiabatic process, the pressure of a mixture of monatomic and diatomic gases is found to be proportional to the cube of the temperature. Find the value of $\gamma = (C_p/C_V)$

Ans: 3/2

8. Calculate the mean free path of air molecules at STP. The diameter of N_2 and O_2 is about $3 \times 10^{-10}\, m$

Ans: $\lambda \approx 9 \times 10^{-8}\, m$

9. A gas made of a mixture of 2 moles of oxygen and 4 moles of argon at temperature T. Calculate the energy of the gas in terms of RT. Neglect the vibrational modes.

Ans: 11RT

10. Estimate the total number of air molecules in a room of capacity 25 m^3 at a temperature of 27°C.

Ans: 6.1×10^{26} molecules

BOOKS FOR REFERENCE

1. Serway and Jewett, Physics for scientist and Engineers with modern physics, Brook/Coole publishers, Eighth edition
2. Paul Tipler and Gene Mosca, Physics for scientist and engineers with modern physics, Sixth edition, W.H.Freeman and Company
3. H.C.Verma, Concepts of physics -Volume 2, Bharati Bhawan Publishers
4. Douglas C. Giancoli, Physics for scientist & Engineers, Pearson Publications, Fourth Edition
5. James Walker, Physics, Addison Wesley, Fourth Edition

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ICT CORNER

Kinetic Theory of Gases

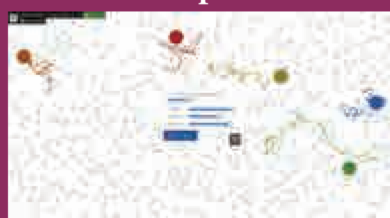
Through this activity you will be able to learn about the Brownian motion of the particles.

STEPS:

- Use the URL or scan the QR code to open 'interactive' simulation on "Brownian motion".
- Observe the movement of particles (Big balls) suspended in the gas at the initial stage. Observe the molecules in the gas by dragging the first slider. 'Drag to see what's actually going on'
- Find the variants such as "Energy", "Size Ratio" and "Mass Ratio" below the first slider. These variants can be lowered or increased.
- By dragging to the appropriate values of the variants, Brownian motion of the particles shall be observed.

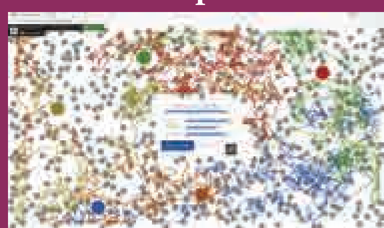
Step1

Step2



Step3

Step4



URL:

http://labs.minutelabs.io/Brownian-Motion/

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

UNIT 10

OSCILLATIONS

Life is a constant oscillation between the sharp horns of a dilemma – H.L. Mencken

LEARNING OBJECTIVES

In this unit, the student is exposed to

- oscillatory motion – periodic motion and non-periodic motion
- simple harmonic motion
- angular harmonic motion
- linear harmonic oscillator – both horizontal and vertical
- combination of springs – series and parallel
- simple pendulum
- expression of energy – potential energy, kinetic energy and total energy
- graphical representation of simple harmonic motion
- types of oscillation – free, damped, maintained and forced oscillations
- concept of resonance

BRTMPX

10.1 INTRODUCTION

Figure 10.1. Thanjavur dancing doll

Have you seen the Thanjavur Dancing Doll (In Tamil, it is called 'Thanjavur thalayatti bommai')?. It is a world famous Indian cultural doll (Figure 10.1). What does this doll do when disturbed? It will dance such that the head and body move continuously in a to and fro motion, until the movement gradually stops. Similarly, when we walk on the road, our hands and legs will move front and back. Again similarly, when a mother swings a cradle to make her child sleep, the cradle is made to move in to and fro motion. All these motions are different from the motion that we have discussed so far. These motions are shown in Figure 10.2. Generally, they are known as oscillatory motion or vibratory motion. A similar motion occurs even at atomic levels. When the temperature is raised, the atoms in a solid vibrate about their rest position (mean position or equilibrium position). The study of vibrational motion is very important in engineering applications, such as, designing the structure of building, mechanical equipments, etc.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Figure 10.2. Motions

10.1.1 Periodic and non-periodic motion

Motion in physics can be classified as repetitive (periodic motion) and non-repetitive (non-periodic motion).

1. **Periodic motion**
   Any motion which repeats itself in a fixed time interval is known as periodic motion.
   Examples : Hands in pendulum clock, swing of a cradle, the revolution of the Earth around the Sun, waxing and waning of Moon, etc.
2. **Non-Periodic motion**
   Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion.
   Example : Occurance of Earth quake, eruption of volcano, etc.

EXAMPLE 10.1

Classify the following motions as periodic and non-periodic motions?.

a. Motion of Halley's comet.
b. Motion of clouds.
c. Moon revolving around the Earth.

Solution

a. Periodic motion
b. Non-periodic motion
c. Periodic motion

EXAMPLE 10.2

Which of the following functions of time represent periodic and non-periodic motion?.

a. $\sin \omega t + \cos \omega t$
b. $\ln \omega t$

Solution

a. Periodic
b. Non-periodic

Question to ponder

Discuss "what will happen if the motion of the Earth around the Sun is not a periodic motion".

10.1.2 Oscillatory motion

When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory (or vibratory). Examples; our heart beat, swinging motion of the wings of an insect, grandfather's clock (pendulum clock), etc. Note that all oscillatory motion are periodic whereas all periodic motions need not be oscillation in nature. see Figure 10.3

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

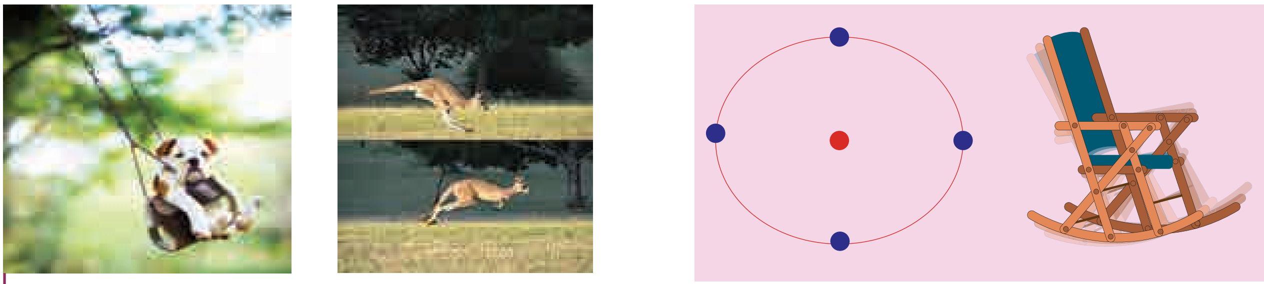


Figure 10.3 **Oscillatory or vibratory motions**

10.2 SIMPLE HARMONIC MOTION (SHM)

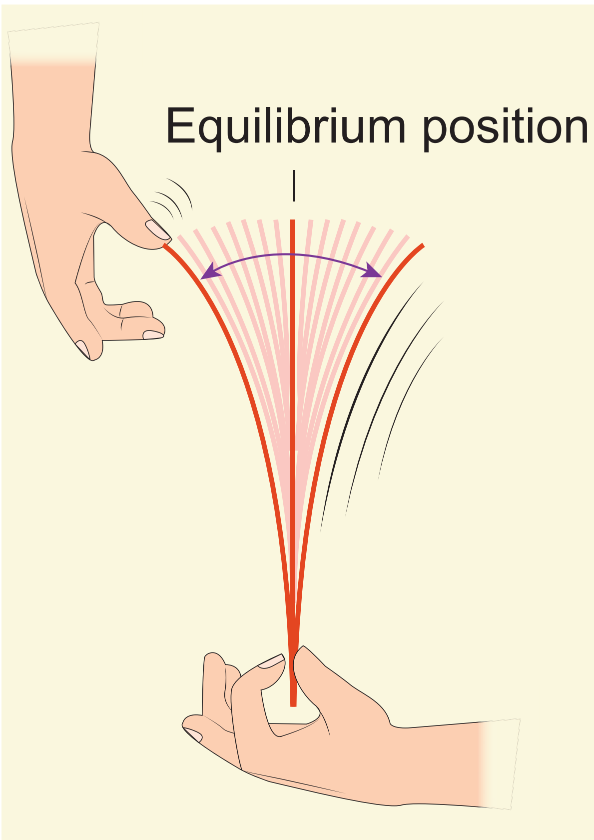


Figure 10.4 Simple Harmonic Motion

> **Note** A simple harmonic motion is a special type of oscillatory motion. But all oscillatory motions need not be simple harmonic .

Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point. In one dimensional case, let x be the displacement of the particle and a_x be the acceleration of the particle, then

$$a_x \propto x \qquad (10.1)$$

$$a_x = -\,b\,x \qquad (10.2)$$

where b is a constant which measures acceleration per unit displacement and dimensionally it is equal to T^{-2}. By multiplying by mass of the particle on both sides of equation (10.2) and from Newton's second law, the force is

$$F_x = -\,k\,x \qquad (10.3)$$

where k is a force constant which is defined as force per unit length. The negative sign indicates that displacement and force (or acceleration) are in opposite directions. This means that when the displacement of the particle is taken towards right of equilibrium position (x takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

displacement of the particle is taken towards left of equilibrium position (x takes negative value), the force (or acceleration) will point towards equilibrium (towards right). This type of force is known as ***restoring force*** *because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position.* This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.

BS3IRK

In order to represent in two or three dimensions, we can write using vector notation

$$\vec{F} = -k\vec{r} \qquad (10.4)$$

where $\vec{r}$ is the displacement of the particle from the chosen origin. Note that the force and displacement have a linear relationship. This means that the exponent of force $\vec{F}$ and the exponent of displacement $\vec{r}$ are unity. The sketch between cause (magnitude of force $|\vec{F}|$) and effect (magnitude of displacement $|\vec{r}|$) is a straight line passing through second and fourth quadrant as shown in Figure 10.5. By measuring slope $\frac{1}{k}$, one can find the numerical value of force constant k.

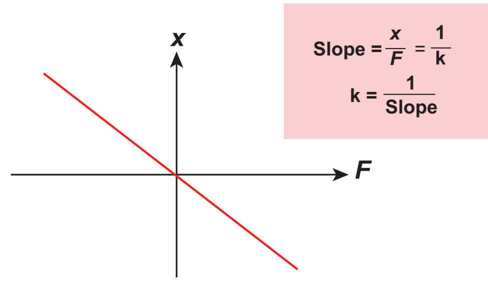


Figure 10.5 Force verses displacement graph

10.2.1 The projection of uniform circular motion on a diameter of SHM

Consider a particle of mass m moving with uniform speed v along the circumference of a circle whose radius is r in anti-clockwise direction (as shown in Figure 10.6). Let us assume that the origin of the coordinate system coincides with the center O of the circle. If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time t, then $\theta = \omega t$. By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate

BSCET8

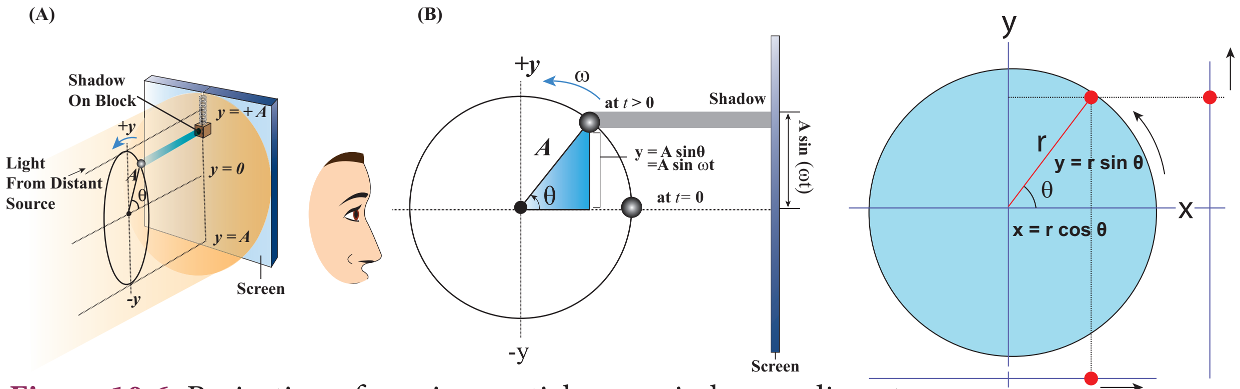


Figure 10.6 Projection of moving particle on a circle on a diameter

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion. Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter as shown in Figure 10.7. Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.

The following figures explain the position of particle at different time :

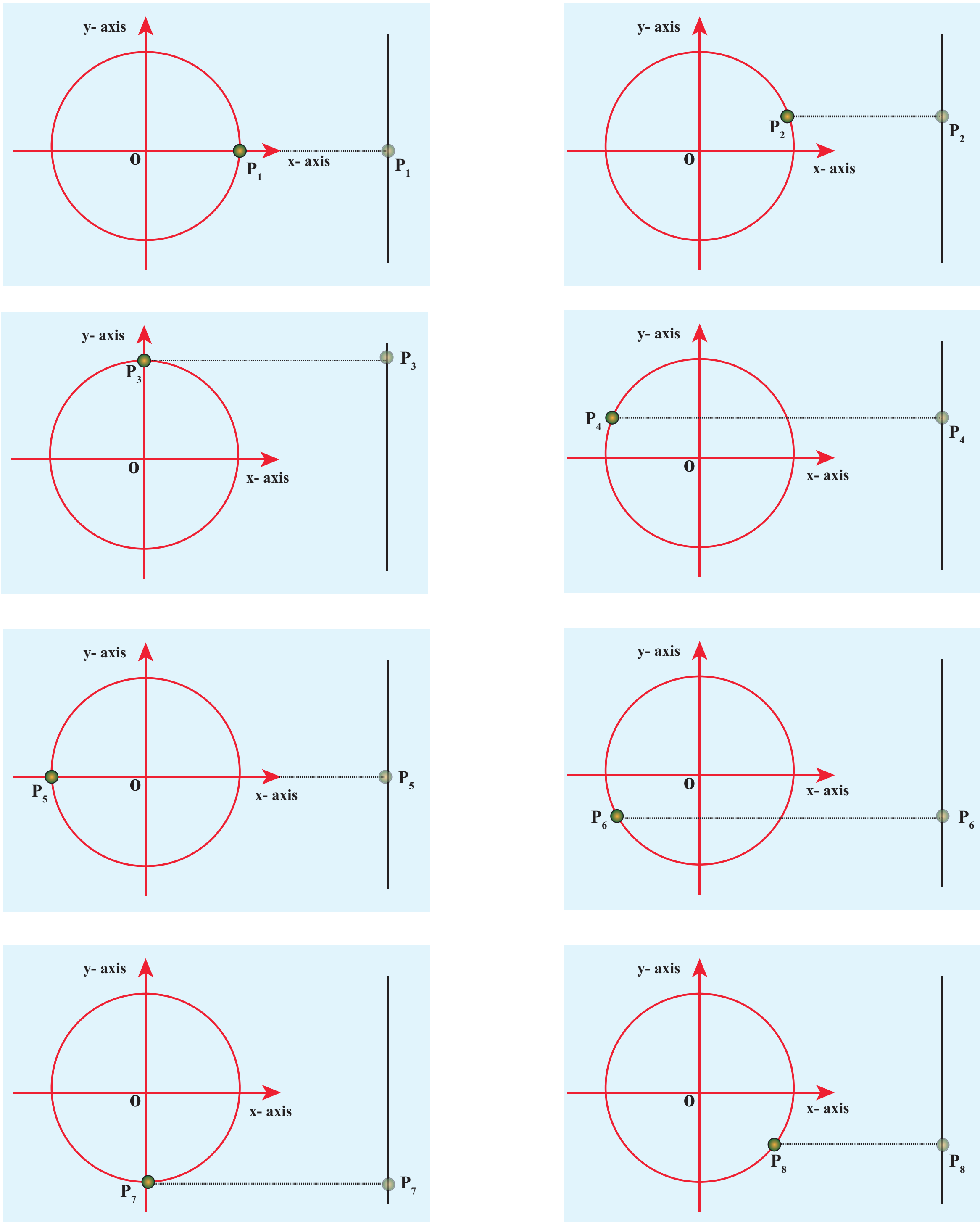


Figure 10.7 The location of a particle at each instant as projected on a vertical axis

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

As a specific example, consider a spring mass system (or oscillation of pendulum) as shown in Figure 10.8. When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion.

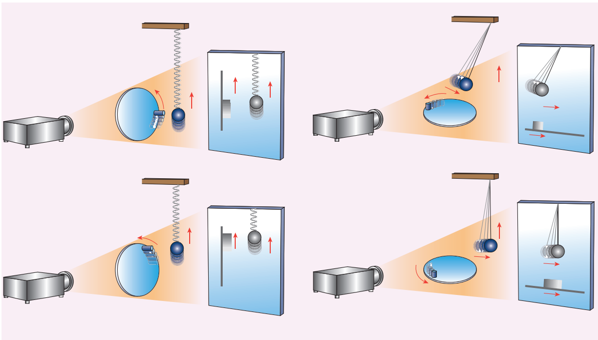


Figure 10.8 Motion of spring mass (or simple pendulum) related to uniform circular motion

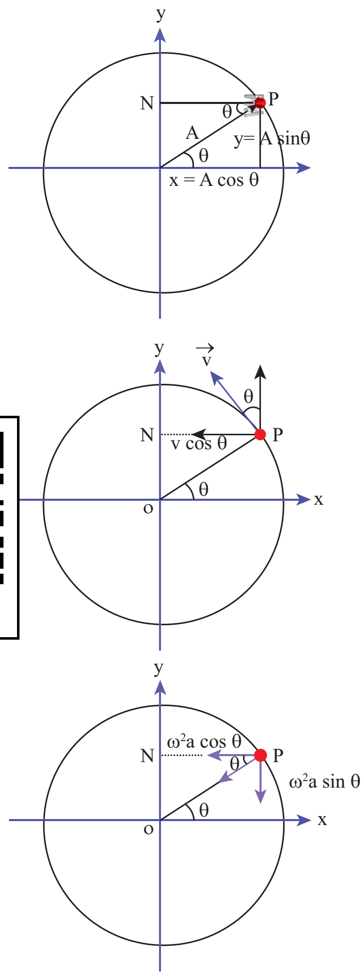
Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straightline motion which is simple harmonic in nature. The circle is known as reference circle of the simple harmonic motion. *The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.*

ACTIVITY

a. Sketch the projection of ***spiral in*** motion as a wave form.

b. Sketch the projection of ***spiral out*** motion as a wave form.

10.2.2 Displacement, velocity, acceleration and its graphical representation - SHM



BSLAUV

Figure 10.9 Displacement, velocity and acceleration of a particle at some instant of time

The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement. Let *P* be the position of the particle on a circle of radius A at some instant of time *t* as shown in Figure 10.9. Then its displacement y at that instant of time *t* can be derived as follows

In ΔOPN

$$\sin\theta = \frac{ON}{OP} \Rightarrow ON = OP\sin\theta \qquad (10.5)$$

But $\theta = \omega t$, ON = y and OP = A

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$y = A \sin \omega t \qquad (10.6)$$

The displacement y takes maximum value (which is equal to A) when $\sin \omega t = 1$. This *maximum displacement from the mean position is known as amplitude (A) of the vibrating particle.* For simple harmonic motion, the amplitude is constant. But, in general, for any motion other than simple harmonic, the amplitude need not be constant, it may vary with time.

Velocity

The rate of change of displacement is velocity. Taking derivative of equation (10.6) with respect to time, we get

$$v = \frac{dy}{dt} = \frac{d}{dt}(A \sin \omega t)$$

For circular motion (of constant radius), amplitude A is a constant and further, for uniform circular motion, angular velocity ω is a constant. Therefore,

$$v = \frac{dy}{dt} = A\,\omega \cos \omega t \qquad (10.7)$$

Using trigonometry identity,

$\sin^2 \omega t + \cos^2 \omega t = 1 \Rightarrow \cos \omega t = \sqrt{1 - \sin^2 \omega t}$

we get

$$v = A\omega\sqrt{1 - \sin^2 \omega t}$$

From equation (10.6),

$$\sin \omega t = \frac{y}{A}$$

$$v = A\omega\sqrt{1 - \left(\frac{y}{A}\right)^2}$$

$$v = \omega\sqrt{A^2 - y^2} \qquad (10.8)$$

From equation (10.8), when the displacement $y = 0$, the velocity $v = \omega A$ (maximum) and for the maximum displacement $y = A$, the velocity $v = 0$ (minimum).

As displacement increases from zero to maximum, the velocity decreases from maximum to zero. This is repeated.

Since velocity is a vector quantity, equation (10.7) can also be deduced by resolving in to components.

Acceleration

The rate of change of velocity is acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(A\omega \cos \omega t\right)$$

$$a = -\omega^2 A \sin \omega t = -\omega^2 y \qquad (10.9)$$

$$\therefore \quad a = \frac{d^2 y}{dt^2} = -\omega^2 y \qquad (10.10)$$

From the Table 10.1 and figure 10.10, we observe that at the mean position

Table 10.1 Displacement, velocity and acceleration at different instant of time.

Time	0	$\frac{T}{4}$	$\frac{2T}{4}$	$\frac{3T}{4}$	$\frac{4T}{4} = T$
ωt	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement $y = A \sin \omega t$	0	A	0	$-A$	0
Velocity $v = A\,\omega \cos \omega t$	$A\,\omega$	0	$-A\,\omega$	0	$A\,\omega$
Acceleration $a = -A\,\omega^2 \sin \omega t$	0	$-A\,\omega^2$	0	$A\,\omega^2$	0

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

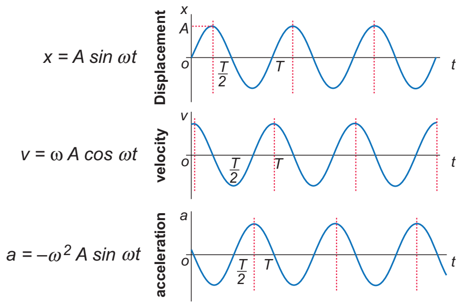


Figure 10.10 Variation of displacement, velocity and acceleration at different instant of time

(y = 0), velocity of the particle is maximum but the acceleration of the particle is zero. At the extreme position ($y = \pm A$), the velocity of the particle is zero but the acceleration is maximum $\pm A\omega^2$ acting in the opposite direction.

EXAMPLE 10.3

Which of the following represent simple harmonic motion?

(i) $x = A \sin \omega t + B \cos \omega t$

(ii) $x = A \sin \omega t + B \cos 2\omega t$

(iii) $x = A\, e^{i\omega t}$

(iv) $x = A \ln \omega t$

Solution

(i) $x = A \sin \omega t + B \cos \omega t$

$$\frac{dx}{dt} = A\,\omega \cos \omega t - B\,\omega \sin \omega t$$

$$\frac{d^2x}{dt^2} = -\,\omega^2\,(A \sin \omega t + B \cos \omega t)$$

$$\frac{d^2x}{dt^2} = -\,\omega^2 x$$

This differential equation is similar to the differential equation of SHM (equation 10.10).

Therefore, $x = A \sin \omega t + B \cos \omega t$ represents SHM.

(ii) $x = A \sin \omega t + B \cos 2\omega t$

$$\frac{dx}{dt} = A\,\omega \cos \omega t - B\,(2\omega)\sin 2\omega t$$

$$\frac{d^2x}{dt^2} = -\,\omega^2\,(A \sin \omega t + 4B \cos 2\omega t)$$

$$\frac{d^2x}{dt^2} \neq -\,\omega^2 x$$

This differential equation is not like the differential equation of a SHM (equation 10.10). Therefore, $x = A \sin \omega t + B \cos 2\omega t$ does not represent SHM.

(iii) $x = A\, e^{i\omega t}$

$$\frac{dx}{dt} = A\,\omega e^{i\omega t}$$

$$\frac{d^2x}{dt^2} = -A\,\omega^2\, e^{i\omega t} = -\omega^2 x$$

This differential equation is like the differential equation of SHM (equation 10.10). Therefore, $x = A\, e^{i\omega t}$ represents SHM.

(iv) $x = A \ln \omega t$

$$\frac{dx}{dt} = \left(\frac{A}{\omega t}\right)\omega = \frac{A}{t}$$

$$\frac{d^2x}{dt^2} = -\frac{A}{t^2} \Rightarrow \frac{d^2x}{dt^2} \neq -\omega^2 x$$

This differential equation is not like the differential equation of a SHM (equation 10.10). Therefore, $x = A \ln \omega t$ does not represent SHM.

EXAMPLE 10.4

Consider a particle undergoing simple harmonic motion. The velocity of the particle at position x_1 is v_1 and velocity of the particle at position x_2 is v_2. Show that the ratio of time period and amplitude is

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

Solution

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Using equation (10.8)

$v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2 (A^2 - x^2)$

Therefore, at position x_1,

$$v_1^2 = \omega^2(A^2 - x_1^2) \qquad (1)$$

Similarly, at position x_2,

$$v_2^2 = \omega^2(A^2 - x_2^2) \qquad (2)$$

Subtrating (2) from (1), we get

$$v_1^2 - v_2^2 = \omega^2(A^2 - x_1^2) - \omega^2(A^2 - x_2^2)$$

$$= \omega^2(x_2^2 - x_1^2)$$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \qquad (3)$$

Dividing (1) and (2), we get

$$\frac{v_1^2}{v_2^2} = \frac{\omega^2\left(A^2 - x_1^2\right)}{\omega^2\left(A^2 - x_2^2\right)} \Rightarrow A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \qquad (4)$$

Dividing equation (3) and equation (4), we have

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

10.2.3 Time period, frequency, phase, phase difference and epoch in SHM.

(i) Time period

The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. For one complete revolution, the time taken is t = T, therefore

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} \qquad (10.11)$$

Then, the displacement of a particle executing simple harmonic motion can be written either as sine function or cosine function.

$$y(t) = A \sin \frac{2\pi}{T} t \text{ or } y(t) = A \cos \frac{2\pi}{T} t$$

where T represents the time period. Suppose the time t is replaced by $t + T$, then the function

$$y(t + T) = A \sin \frac{2\pi}{T}(t + T)$$

$$= A \sin\left(\frac{2\pi}{T} t + 2\pi\right)$$

$$= A \sin \frac{2\pi}{T} t = y(t)$$

$$y(t + T) = y(t)$$

Thus, the function repeats after one time period. This $y(t)$ is an example of periodic function.

(ii) Frequency and angular frequency

The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is s^{-1} or hertz (In symbol, Hz).

Mathematically, frequency is related to time period by

$$f = \frac{1}{T} \qquad (10.12)$$

The number of cycles (or revolutions) per second is called angular frequency. It is usually denoted by the Greek small letter 'omega', ω.

Comparing equation (10.11) and equation (10.12), angular frequency and frequency are related by

$$\omega = 2\pi f \qquad (10.13)$$

SI unit for angular frequency is rad s^{-1}. (read it as radian per second)

(iii) Phase

The phase of a vibrating particle at any instant completely specifies the state of the particle. It expresses the position and direction of motion of the particle at that instant with respect to its mean position (Figure 10.11).

$$y = A \sin(\omega t + \varphi_0) \qquad (10.14)$$

where $\omega t + \varphi_0 = \varphi$ *is called the phase of the vibrating particle. At time* $t = 0\,s$ *(initial time), the phase* $\varphi = \varphi_0$ *is called epoch (initial phase) where* φ_0 *is called the angle of epoch.*

Phase difference: Consider two particles executing simple harmonic motions. Their

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

equations are $y_1 = A\sin(\omega t + \varphi_1)$ and $y_2 = A\sin(\omega t + \varphi_2)$, then the phase difference $\Delta\varphi = (\omega t + \varphi_2) - (\omega t + \varphi_1) = \varphi_2 - \varphi_1$.

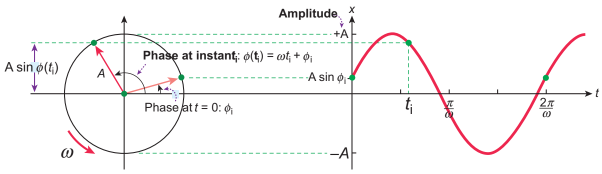


Figure 10.11 The phase of vibrating particle at two instant of time.

EXAMPLE 10.5

A nurse measured the average heart beats of a patient and reported to the doctor in terms of time period as 0.8 *s*. Express the heart beat of the patient in terms of number of beats measured per minute.

Solution

Let the number of heart beats measured be *f*. Since the time period is inversely proportional to the heart beat, then

$$f = \frac{1}{T} = \frac{1}{0.8} = 1.25\, s^{-1}$$

One minute is 60 second,

$$(1 \text{ second} = \frac{1}{60} \text{ minute} \Rightarrow 1\ s^{-1} = 60 \text{ min}^{-1})$$

$f = 1.25\ s^{-1} \Rightarrow f = 1.25 \times 60 \text{ min}^{-1} = 75$ beats per minute

EXAMPLE 10.6

Calculate the amplitude, angular frequency, frequency, time period and initial phase for the simple harmonic oscillation given below

a. $y = 0.3\sin(40\pi t + 1.1)$

b. $y = 2\cos(\pi t)$

c. $y = 3\sin(2\pi t - 1.5)$

Solution

Simple harmonic oscillation equation is $y = A\sin(\omega t + \varphi_0)$ or $y = A\cos(\omega t + \varphi_0)$

a. For the wave, $y = 0.3\sin(40\pi t + 1.1)$

Amplitude is $A = 0.3$ unit

Angular frequency $\omega = 40\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{20} = 0.05\,s$

Initial phase is $\varphi_0 = 1.1$ rad

b. For the wave, $y = 2\cos(\pi t)$

Amplitude is $A = 2$ unit

Angular frequency $\omega = \pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = 0.5\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{0.5} = 2\,s$

Initial phase is $\varphi_0 = 0$ rad

c. For the wave, $y = 3\sin(2\pi t + 1.5)$

Amplitude is $A = 3$ unit

Angular frequency $\omega = 2\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{1} = 1\,s$

Initial phase is $\varphi_0 = 1.5$ rad

EXAMPLE 10.7

Show that for a simple harmonic motion, the phase difference between

a. displacement and velocity is $\frac{\pi}{2}$ radian or 90°.

b. velocity and acceleration is $\frac{\pi}{2}$ radian or 90°.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

c. displacement and acceleration is π radian or 180°.

Solution

a. The displacement of the particle executing simple harmonic motion
$y = A \sin\omega t$

Velocity of the particle is

$$v = A\,\omega\,\cos\omega t = A\,\omega\,\sin\left(\omega t + \frac{\pi}{2}\right)$$

The phase difference between displacement and velocity is $\frac{\pi}{2}$.

b. The velocity of the particle is

$$v = A\,\omega\,\cos\,\omega t$$

Acceleration of the particle is

$$a = -A\omega^2\,\sin\omega t = A\omega^2\,\cos\left(\omega t + \frac{\pi}{2}\right)$$

The phase difference between velocity and acceleration is $\frac{\pi}{2}$.

c. The displacement of the particle is
$y = A \sin\omega t$

Acceleration of the particle is

$a = -A\,\omega^2 \sin\,\omega t = A\,\omega^2 \sin(\omega t + \pi)$

The phase difference between displacement and acceleration is π.

10.3 ANGULAR SIMPLE HARMONIC MOTION

10.3.1 Time period and frequency of angular SHM

When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation. The point

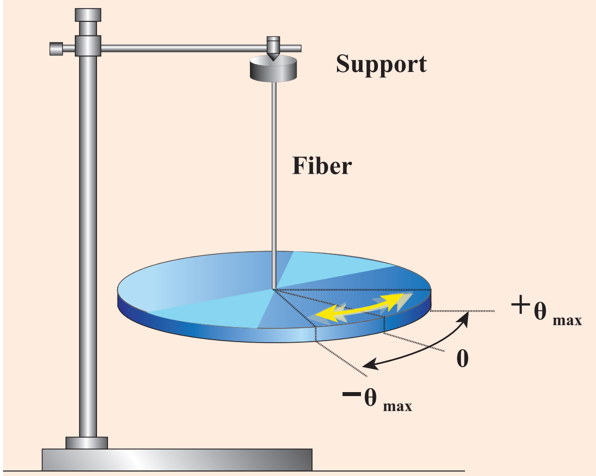


Figure 10.12 A body (disc) allowed to rotate freely about an axis

at which the resultant torque acting on the body is taken to be zero is called mean position. If the body is displaced from the mean position, then the resultant torque acts such that it is proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position. (Note: Torque is explained in unit 5)

Let $\vec{\theta}$ be the angular displacement of the body and the resultant torque $\vec{\tau}$ acting on the body is

$$\vec{\tau} \propto \vec{\theta} \qquad (10.15)$$

$$\vec{\tau} = -\kappa\vec{\theta} \qquad (10.16)$$

κ is the restoring torsion constant, which is torque per unit angular displacement. If I is the moment of inertia of the body and $\vec{\alpha}$ is the angular acceleration then

$$\vec{\tau} = I\vec{\alpha} = -\kappa\vec{\theta}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

But $\vec{\alpha}=\frac{d^2\vec{\theta}}{dt^2}$ and therefore,

$$\frac{d^2\vec{\theta}}{dt^2}=-\frac{\kappa}{I}\vec{\theta} \qquad (10.17)$$

This differential equation resembles simple harmonic differential equation.

So, comparing equation (10.17) with simple harmonic motion given in equation (10.10), we have

$$\omega=\sqrt{\frac{\kappa}{I}}\, rad\, s^{-1} \qquad (10.18)$$

The frequency of the angular harmonic motion (from equation 10.13) is

$$f=\frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}\, Hz \qquad (10.19)$$

The time period (from equation 10.12) is

$$T=2\pi\sqrt{\frac{I}{\kappa}} \text{ second} \qquad (10.20)$$

10.3.2 Comparison of Simple Harmonic Motion and Angular Simple Harmonic Motion

In linear simple harmonic motion, the displacement of the particle is measured in terms of linear displacement $\vec{r}$ The restoring force is $\vec{F}=-k\vec{r}$, where k is a spring constant or force constant which is force per unit displacement. In this case, the inertia factor is mass of the body executing simple harmonic motion.

In angular simple harmonic motion, the displacement of the particle is measured in terms of angular displacement $\vec{\theta}$. Here, the spring factor stands for torque constant i.e., the moment of the couple to produce unit angular displacement or the restoring torque per unit angular displacement. In this case, the inertia factor stands for moment of inertia of the body executing angular simple harmonic oscillation.

Table 10.2 Comparision of simple harmonic motion and angular harmonic motion

S.No	Simple Harmonic Motion	Angular Harmonic Motion
1.	The displacement of the particle is measured in terms of linear displacement $\vec{r}$.	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$ (also known as angle of twist).
2.	Acceleration of the particle is $\vec{a}=-\omega^2\vec{r}$	Angular acceleration of the particle is $\vec{\alpha}=-\omega^2\vec{\theta}$.
3.	Force, $\vec{F}=m\vec{a}$, where m is called mass of the particle.	Torque, $\vec{\tau}=I\vec{\alpha}$, where I is called moment of inertia of a body.
4.	The restoring force $\vec{F}=-k\vec{r}$, where k is restoring force constant.	The restoring torque $\vec{\tau}=-\kappa\vec{\theta}$, where the symbol κ (Greek alphabet is pronounced as '*kappa*') is called restoring torsion constant. It depends on the property of a particular torsion fiber.
5.	Angular frequency, $\omega=\sqrt{\frac{k}{m}}$ rad s^{-1}	Angular frequency, $\omega=\sqrt{\frac{\kappa}{I}}$ rad s^{-1}

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

10.4 LINEAR SIMPLE HARMONIC OSCILLATOR (LHO)

10.4.1 Horizontal oscillations of a spring-mass system

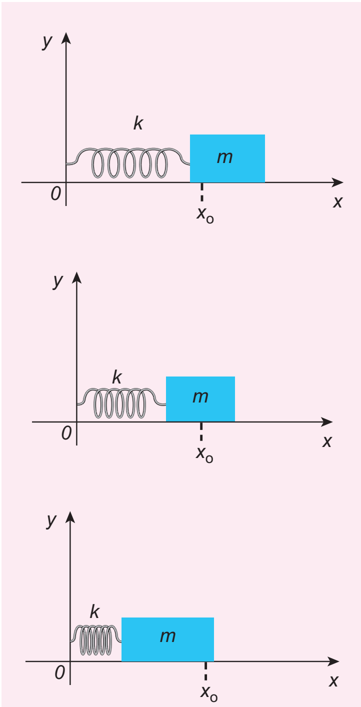


Figure 10.13 Horizontal oscillation of a spring-mass system

Consider a system containing a block of mass m attached to a massless spring with stiffness constant or force constant or spring constant k placed on a smooth horizontal surface (frictionless surface) as shown in Figure 10.13. Let x_0 be the equilibrium position or mean position of mass m when it is left undisturbed. Suppose the mass is displaced through a small displacement x towards right from its equilibrium position and then released, it will oscillate back and forth about its mean position x_0. Let F be the restoring force (due to stretching of the spring) which is proportional to the amount of displacement of block. For one dimensional motion, mathematically, we have

$$F \propto x$$
$$F = -k\,x$$

where negative sign implies that the restoring force will always act opposite to the direction of the displacement. This equation is called Hooke's law (refer to unit 7). Notice that, the restoring force is linear with the displacement (i.e., the exponent of force and displacement are unity). This is not always true; in case if we apply a very large stretching force, then the amplitude of oscillations becomes very large (which means, force is proportional to displacement containing higher powers of x) and therefore, the oscillation of the system is not linear and hence, it is called non-linear oscillation. We restrict ourselves only to linear oscillations throughout our discussions, which means Hooke's law is valid (force and displacement have a linear relationship).

From Newton's second law, we can write the equation for the particle executing simple harmonic motion

$$m\frac{d^2x}{dt^2} = -k\,x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad (10.21)$$

Comparing the equation (10.21) with simple harmonic motion equation (10.10), we get

$$\omega^2 = \frac{k}{m}$$

which means the angular frequency or natural frequency of the oscillator is

$$\omega = \sqrt{\frac{k}{m}}\ \text{rad } s^{-1} \qquad (10.22)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The frequency of the oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ Hertz} \qquad (10.23)$$

and the time period of the oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \text{ seconds} \qquad (10.24)$$

Notice that ***in simple harmonic motion, the time period of oscillation is independent of amplitude***. This is valid only if the amplitude of oscillation is small.
The solution of the differential equation of a SHM may be written as

$$x(t) = A\sin(\omega t + \varphi) \qquad (10.25)$$

Or

$$x(t) = A\cos(\omega t + \varphi) \qquad (10.26)$$

where A, ω and ϕ are constants. General solution for differential equation 10.21 is $x(t) = A\sin(\omega t + \varphi) + B\cos(\omega t + \varphi)$ where A and B are contants.

Note

(a) Since, mass is inertial porperty and spring constant is an elastic property.

Time period is $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{\text{Inertial property}}{\text{Elasticproperty}}} = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|}$$

(b) $\frac{\text{Displacement}}{\text{acceleration}} = \frac{x}{\frac{d^2x}{dt^2}} = -\frac{m}{k}$, whose

modulus value or magnitude is $\frac{m}{k}$

hence, time period $T = 2\pi\sqrt{\frac{m}{k}}$

10.4.2 Vertical oscillations of a spring

Figure 10.14 Springs

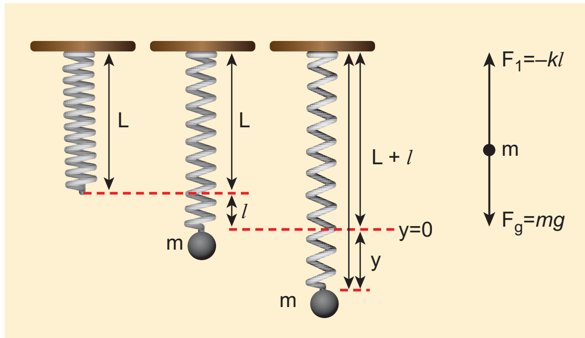


Figure 10.15 A massless spring with stiffness constant k

Let us consider a massless spring with stiffness constant or force constant k attached to a ceiling as shown in Figure 10.15. Let the length of the spring before loading mass m be L. If the block of mass m is attached to the other end of spring, then the spring elongates by a length l. Let F_1 be the restoring force due to stretching of spring. Due to mass m, the gravitational force acts vertically downward. We can draw free-body diagram for this system as shown in Figure 10.15. When the system is under equilibrium,

$$F_1 + mg = 0 \qquad (10.27)$$

But the spring elongates by small displacement l, therefore,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$F_1 \propto l \Rightarrow F_1 = -k\,l \qquad (10.28)$$

Substituting equation (10.28) in equation (10.27), we get

$$-k\,l + mg = 0$$
$$mg = kl$$
or
$$\frac{m}{k} = \frac{l}{g} \qquad (10.29)$$

Suppose we apply a very small external force on the mass such that the mass further displaces downward by a displacement y, then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is $y + l$) is

$$F_2 \propto (y + l)$$
$$F_2 = -k\,(y + l) = -ky - kl \qquad (10.30)$$

Since, the mass moves up and down with acceleration $\frac{d^2y}{dt^2}$, by drawing the free body diagram for this case, we get

$$-ky - kl + mg = m\frac{d^2y}{dt^2} \qquad (10.31)$$

The net force acting on the mass due to this stretching is

$$F = F_2 + mg$$
$$F = -ky - kl + mg \qquad (10.32)$$

The gravitational force opposes the restoring force. Substituting equation (10.29) in equation (10.32), we get

$$F = -ky - kl + kl = -ky$$

Applying Newton's law, we get

$$m\frac{d^2y}{dt^2} = -k\,y$$
$$\frac{d^2y}{dt^2} = -\frac{k}{m}y \qquad (10.33)$$

The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ second} \qquad (10.34)$$

Note The time period obtained for horizontal oscillations of spring and for vertical oscillations of spring are found to be equal.

The time period can be rewritten using equation (10.29)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}} \text{ second} \qquad (10.35)$$

The acceleration due to gravity g can be computed from the formula

$$g = 4\pi^2\left(\frac{l}{T^2}\right) \text{ m s}^{-2} \qquad (10.36)$$

EXAMPLE 10.8

A spring balance has a scale which ranges from 0 to 25 kg and the length of the scale is 0.25m. It is taken to an unknown planet X where the acceleration due to gravity is 11.5 m s^{-1}. Suppose a body of mass M kg is suspended in this spring and made to oscillate with a period of 0.50 s. Compute the gravitational force acting on the body.

Solution

Let us first calculate the stiffness constant of the spring balance by using equation (10.29),

$$k = \frac{mg}{l} = \frac{25 \times 11.5}{0.25} = 1150\ N\,m^{-1}$$

The time period of oscillations is given by $T = 2\pi\sqrt{\frac{M}{k}}$, where M is the mass of the body.

Since, M is unknown, rearranging, we get

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$M = \frac{kT^2}{4\pi^2} = \frac{(1150)(0.5)^2}{4\pi^2} = 7.3\,kg$$

The gravitational force acting on the body is $W = Mg = 7.3 \times 11.5 = 83.95\ N \approx 84\ N$

10.4.3 Combinations of springs

Figure 10.16 Combination of spring as a shock-absorber in the motor cycle

Spring constant or force constant, also called as stiffness constant, is a measure of the stiffness of the spring. Larger the value of the spring constant, stiffer is the spring. This implies that we need to apply more force to compress or elongate the spring. Similarly, smaller the value of spring constant, the spring can be stretched (elongated) or compressed with lesser force. Springs can be connected in two ways. Either the springs can be connected end to end, also known as series connection, or alternatively, connected in parallel. In the following subsection, we compute the effective spring constant when

a. Springs are connected in series
b. Springs are connected in parallel

a. **Springs connected in series**

When two or more springs are connected in series, we can replace (by removing) all the springs in series with an equivalent spring (effective spring) whose net effect is the same as if all the springs are in series connection. Given the value of individual spring constants k_1, k_2, k_3,... (known quantity), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_s (unknown quantity). For simplicity, let us consider only two springs whose spring constant are k_1 and k_2 and which can be attached to a mass m as shown in Figure 10.17. The results thus obtained can be generalized for any number of springs in series.

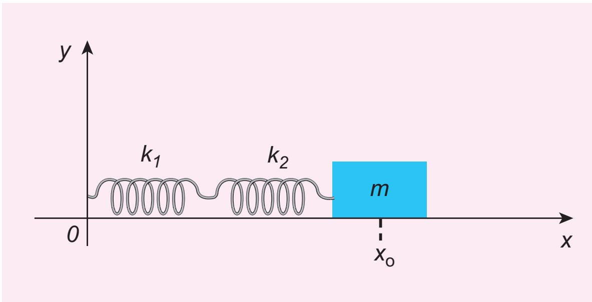


Figure 10.17 Springs are connected in series

Let F be the applied force towards right as shown in Figure 10.18. Since the spring constants for different spring are different and the connection points between them is not rigidly fixed, the strings can stretch in different lengths. Let x_1 and x_2 be the elongation of springs from their equilibrium position (un-stretched position) due to the applied force F. Then, the net displacement of the mass point is

$$x = x_1 + x_2 \qquad (10.37)$$

From Hooke's law, the net force

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

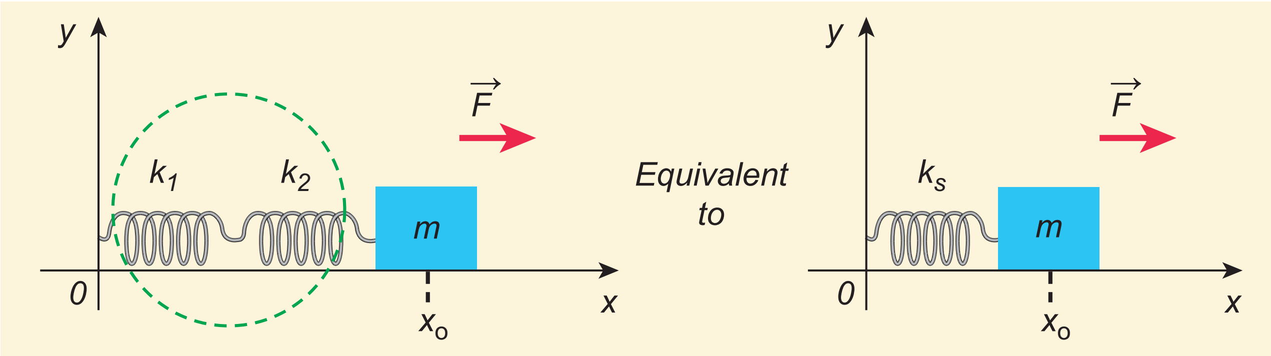


Figure 10.18 Effective spring constant in series connection

$$F = -k_s(x_1 + x_2) \Rightarrow x_1 + x_2 = -\frac{F}{k_s} \quad (10.38)$$

For springs in series connection

$$-k_1x_1 = -k_2x_2 = F$$

$$\Rightarrow x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2} \quad (10.39)$$

Therefore, substituting equation (10.39) in equation (10.38), the *effective spring constant* can be calculated as

$$-\frac{F}{k_1} - \frac{F}{k_2} = -\frac{F}{k_s}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

Or

$$k_s = \frac{k_1k_2}{k_1 + k_2} \text{ Nm}^{-1} \quad (10.40)$$

Suppose we have n springs connected in series, the effective spring constant in series is

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} = \sum_{i=1}^{n}\frac{1}{k_i} \quad (10.41)$$

If all spring constants are identical i.e., $k_1 = k_2 = \ldots = k_n = k$ then

$$\frac{1}{k_s} = \frac{n}{k} \Rightarrow k_s = \frac{k}{n} \quad (10.42)$$

This means that the effective spring constant reduces by the factor n. Hence, for springs in series connection, the effective spring constant is lesser than the individual spring constants.

From equation (10.39), we have,

$$k_1x_1 = k_2x_2$$

Then the ratio of compressed distance or elongated distance x_1 and x_2 is

$$\frac{x_2}{x_1} = \frac{k_1}{k_2} \quad (10.43)$$

The elastic potential energy stored in first and second springs are $V_1 = \frac{1}{2}k_1x_1^2$ and $V_2 = \frac{1}{2}k_2x_2^2$ respectively. Then, their ratio is

$$\frac{V_1}{V_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1}{k_2}\left(\frac{x_1}{x_2}\right)^2 = \frac{k_2}{k_1} \quad (10.44)$$

Note

The reciprocal of stiffness constant is called flexibility constant or compliance, denoted by C. It is measured in m N^{-1}

If n springs are connected in series :

$$\text{net compliance } C_s = \sum_{i=1}^{n} C_i$$

If n springs are connected in parallel :

$$\frac{1}{C_p} = \sum_{i=1}^{n}\frac{1}{C_i}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 10.9

Consider two springs whose force constants are 1 N m^{-1} and 2 N m^{-1} which are connected in series. Calculate the effective spring constant (k_s) and comment on k_s.

Solution

$$k_1 = 1\text{ N m}^{-1},\ k_2 = 2\text{ N m}^{-1}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2}\text{ N m}^{-1}$$

$$k_s = \frac{1\times 2}{1+2} = \frac{2}{3}\text{ N m}^{-1}$$

$$k_s < k_1 \text{ and } k_s < k_2$$

Therefore, the effective spring constant is lesser than both k_1 and k_2.

b. Springs connected in parallel

When two or more springs are connected in parallel, we can replace (by removing) all these springs with an equivalent spring (effective spring) whose net effect is same as if all the springs are in parallel connection. Given the values of individual spring constants to be k_1, k_2, k_3, ... (known quantities), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_p (unknown quantity). For simplicity, let us consider only two springs of spring constants k_1 and k_2 attached to a mass m as shown in Figure 10.19. The results can be generalized to any number of springs in parallel.

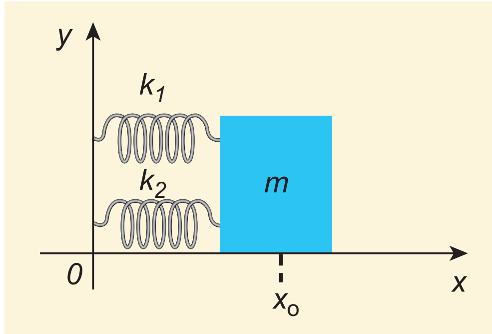


Figure 10.19 Springs connected in parallel

Let the force F be applied towards right as shown in Figure 10.20. In this case, both the springs elongate or compress by the same amount of displacement. Therefore, net force for the displacement of mass m is

$$F = -k_p x \qquad (10.45)$$

where k_p is called ***effective spring constant***. Let the first spring be elongated by a displacement x due to force F_1 and second spring be elongated by the same displacement x due to force F_2, then the net force

$$F = -k_1 x - k_2 x \qquad (10.46)$$

Equating equations (10.46) and (10.45), we get

$$k_p = k_1 + k_2 \qquad (10.47)$$

Generalizing, for n springs connected in parallel,

$$k_p = \sum_{i=1}^{n} k_i \qquad (10.48)$$

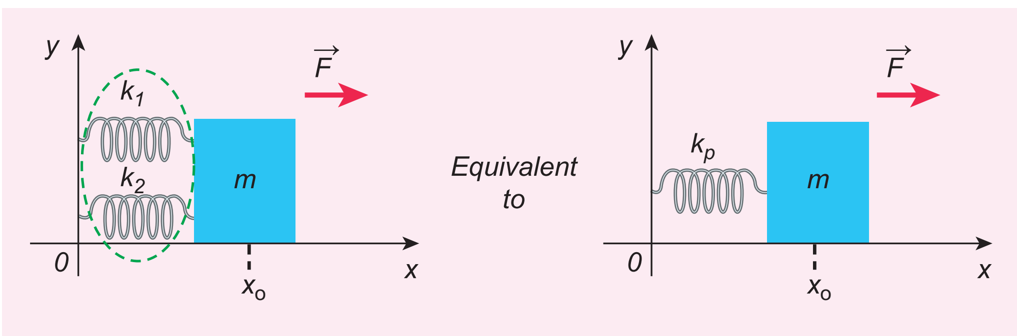


Figure 10.20 Effective spring constant in parallel connection

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

If all spring constants are identical i.e., $k_1 = k_2 = ... = k_n = k$ then

$$k_p = n\,k \qquad (10.49)$$

This implies that the effective spring constant increases by a factor n. Hence, for the springs in parallel connection, the effective spring constant is greater than individual spring constant.

Note The spring constant is inversely proportional to the length of the spring

$$k \propto \frac{1}{\textit{length of the spring}}$$

If the spring is cut into two pieces, one piece with length l_1 and other with length l_2, such that $l_1 = nl_2$, then spring constant of first length is $k_1 = \frac{k(n+1)}{n}$ and spring constant of second length is $k_2 = (n+1)\,k$, where k is the original spring constant before cutting into pieces.

EXAMPLE 10.10

Consider two springs with force constants $1\ \text{N m}^{-1}$ and $2\ \text{N m}^{-1}$ connected in parallel. Calculate the effective spring constant (k_p) and comment on k_p.

Solution

$$k_1 = 1\ \text{N m}^{-1},\ k_2 = 2\ \text{N m}^{-1}$$

$$k_p = k_1 + k_2\ \text{N m}^{-1}$$

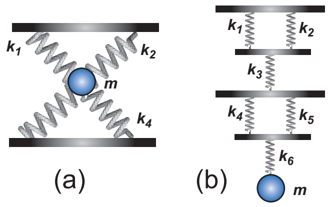
$$k_p = 1 + 2 = 3\ \text{N m}^{-1}$$

$$k_p > k_1 \text{ and } k_p > k_2$$

Therefore, the effective spring constant is greater than both k_1 and k_2.

EXAMPLE 10.11

Calculate the equivalent spring constant for the following systems and also compute if all the spring constants are equal:



(a) (b)

Solution

a. Since k_1 and k_2 are parallel, $k_u = k_1 + k_2$
Similarly, k_3 and k_4 are parallel, therefore, $k_d = k_3 + k_4$
But k_u and k_d are in series,

$$\text{therefore, } k_{eq} = \frac{k_u k_d}{k_u + k_d}$$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k$
Which means, $k_u = 2k$ and $k_d = 2k$

$$\text{Hence, } k_{eq} = \frac{4k^2}{4k} = k$$

b. Since k_1 and k_2 are parallel, $k_A = k_1 + k_2$
Similarly, k_4 and k_5 are parallel, therefore, $k_B = k_4 + k_5$
But k_A, k_3, k_B, and k_6 are in series,

$$\text{therefore, } \frac{1}{k_{eq}} = \frac{1}{k_A} + \frac{1}{k_3} + \frac{1}{k_B} + \frac{1}{k_6}$$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k$
which means, $k_A = 2k$ and $k_B = 2k$

$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{2k} + \frac{1}{k} = \frac{3}{k}$$

$$k_{eq} = \frac{k}{3}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 10.12

A mass m moves with a speed v on a horizontal smooth surface and collides with a nearly massless spring whose spring constant is k. If the mass stops after collision, compute the maximum compression of the spring.

Solution

When the mass collides with the spring, from the law of conservation of energy "the loss in kinetic energy of mass is gain in elastic potential energy by spring".

Let x be the distance of compression of spring, then the law of conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{k}}$$

10.4.4 Oscillations of a simple pendulum in SHM and laws of simple pendulum

Simple pendulum

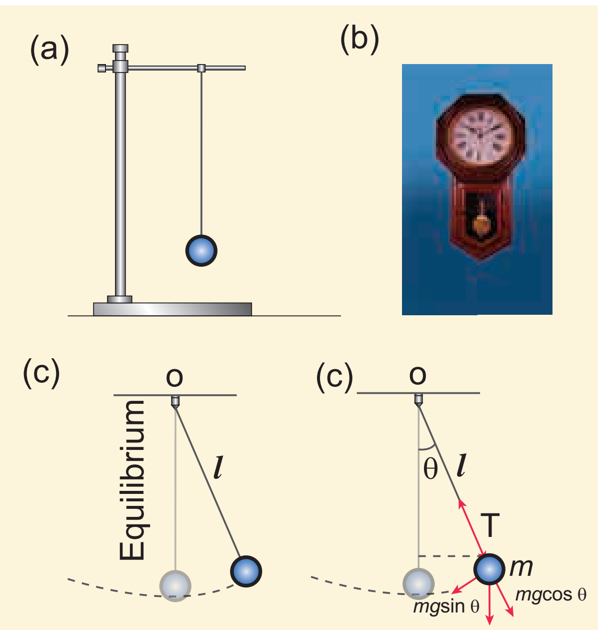


Figure 10.21 Simple pendulum

A pendulum is a mechanical system which exhibits periodic motion. It has a bob with mass m suspended by a long string (assumed to be massless and inextensible string) and the other end is fixed on a stand as shown in Figure 10.21 (a). At equilibrium, the pendulum does not oscillate and hangs vertically downward. Such a position is known as mean position or equilibrium position. When a pendulum is displaced through a small displacement from its equilibrium position and released, the bob of the pendulum executes to and fro motion. Let l be the length of the pendulum which is taken as the distance between the point of suspension and the centre of gravity of the bob. Two forces act on the bob of the pendulum at any displaced position, as shown in the Figure 10.21 (d),

(i) The gravitational force acting on the body ($\vec{F} = m\vec{g}$) which acts vertically downwards.

(ii) The tension in the string $\vec{T}$ which acts along the string to the point of suspension.

Resolving the gravitational force into its components:

a. **Normal component:** The component along the string but in opposition to the direction of tension, $F_{as} = mg\cos\theta$.

b. **Tangential component:** The component perpendicular to the string i.e., along tangential direction of arc of swing, $F_{ps} = mg\sin\theta$.

Therefore, The normal component of the force is, along the string,

$$T - W_{as} = m\frac{v^2}{l}$$

Here v is speed of bob

$$T - mg\cos\theta = m\frac{v^2}{l} \quad (10.50)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Note From Newton's 2nd law, $\vec{F} = m\vec{a}$ Here, the net force on the L.H.S is T-W_{as}
In R.H.S, $m\vec{a}$ is equivalent to the centripetal force = $\frac{mv^2}{l}$ which makes the bob oscillate.

From the Figure 10.21, we can observe that the tangential component W_{ps} of the gravitational force always points towards the equilibrium position i.e., the direction in which it always points opposite to the direction of displacement of the bob from the mean position. Hence, in this case, the tangential force is nothing but the restoring force. Applying Newton's second law along tangential direction, we have

$$m\frac{d^2s}{dt^2} + F_{ps} = 0 \Rightarrow m\frac{d^2s}{dt^2} = -F_{ps}$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta \qquad (10.51)$$

where, s is the position of bob which is measured along the arc. Expressing arc length in terms of angular displacement i.e.,

$$s = l\,\theta \qquad (10.52)$$

then its acceleration,

$$\frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2} \qquad (10.53)$$

Substituting equation (10.53) in equation (10.51), we get

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \qquad (10.54)$$

Because of the presence of sin θ in the above differential equation, it is a non-linear differential equation (Here, homogeneous second order). Assume "the small oscillation approximation", sin $\theta \approx \theta$, the above differential equation becomes linear differential equation.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \qquad (10.55)$$

This is the well known oscillatory differential equation. Therefore, the angular frequency of this oscillator (natural frequency of this system) is

$$\omega^2 = \frac{g}{l} \qquad (10.56)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{l}} \text{ in rad } s^{-1} \qquad (10.57)$$

The frequency of oscillations is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \text{ in Hz} \qquad (10.58)$$

and time period of oscillations is

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ in second} \qquad (10.59)$$

Laws of simple pendulum

The time period of a simple pendulum

a. Depends on the following laws

(i) Law of length

For a given value of acceleration due to gravity, the time period of a simple pendulum is directly proportional to the square root of length of the pendulum.

$$T \propto \sqrt{l} \qquad (10.60)$$

(ii) Law of acceleration

For a fixed length, the time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity.

$$T \propto \frac{1}{\sqrt{g}} \qquad (10.61)$$

b. Independent of the following factors

(i) Mass of the bob

The time period of oscillation is independent of mass of the simple

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

pendulum. This is similar to free fall. Therefore, in a pendulum of fixed length, it does not matter whether an elephant swings or an ant swings. Both of them will swing with the same time period.

(ii) Amplitude of the oscillations

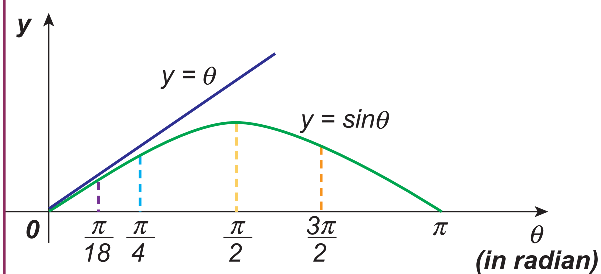
For a pendulum with small angle approximation (angular displacement is very small), the time period is independent of amplitude of the oscillation.

EXAMPLE 10.13

In simple pendulum experiment, we have used small angle approximation . Discuss the small angle approximation.

θ (in degrees)	θ(in radian)	sin θ
0	0	0
5	0.087	0.087
10	0.174	0.174
15	0.262	0.256
20	0.349	0.342
25	0.436	0.422
30	0.524	0.500
35	0.611	0.574
40	0.698	0.643
45	0.785	0.707

For θ in radian, $\sin\theta \approx \theta$ for very small angles



This means that "for θ as large as 10 degrees, $\sin\theta$ is nearly the same as θ when θ is expressed in radians". As θ increases in value $\sin\theta$ gradually becomes different from θ

Pendulum length due to effect of temperature

Suppose the suspended wire is affected due to change in temperature. The rise in temperature affects length by

$$l = l_o(1 + \alpha\,\Delta t)$$

where l_o is the original length of the wire and l is final length of the wire when the temperature is raised. Let Δt is the change in temperature and α is the co-efficient of linear expansion.

Then, $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l_0(1+\alpha\Delta t)}{g}}$

$$= 2\pi\sqrt{\frac{l_0}{g}}\sqrt{(1+\alpha\Delta t)}$$

$$T = T_0\,(1+\alpha\Delta t)^{\frac{1}{2}} \approx T_0\,(1+\frac{1}{2}\alpha\Delta t)$$

$$\Rightarrow \quad \frac{T}{T_0} - 1 = \frac{T-T_0}{T_0} = \frac{\Delta T}{T_0} = \frac{1}{2}\alpha\Delta t$$

where ΔT is the change in time period due to the effect of temperature and T_0 is the time period of the simple pendulum with original length l_0.

EXAMPLE 10.14

If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum.

Solution

Since

$$T \propto \sqrt{l}$$

Therefore,

$$T = \text{constant}\ \sqrt{l}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$\frac{T_f}{T_i}=\sqrt{\frac{l+\frac{44}{100}l}{l}}=\sqrt{1.44}=1.2$$

Therefore, $T_f = 1.2\ T_i = T_i + 20\%\ T_i$

Oscillation of liquid in a U-tube:

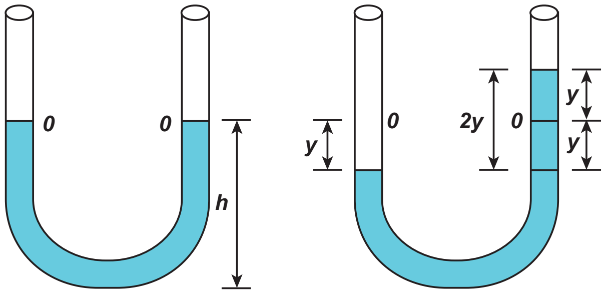


Figure 10.22 U-shaped glass tube

Consider a U-shaped glass tube which consists of two open arms with uniform cross-sectional area A. Let us pour a non-viscous uniform incompressible liquid of density ρ in the U-shaped tube to a *height* *h* as shown in the Figure 10.22. If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position *O*. It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm (edge of the tube on either side), which balances with the atmospheric pressure. Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm, and the liquid gets disturbed from equilibrium position *O*, which means, the pressure at blown arm is higher than the other arm. This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest.

Time period of the oscillation is

$$T = 2\pi\sqrt{\frac{l}{2g}}\ \text{second} \qquad (10.62)$$

10.5 ENERGY IN SIMPLE HARMONIC MOTION

a. Expression for Potential Energy

For the simple harmonic motion, the force and the displacement are related by Hooke's law

$$\vec{F} = -k\vec{r}$$

Since force is a vector quantity, in three dimensions it has three components. Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case

$$F = -\,k\,x \qquad (10.63)$$

As we have discussed in unit 4 of volume I, the work done by the conservative force field is independent of path. The potential energy U can be calculated from the following expression.

$$F = -\frac{dU}{dx} \qquad (10.64)$$

Comparing (10.63) and (10.64), we get

$$-\frac{dU}{dx} = -\,kx$$

$$dU = k\,x\,dx$$

BSV6WI

Dummy variable

The integrating variable x' (read x' as "x prime")is a dummy variable

$$\int_0^y tdt=\int_0^y xdx=\int_0^y pdp=\frac{y^2}{2}$$

Notice that the integrating variables like *t*, *x* and *p* are dummy variables because, in this integration, whether we put *t* or *x* or *p* as variable for integration, we get the same answer.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

This work done by the force F during a small displacement dx stores as potential energy

$$U(x)=\int_0^x k\,x'\,dx'=\frac{1}{2}k\left(x'\right)^2\Big|_0^x=\frac{1}{2}kx^2 \quad (10.65)$$

From equation (10.22), we can substitute the value of force constant $k = m\,\omega^2$ in equation (10.65),

$$U\left(x\right)=\frac{1}{2}m\omega^2x^2 \quad (10.66)$$

where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation (10.6), we get

$$x = A \sin \omega t$$

$$U(t)=\frac{1}{2}m\omega^2A^2\sin^2\omega t \quad (10.67)$$

This variation of U is shown below.

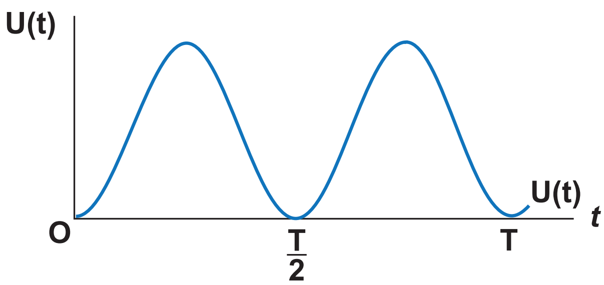


Figure 10.23 Variation of potential energy with time t

Question to think over

"If the potential energy is minimum then its second derivative is positive, why?"

b. Expression for Kinetic Energy

Kinetic energy

$$KE=\frac{1}{2}mv_x^2=\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \quad (10.68)$$

Since the particle is executing simple harmonic motion, from equation (10.6)

$$x = A \sin \omega t$$

Therefore, velocity is

$$v_x=\frac{dx}{dt}=A\omega\cos\omega t \quad (10.69)$$

$$=A\omega\sqrt{1-\left(\frac{x}{A}\right)^2}$$

$$v_x=\omega\sqrt{A^2-x^2} \quad (10.70)$$

Hence,

$$KE=\frac{1}{2}mv_x^2=\frac{1}{2}m\omega^2\left(A^2-x^2\right) \quad (10.71)$$

$$KE=\frac{1}{2}m\omega^2A^2\cos^2\omega t \quad (10.72)$$

This variation with time is shown below.

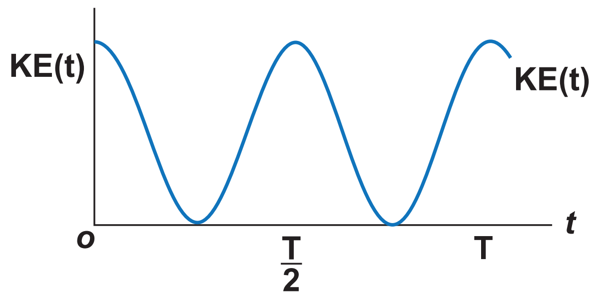


Figure 10.24 Variation of kinetic energy with time t.

c. Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy

$$E = KE + U \quad (10.73)$$

$$E=\frac{1}{2}m\omega^2(A^2-x^2)+\frac{1}{2}m\omega^2x^2$$

Hence, cancelling x^2 term,

$$E=\frac{1}{2}m\omega^2A^2= \text{constant} \quad (10.74)$$

Alternatively, from equation (10.67) and equation (10.72), we get the total energy as

$$E=\frac{1}{2}m\omega^2A^2\sin^2\omega t+\frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$=\frac{1}{2}m\omega^2A^2(\sin^2\omega t+\cos^2\omega t)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

From trigonometry identity,
$(\sin^2 \omega t + \cos^2 \omega t) = 1$

$$E = \frac{1}{2} m\omega^2 A^2 = \text{constant}$$

which gives the law of conservation of total energy. This is depicted in Figure 10.26

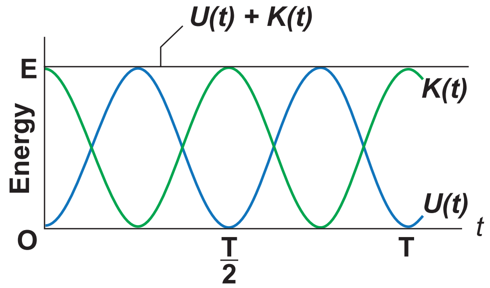


Figure 10.25 Both kinetic energy and potential energy vary but total energy is constant

Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}} \qquad (10.75)$$

EXAMPLE 10.15

Write down the kinetic energy and total energy expressions in terms of linear momentum, For one-dimensional case.

Solution

Kinetic energy is $KE = \frac{1}{2} m v_x^2$

Multiply numerator and denominator by m

$$KE = \frac{1}{2m} m^2 v_x^2 = \frac{1}{2m} (m v_x)^2 = \frac{1}{2m} p_x^2$$

where, p_x is the linear momentum of the particle executing simple harmonic motion.

Total energy can be written as sum of kinetic energy and potential energy, therefore, from equation (10.73) and also from equation (10.75), we get

$$E = KE + U(x) = \frac{1}{2m} p_x^2 + \frac{1}{2} m\omega^2 x^2 = \text{constant}$$

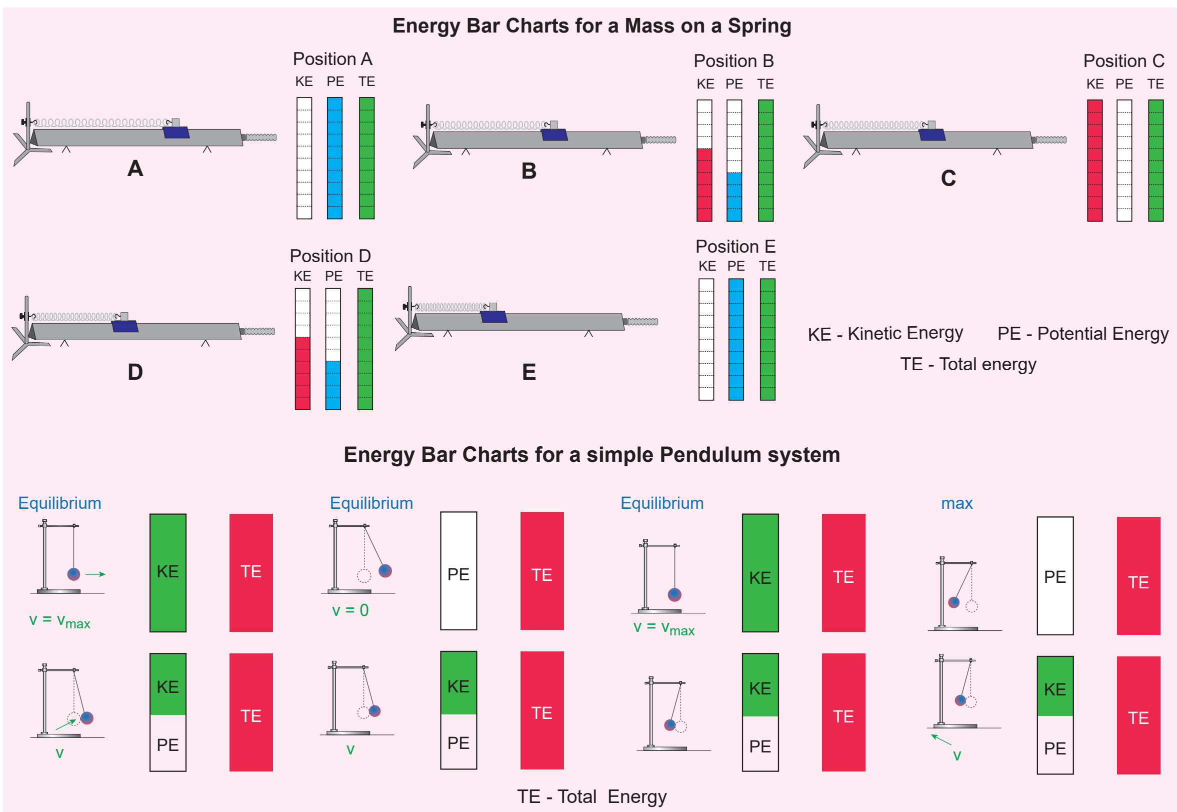


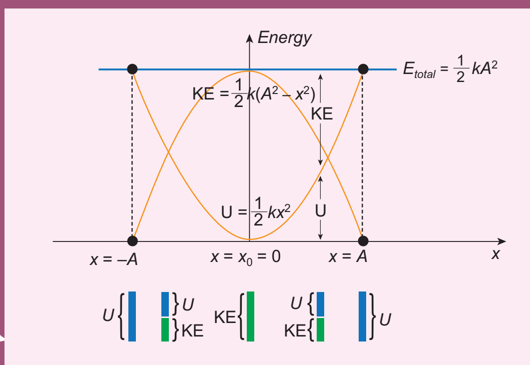
Figure 10.26 Conservation of energy – spring mass system and simple pendulum system

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Note

Conservation of energy

Both the kinetic energy and potential energy are periodic functions, and repeat their values after a time period $\frac{T}{2}$. But total energy is constant for all the values of x or t. The kinetic energy and the potential energy for a simple harmonic motion are always positive. Note that kinetic energy cannot take negative value because it is proportional to the square of velocity. The measurement of any physical quantity must be a real number. Therefore, if kinetic energy is negative then the numerical value of velocity becomes an imaginary number, which is physically not acceptable. At equilibrium, it is purely kinetic energy and at extreme positions it is purely potential energy.



EXAMPLE 10.16

Compute the position of an oscillating particle when its kinetic energy and potential energy are equal.

Solution

Since the kinetic energy and potential energy of the oscillating particle are equal,

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$$A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

10.6 TYPES OF OSCILLATIONS:

10.6.1 Free oscillations

When the oscillator is allowed to oscillate by displacing its position from equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator. Such an oscillation or vibration is known as free oscillation or free vibration. In this case, the amplitude, frequency and the energy of the vibrating object remains constant.

Examples:

(i) Vibration of a tuning fork.
(ii) Vibration in a stretched string.
(iii) Oscillation of a simple pendulum.
(iv) Oscillationsof a spring-mass system.

10.6.2 Damped oscillations

During the oscillation of a simple pendulum (in previous case), we have assumed that the amplitude of the oscillation is constant and also the total energy of the oscillator is constant. But in reality, in a medium, due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses. It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy. The energy lost is absorbed by the surrounding medium. This type of

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

oscillatory motion is known as damped oscillation. In other words, if an oscillator moves in a resistive medium, its amplitude goes on decreasing and the energy of the oscillator is used to do work against the resistive medium. The motion of the oscillator is said to be damped and in this case, the resistive force (or damping force) is proportional to the velocity of the oscillator.

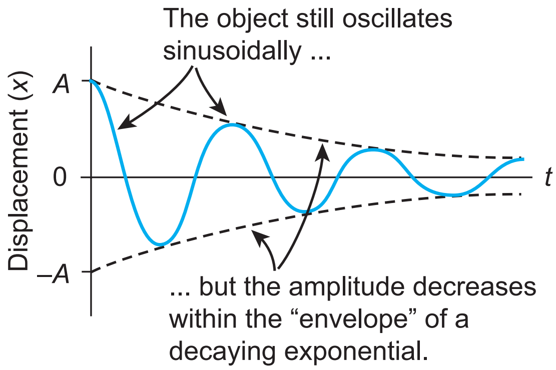


Figure 10.27 Damped harmonic oscillator – amplitude decreases as time increases.

Examples

(i) The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container.

(ii) Electromagnetic oscillations in a tank circuit.

(iii) Oscillations in a dead beat and ballistic galvanometers.

10.6.3 Maintained oscillations

While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example:

The vibration of a tuning fork getting energy from a battery or from external power supply.

10.6.4 Forced oscillations

Any oscillator driven by an external periodic agency to overcome the damping is known as forced oscillator or driven oscillator. -

In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations.

Example:

Sound boards of stringed instruments.

10.6.5 Resonance

It is a special case of forced vibrations where the frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven). As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

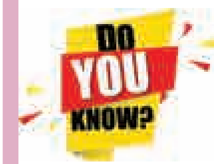
Example

The breaking of glass due to sound

Note

The concept of resonance is used in Tuning of station (or channel) in a radio (or Television) circuits.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181



Soliders are not allowed to march on a bridge.

This is to avoid resonant vibration of the bridge.

While crossing a bridge, if the period of stepping on the ground by marching soldiers equals the natural frequency of the bridge, it may result in resonance vibrations. This may be so large that the bridge may collapse.

Extra:

Pendulum in a lift:

(i) Lift moving upwards with acceleration a:

Effective acceleration due to gravity is $g_{eff} = g + a$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g+a)}}$

Since the time period is inversely related to acceleration due to gravity, time period will decrease when lift moves upward.

(ii) Lift moving downwards with acceleration a:

Effective acceleration due to gravity is $g_{eff} = g - a$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g-a)}}$

Since the time period is inversely related to acceleration due to gravity, time period will increase when lift moves downward.

(iii) Lift falls with acceleration $a > g$:

The effective acceleration is $g_{eff} = a - g$

Then time period is $T = \frac{1}{2\pi}\sqrt{\frac{l}{g_{eff}}} = \frac{1}{2\pi}\sqrt{\frac{l}{(a-g)}}$

in this case, the pendulum will turn upside down and will oscillate about higest point.

(iv) Lift falls with acceleration $a = g$:

The effective acceleration is $g_{eff} = g - g = 0$

Then time period is $T \rightarrow \infty$ which means pendulum does not oscillate and its motion is arrested.

(v) If the simple pendulum is kept in a car which moves horizontally with acceleration a:

The effective acceleration is $g_{eff} = \sqrt{g^2 + a^2}$

Time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SUMMARY

- When an object or a particle moves back and forth repeatedly about a reference point for some duration of time it is said to have Oscillatory (or vibratory) motion.
- For a SHM, the acceleration or force on the particle is directly proportional to its displacement from a fixed point and always directed towards that fixed point. The force is
$$F_x = -k\,x$$
where k is a constant whose dimension is force per unit length, called as force constant.
- In Simple harmonic motion, the displacement, $y = A \sin \omega t$.
- In Simple harmonic motion, the velocity, $v = A\,\omega \cos \omega t = \omega\sqrt{A^2 - y^2}$.
- In Simple harmonic motion, the acceleration, $a = \frac{d^2 y}{dt^2} = -\omega^2 y$.
- The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. Time period $T = \frac{2\pi}{\omega}$.
- The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is S^{-1} or hertz (In symbol, Hz). Mathematically, frequency is related to time period by $f = \frac{1}{T}$.
- The frequency of the angular harmonic motion is $f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$ Hz
- For n springs connected in series, the effective spring constant in series is
$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} = \sum_{i=0}^{n} \frac{1}{k_i}$$
- For n springs connected in parallel, the effective spring constant is
$$k_p = \sum_{i=1}^{n} k_i$$
- The time period for U-tube oscillation is $T = 2\pi\sqrt{\frac{l}{2g}}$ second.
- For a conservative system in one dimension, the force field can be derived from a scalar potential energy: $F = -\frac{dU}{dx}$.
- In a simple harmonic motion, potential energy is $U(x) = \frac{1}{2}m\omega^2 x^2$.
- In a simple harmonic motion, kinetic energy is $KE = \frac{1}{2}mv_x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$.
- Total energy for a simple harmonic motion is $E = \frac{1}{2}m\omega^2 A^2 = $ constant.
- Types of oscillations – Free oscillations, Damped oscillations, Maintained oscillations and Forced oscillations.
- Resonance is a special case of forced oscillations.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CONCEPT MAP

Oscillation

Simple Harmonic Motion (SHM)

$F = -kx \quad x = +A\sin\omega t$

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

$a = -\omega^2 A \sin\omega t$

$a = -\omega^2 y$

$F = -\frac{dU}{dx}$ — $U = \frac{1}{2}kx^2$

$TE = \frac{1}{2}kA^2$ — $KE = \frac{1}{2}mv^2$

$V = A\omega\cos\omega t$

$v = \omega\sqrt{A^2 - y^2}$

$T = \frac{2\pi}{\omega}$

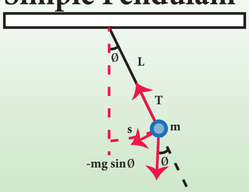
Angular SHM

$\omega = \sqrt{\frac{\kappa}{I}}$

Linear SHM

$\omega = \sqrt{\frac{k}{m}}$

Simple Pendulam



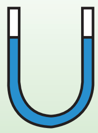
Differential equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

Time Period:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

U - Tube



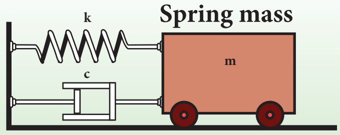
Differential equation:

$$\frac{d^2y}{dt^2} = -\frac{2g}{l}y$$

Time Period:

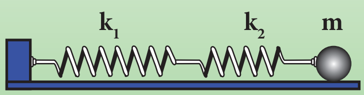
$$T = 2\pi\sqrt{\frac{l}{2g}}$$

Spring mass



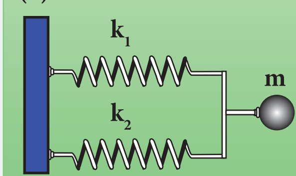
Combination of Springs

(1) Series:



$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

(2) Parallel:



$$k_p = k_1 + k_2$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

I. Multiple Choice Questions

1. In a simple harmonic oscillation, the acceleration against displacement for one complete oscillation will be

(model NSEP 2000-01)

a) an ellipse　　b) a circle

c) a parabola　　d) a straight line

2. A particle executing SHM crosses points A and B with the same velocity. Having taken 3 s in passing from A to B, it returns to B after another 3 s. The time period is

a) 15 s　　b) 6 s

c) 12 s　　d) 9 s

3. The length of a second's pendulum on the surface of the Earth is 0.9 m. The length of the same pendulum on surface of planet X such that the acceleration of the planet X is n times greater than the Earth is

a) $0.9n$　　b) $\frac{0.9}{n} m$

c) $0.9n^2 m$　　d) $\frac{0.9}{n^2}$

4. A simple pendulum is suspended from the roof of a school bus which moves in a horizontal direction with an acceleration a, then the time period is

a) $T \alpha \frac{1}{g^2 + a^2}$　　b) $T \alpha \frac{1}{\sqrt{g^2 + a^2}}$

c) $T \alpha \sqrt{g^2 + a^2}$　　d) $T \alpha (g^2 + a^2)$

5. Two bodies A and B whose masses are in the ratio 1:2 are suspended from two separate massless springs of force constants k_A and k_B respectively. If the two bodies oscillate vertically such that their maximum velocities are in the ratio 1:2, the ratio of the amplitude A to that of B is

a) $\sqrt{\frac{k_B}{2k_A}}$　　b) $\sqrt{\frac{k_B}{8k_A}}$

c) $\sqrt{\frac{2k_B}{k_A}}$　　d) $\sqrt{\frac{8k_B}{k_A}}$

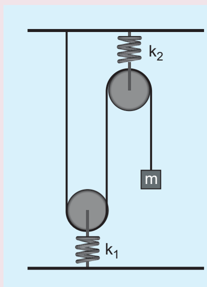
6. A spring is connected to a mass m suspended from it and its time period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is

a) $T' = \sqrt{2}T$　　b) $T' = \frac{T}{\sqrt{2}}$

c) $T' = \sqrt{2T}$　　d) $T' = \sqrt{\frac{T}{2}}$

7. The time period for small vertical oscillations of block of mass m when the masses of the pulleys are negligible and spring constant k_1 and k_2 is

BT52Y6



a) $T = 4\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$

b) $T = 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$

c) $T = 4\pi\sqrt{m(k_1 + k_2)}$

d) $T = 2\pi\sqrt{m(k_1 + k_2)}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

8. A simple pendulum has a time period T_1. When its point of suspension is moved vertically upwards according as $y = k t^2$, where y is vertical distance covered and $k = 1\ ms^{-2}$, its time period becomes T_2. Then, $\frac{T_1^2}{T_2^2}$ is (g = 10 m s^{-2}) (IIT 2005)

a) $\frac{5}{6}$ b) $\frac{11}{10}$

c) $\frac{6}{5}$ d) $\frac{5}{4}$

9. An ideal spring of spring constant k, is suspended from the ceiling of a room and a block of mass M is fastened to its lower end. If the block is released when the spring is un-stretched, then the maximum extension in the spring is (IIT 2002)

a) $4\frac{Mg}{k}$ b) $\frac{Mg}{k}$

c) $2\frac{Mg}{k}$ d) $\frac{Mg}{2k}$

10. A pendulum is hung in a very high building oscillates to and fro motion freely like a simple harmonic oscillator. If the acceleration of the bob is 16 ms^{-2} at a distance of 4 m from the mean position, then the time period is (NEET 2018 model)

a) 2 s b) 1 s

c) 2πs (d) πs

11. A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will

a) first increase and then decrease

b) first decrease and then increase

c) increase continuously

d) decrease continuously

12. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are (AIPMT 2012)

a) kg m s^{-1} b) kg m s^{-2}

c) kg s^{-1} (d) kg s

13. When a damped harmonic oscillator completes 100 oscillations, its amplitude is reduced to $\frac{1}{3}$ of its initial value. What will be its amplitude when it completes 200 oscillations?.

a) $\frac{1}{5}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{9}$

14. Which of the following differential equations represents a damped harmonic oscillator ?

a) $\frac{d^2y}{dt^2}+y=0$ b) $\frac{d^2y}{dt^2}+\gamma\frac{dy}{dt}+y=0$

c) $\frac{d^2y}{dt^2}+k^2y=0$ d) $\frac{dy}{dt}+y=0$

15. If the inertial mass and gravitational mass of the simple pendulum of length l are not equal, then the time period of the simple pendulum is

a) $T=2\pi\sqrt{\frac{m_i l}{m_g g}}$

b) $T=2\pi\sqrt{\frac{m_g l}{m_i g}}$

c) $T=2\pi\frac{m_g}{m_i}\sqrt{\frac{l}{g}}$

d) $T=2\pi\frac{m_i}{m_g}\sqrt{\frac{l}{g}}$

Answers:

1) d	2) c	3) a	4) b
5) b	6) b	7) a	8) c
9) c	10) d	11) a	12) c
13) d	14) b	15) a	

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

II. Short Answers Questions

1. What is meant by periodic and non-periodic motion?. Give any two examples, for each motion.
2. What is meant by force constant of a spring?.
3. Define time period of simple harmonic motion.
4. Define frequency of simple harmonic motion.
5. What is an epoch?.
6. Write short notes on two springs connected in series.
7. Write short notes on two springs connected in parallel.
8. Write down the time period of simple pendulum.
9. State the laws of simple pendulum?.
10. Write down the equation of time period for linear harmonic oscillator.
11. What is meant by free oscillation?.
12. Explain damped oscillation. Give an example.
13. Define forced oscillation. Give an example.
14. What is meant by maintained oscillation?. Give an example.
15. Explain resonance. Give an example.

III. Long Answers Questions

1. What is meant by simple harmonic oscillation?. Give examples and explain why every simple harmonic motion is a periodic motion whereas the converse need not be true.
2. Describe Simple Harmonic Motion as a projection of uniform circular motion.
3. What is meant by angular harmonic oscillation?. Compute the time period of angular harmonic oscillation.
4. Write down the difference between simple harmonic motion and angular simple harmonic motion.
5. Discuss the simple pendulum in detail.
6. Explain the horizontal oscillations of a spring.
7. Describe the vertical oscillations of a spring.
8. Write short notes on the oscillations of liquid column in U-tube.
9. Discuss in detail the energy in simple harmonic motion.
10. Explain in detail the four different types of oscillations.

IV. Numerical Problems

1. Consider the Earth as a homogeneous sphere of radius R and a straight hole is bored in it through its centre. Show that a particle dropped into the hole will execute a simple harmonic motion such that its time period is

$$T = 2\pi\sqrt{\frac{R}{g}}$$

2. Calculate the time period of the oscillation of a particle of mass m moving in the potential defined as

$$U(x) = \begin{cases} \frac{1}{2}kx^2, x < 0 \\ mgx, x > 0 \end{cases}$$

Answer: $\pi\sqrt{\frac{m}{k}} + 2\sqrt{\frac{2E}{g^2 m}}$, where E is the total energy of the particle.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

3. Consider a simple pendulum of length $l = 0.9\ m$ which is properly placed on a trolley rolling down on a inclined plane which is at $\theta = 45°$ with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

Answer: 0.86 s

4. A piece of wood of mass m is floating erect in a liquid whose density is ρ. If it is slightly pressed down and released, then executes simple harmonic motion. Show that its time period of oscillation is $T = 2\pi\sqrt{\frac{m}{Ag\rho}}$

5. Consider two simple harmonic motion along x and y-axis having same frequencies but different amplitudes as $x = A\sin(\omega t + \varphi)$ (along x axis) and $y = B\sin\omega t$ (along y axis). Then show that

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\varphi = \sin^2\varphi$$

and also discuss the special cases when

a. $\varphi = 0$ b. $\varphi = \pi$ c. $\varphi = \frac{\pi}{2}$

d. $\varphi = \frac{\pi}{2}$ and $A = B$ (e) $\varphi = \frac{\pi}{4}$

Note: *when a particle is subjected to two simple harmonic motion at right angle to each other the particle may move along different paths. Such paths are called* ***Lissajous figures***.

Answer :

a. $y = \frac{B}{A}x$, equation is a straight line passing through origin with positive slope.

b. $y = -\frac{B}{A}x$ equation is a straight line passing through origin with negative slope.

c. $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, equation is an ellipse whose center is origin.

d. $x^2 + y^2 = A^2$, equation is a circle whose center is origin .

e. $\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\frac{1}{\sqrt{2}} = \frac{1}{2}$, equation is an ellipse (oblique ellipse which means tilted ellipse)

6. Show that for a particle executing simple harmonic motion

a. the average value of kinetic energy is equal to the average value of potential energy.

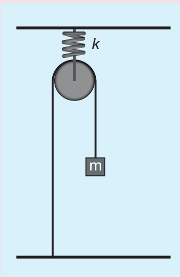
b. average potential energy = average kinetic energy = $\frac{1}{2}$(total energy)

Hint : *average kinetic energy = <kinetic energy>* = $\frac{1}{T}\int_0^T (Kinetic\,energy)\,dt$

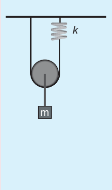
and

average Potential energy = <Potential energy> = $\frac{1}{T}\int_0^T (Potential\ energy)\,dt$

7. Compute the time period for the following system if the block of mass m is slightly displaced vertically down from its equilibrium position and then released. Assume that the pulley is light and smooth, strings and springs are light.



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181



Hint and answer:

Case(a)

Pulley is fixed rigidly here. When the mass displace by y and the spring will also stretch by y. Therefore, $F = T = ky$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case(b)

Mass displace by y, pulley also displaces by y. $T = 4ky$.

$$T = 2\pi\sqrt{\frac{m}{4k}}$$

BOOKS FOR REFERENCE

1. Vibrations and Waves – A. P. French, CBS publisher and Distributors Pvt. Ltd.
2. Concepts of Physics – H. C. Verma, Volume 1 and Volume 2, Bharati Bhawan Publisher.
3. Fundamentals of Physics – Halliday, Resnick and Walker, Wiley Publishers, 10th edition.
4. Physics for Scientist and Engineers with Modern Physics – Serway and Jewett, Brook/ Coole Publishers, Eighth Edition.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ICT CORNER

Oscillations

Through this activity you will be able to learn about the resonance.

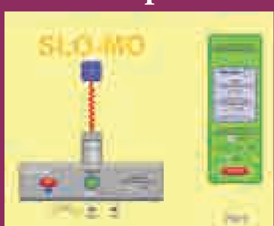
STEPS:

- Use the URL or scan the QR code to open ‘PhET’ simulation on ‘Resonance’. Click the play button.
- In the activity window a diagram of resonator is given. Click the play icon and move the slider on ‘sim speed’ given below to see the resonance.
- Move the slider to change ‘Number of Resonators’, ‘Mass’ and ‘Spring constant’ on the right side window and see the ‘frequency’.
- Select the ‘On’, ‘Off’ button on ‘Gravity’ to see the different resonance.

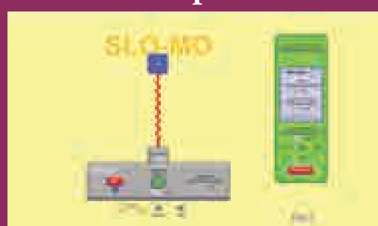
Step1

Step2

Step3



Step4



URL:

https://phet.colorado.edu/en/simulation/legacy/resonance

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

UNIT 11

WAVES

We are slowed down sound and light waves, a walking bundle of frequencies tuned into the cosmos. We are souls dressed up in sacred biochemical garments and our bodies are the instruments through which our souls play their music – Albert Einstein

LEARNING OBJECTIVES

In this unit, the student is exposed to

- waves and their types (transverse and longitudinal)
- basic terms like wavelength, frequency, time period and amplitude of a wave
- velocity of transverse waves and longitudinal waves
- velocity of sound waves
- reflection of sound waves from plane and curved surfaces and its applications
- progressive waves and their graphical representation
- superposition principle, interference of waves, beats and standing waves
- characteristics of stationary waves, sonometer
- fundamental frequency, harmonics and overtones
- intensity and loudness
- vibration of air column – closed organ pipe, open organ pipe and resonance air column
- Doppler effect and its applications

BTDXZT

11.1 INTRODUCTION

In the previous chapter, we have discussed the oscillation of a particle. Consider a medium which consists of a collection of particles. If the disturbance is created at one end, it propagates and reaches the other end. That is, the disturbance produced at the first mass point is transmitted to the next neighbouring mass point, and so on. Notice that here, only the disturbance is transmitted, not the mass points. Similarly, the speech we deliver is due to the vibration of our vocal chord inside the throat. This leads to the vibration of the surrounding air molecules and hence, the effect of speech (information) is transmitted from one point in space to another point in space without the medium carrying the particles. Thus, *the disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium is known as a* wave.

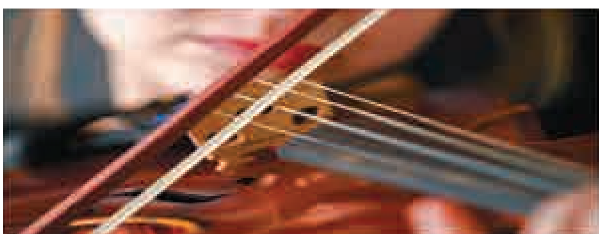


Figure 11.1 Standing waves in a violin

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

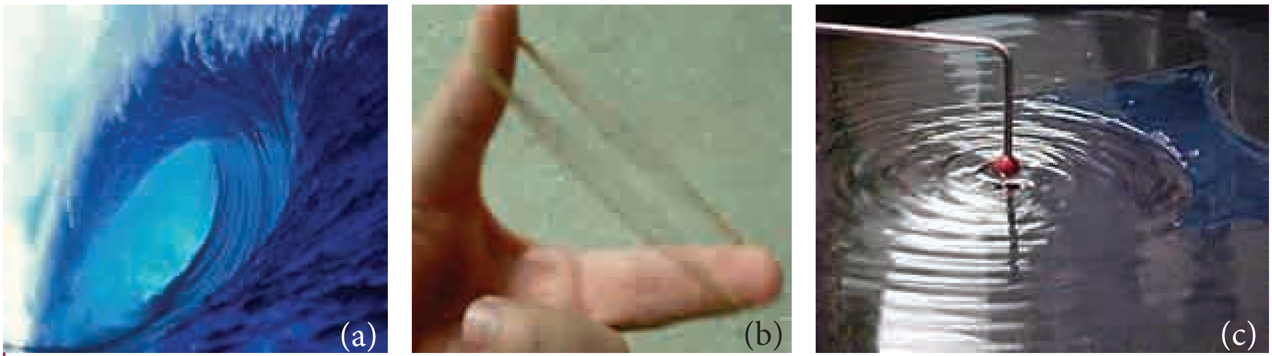


Figure 11.2 Waves formed in (a) ocean, (b) standing waves in plucking rubber band and (c) ripples formed on water surface

Standing near a beach, one can observe tides in the ocean reaching the seashore with a similar wave pattern; hence they are called ocean waves. A rubber band when plucked vibrates like a wave which is an example of a standing wave. These are shown in Figure 11.2. Other examples of waves are light waves (electromagnetic waves), through which we observe and enjoy the beauty of nature and sound waves using which we hear and enjoy pleasant melodious songs. Day to day applications of waves are numerous, as in mobile phone communication, laser surgery, etc.

11.1.1 Ripples and wave formation on the water surface

Figure 11.3 Ripples formed on the surface of water

Suppose we drop a stone in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface as shown in Figure 11.3. We find that this disturbance spreads out (diverges out) in the form of concentric circles of ever increasing radii (ripples) and strike the boundary of the trough. This is because some of the kinetic energy of the stone is transmitted to the water molecules on the surface. Actually the particles of the water (medium) themselves do not move outward with the disturbance. This can be observed by keeping a paper strip on the water surface. The strip moves up and down when the disturbance (wave) passes on the water surface. This shows that the water molecules only undergo vibratory motion about their mean positions.

11.1.2 Formation of waves on stretched string

Let us take a long string and tie one end of the string to the wall as shown in Figure 11.4 (a). If we give a quick jerk, a bump (like pulse) is produced in the string as shown in Figure 11.4 (b). Such a disturbance is sudden and it lasts for a short duration, hence it is known as a wave pulse. If jerks are given continuously then the waves produced are standing waves. Similar waves are produced by a plucked string in a guitar.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

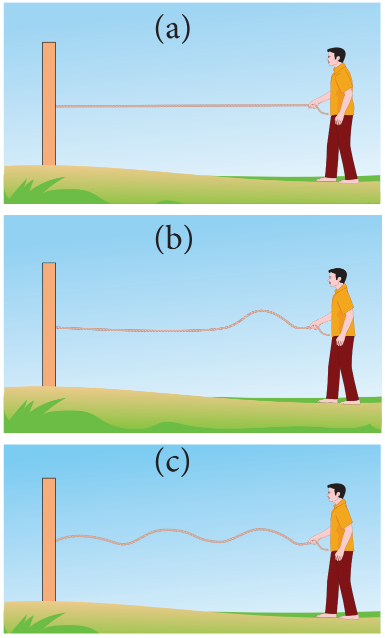


Figure 11.4: Wave pulse created during jerk produced on one end of the string

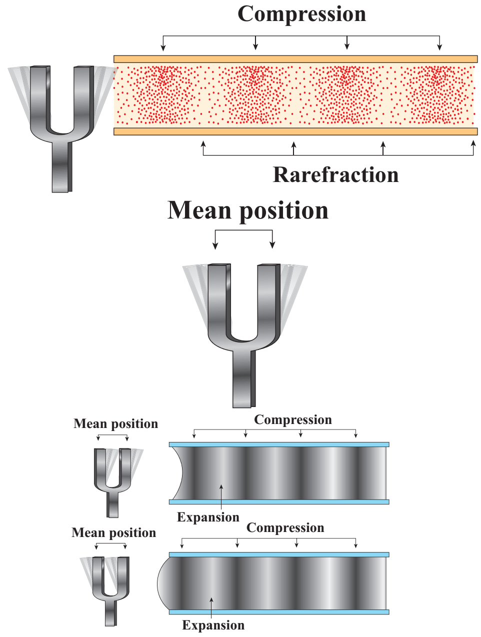


Figure 11.5 Waves due to strike of a tuning fork on a rubber pad

11.1.3. Formation of waves in a tuning fork

When we strike a tuning fork on a rubber pad, the prongs of the tuning fork vibrate about their mean positions. The prong vibrating about a mean position means moving outward and inward, as indicated in the Figure 11.5. When a prong moves outward, it pushes the layer of air in its neighbourhood which means there is more accumulation of air molecules in this region. Hence, the density and also the pressure increase. These regions are known as compressed regions or compressions. This compressed air layer moves forward and compresses the next neighbouring layer in a similar manner. Thus a wave of compression advances or passes through air. When the prong moves inwards, the particles of the medium are moved to the right. In this region both density and pressure are low. It is known as a rarefaction or elongation.

11.1.4 Characteristics of wave motion

- For the propagation of the waves, the medium must possess both inertia and elasticity, which decide the velocity of the wave in that medium.
- In a given medium, the velocity of a wave is a constant whereas the constituent particles in that medium move with different velocities at different positions. Velocity is maximum at their mean position and zero at extreme positions.
- Waves undergo reflections, refraction, interference, diffraction and polarization.

Point to ponder

The medium possesses both inertia and elasticity for propagation of waves.

Light is an electromagnetic wave. what is the medium for its transmission?

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

11.1.5 Mechanical wave motion and its types

Wave motion can be classified into two types

a. **Mechanical wave** – Waves which require a medium for propagation are known as mechanical waves.

   Examples: sound waves, ripples formed on the surface of water, etc.

b. **Non mechanical wave** – Waves which do not require any medium for propagation are known as non-mechanical waves.

   Example: light

Further, waves can be classified into two types

a. Transverse waves

b. Longitudinal waves

11.1.6 Transverse wave motion

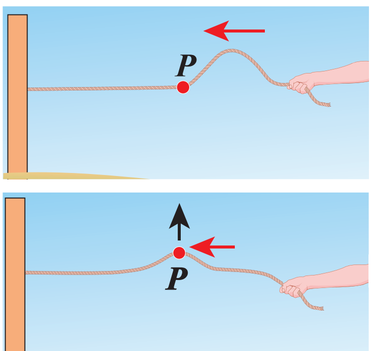


Figure 11.6 Transverse wave

In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves as shown in Figure 11.6.

Example: light (electromagnetic waves)

11.1.7 Longitudinal wave motion

In longitudinal wave motion, the constituent of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation (direction of energy transfer) of waves as shown in Figure 11.7.

Example: Sound waves travelling in air.

Discuss with your Teacher

- Tsunami (pronounced soo-nah-mee in Japanese) means Harbour waves. A tsunami is a series of huge and giant waves which come with great speed and huge force. What happened on 26th December2004 in southern part of India? - Discuss
- Gravitational waves - LIGO (Laser Interferometer Gravitational wave Observatory) experiment
  Nobel Prize winners in Physics 2017
  i) Prof. Rainer Weiss
  ii) Prof. Barry C. Barish
  iii) Prof. Kip S. Thorne

"For decisive contributions to the LIGO detector and observation of gravitational forces"

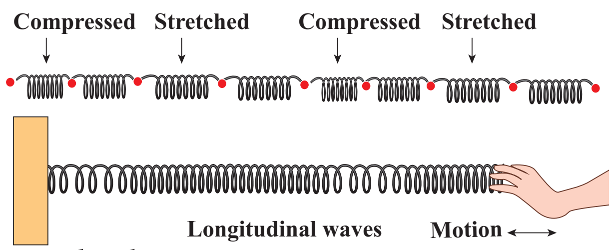


Figure 11.7 Longitudinal waves

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Table 11.1: Comparison of transverse and longitudinal waves

S.No.	Transverse waves	Longitudinal waves
1.	The direction of vibration of particles of the medium is perpendicular to the direction of propagation of waves.	The direction of vibration of particles of the medium is parallel to the direction of propagation of waves.
2.	The disturbances are in the form of crests and troughs.	The disturbances are in the form of compressions and rarefactions.
3.	Transverse waves are possible in elastic medium.	Longitudinal waves are possible in all types of media (solid, liquid and gas).

NOTE:
1. Absence of medium is also known as vacuum. Only electromagnetic waves can travel through vacuum.
2. Rayleigh waves are considered to be mixture of transverse and longitudinal.

11.2 TERMS AND DEFINITIONS USED IN WAVE MOTION

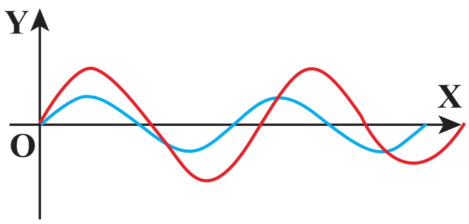


Figure 11.8 Two different sinusoidal waves

Suppose we have two waves as shown in Figure 11.8. Are these two waves identical?. No. Though, the two waves are both sinusoidal, there are many difference between them. Therefore, we have to define some basic terminologies to distinguish one wave from another.

Consider a wave produced by a stretched string as shown in Figure 11.9.

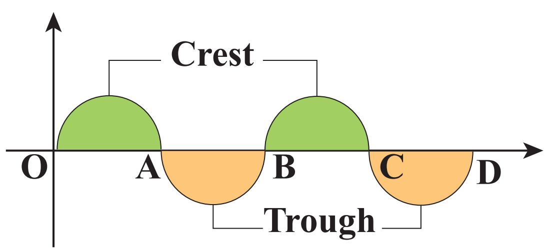


Figure 11.9 Crest and Trough of a wave

If we are interested in counting the number of waves created, let us put a reference level (mean position) as shown in Figure 11.9. Here the mean position is the horizontal line shown. The highest point in the shaded portion is called *crest*. With respect to the reference level, the lowest point on the un-shaded portion is called *trough*. This wave contains repetition of a section O to B and hence we define the length of the smallest section without repetition as one *wavelength* as shown in Figure 11.10. In Figure 11.10 the length OB or length BD is one wavelengh. A Greek letter lambda λ is used to denote one wavelength.

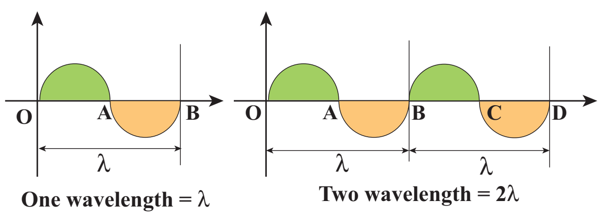


Figure 11.10 Defining wavelength

For transverse waves (as shown in Figure 11.11), the distance between two neighbouring crests or troughs is known as the *wavelength*.

For longitudinal waves, (as shown in Figure 11.12) the distance between two neighbouring compressions or rarefactions is known as the wavelength. The SI unit of wavelength is *meter*.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

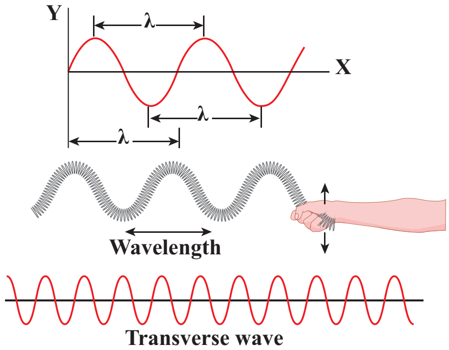


Figure 11.11 Wavelength for transverse waves

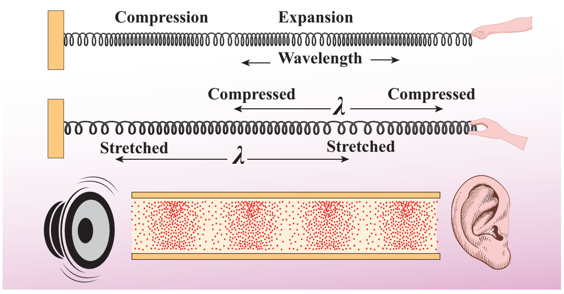
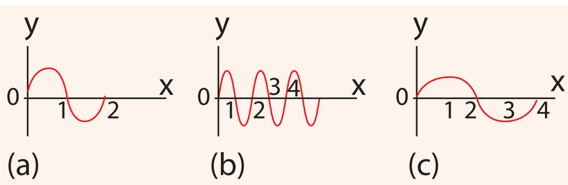


Figure 11.12 Wavelength for longitudinal waves

EXAMPLE 11.1

Which of the following has longer wavelength?



Answer is (c)

In order to understand frequency and time period, let us consider waves (made of three wavelengths) as shown in Figure 11.13 (a). At time $t = 0$ *s*, the wave reaches the point A from left. After time $t = 1$ *s* (shown in figure 11.13(b)), the number of waves which have crossed the point A is two. Therefore, the frequency is defined as "the number of waves crossing a point per second" It is measured in *hertz* whose symbol is *Hz*. In this example,

$$f = 2 \text{ Hz} \qquad (11.1)$$

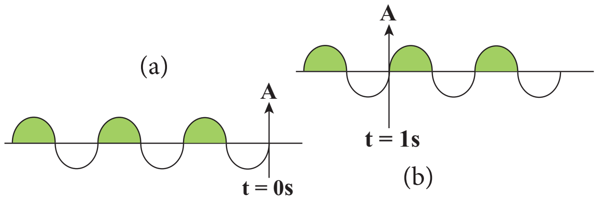


Figure 11.13 A wave consisting of three wavelengths passing a point A at time (a) $t = 0$ s and (b) after time $t = 1$ s

If two waves take one second (time) to cross the point A then the time taken by one wave to cross the point A is half a second. This defines the time period T as

$$T = \frac{1}{2} = 0.5 \text{ s} \qquad (11.2)$$

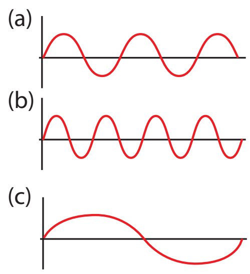
From equation (11.1) and equation (11.2), *frequency and time period are inversely related* i.e.,

$$T = \frac{1}{f} \qquad (11.3)$$

Time period is defined as *the time taken by one wave to cross a point.*

EXAMPLE 11.2

Three waves are shown in the figure below



Write down

(a) the frequency in ascending order

(b) the wavelength in ascending order

Solution

(a) $f_c < f_a < f_b$

(b) $\lambda_b < \lambda_a < \lambda_c$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

From the example 11.2, we observe that the frequency is inversely related to the wavelength, $f \propto \frac{1}{\lambda}$

Then, $f\lambda$ is equal to what?

$$[(\text{i.e})\ f\lambda = ?]$$

A simple dimensional argument will help us to determine this unknown physical quantity.

Dimension of wavelength is, $[\lambda] = L$

Frequency $f = \frac{1}{Time\ period}$, which implies that the dimension of frequency is,

$$[f] = \frac{1}{[T]} = T^{-1}$$

$$\Rightarrow [\lambda f] = [\lambda][f] = LT^{-1} = [\text{velocity}]$$

Therefore,

$$\text{Velocity,} \quad \lambda f = v \qquad (11.4)$$

where v is known as the *wave velocity* or *phase velocity*. This is the velocity with which the wave propagates. *Wave velocity* is the distance travelled by a wave in one second.

Note:

1. The number of cycles (or revolutions) per unit time is called *angular frequency*.

   Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi f$ (unit is radians/second)

2. The number of cycles per unit distance or number of waves per unit distance is called *wave number*.

   wave number, $k = \frac{2\pi}{\lambda}$ (unit is radians/meter)

   In two, three or higher dimensional case, the wave number is the magnitude of a vector called *wave vector*. The points in space of wave vectors are called *reciprocal vectors*, $\vec{k}$.

   Dimensions of $\vec{k}$ is L^{-1}.

   The velocity v, angular frequency ω and wave number λ are related as:

   $$velocity,\ v = \lambda f = \frac{\lambda}{2\pi}(2\pi f) = \frac{(2\pi f)}{2\pi/\lambda} = \omega/k$$

   Wave numbers and wave vectors play an essential role in optics and scattering theory.

EXAMPLE 11.3

The average range of frequencies at which human beings can hear sound waves varies from 20 Hz to 20 kHz. Calculate the wavelength of the sound wave in these limits. (Assume the speed of sound to be 340 m s^{-1}.

Solution

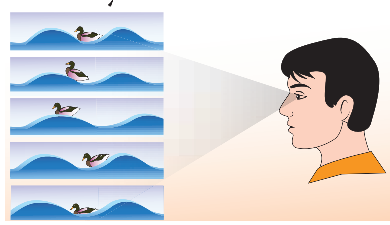
$$\lambda_1 = \frac{v}{f_1} = \frac{340}{20} = 17\,\text{m}$$

$$\lambda_2 = \frac{v}{f_2} = \frac{340}{20 \times 10^3} = 0.017\,\text{m}$$

Therefore, the audible wavelength region is from 0.017 m to 17 m when the velocity of sound in that region is 340 m s^{-1}.

EXAMPLE 11.4

A man saw a toy duck on a wave in an ocean. He noticed that the duck moved up and down 15 times per minute. He roughly measured the wavelength of the ocean wave as 1.2 m. Calculate the time taken by the toy duck for going one time up and down and also the velocity of the ocean wave.



Solution

Given that the number of times the toy duck moves up and down is 15 times per minute. This information gives us frequency (the number of times the toy duck moves up and down)

$$f = \frac{\text{15 times toy duck moves up and down}}{\text{one minute}}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

But one minute is 60 second, therefore, expressing time in terms of second

$$f = \frac{15}{60} = \frac{1}{4} = 0.25\,\text{Hz}$$

The time taken by the toy duck for going one time up and down is time period which is inverse of frequency

$$T = \frac{1}{f} = \frac{1}{0.25} = 4\text{ s}$$

The velocity of ocean wave is

$v = \lambda f = 1.2 \times 0.25 = 0.3\text{ m s}^{-1}$.

Amplitude of a wave:

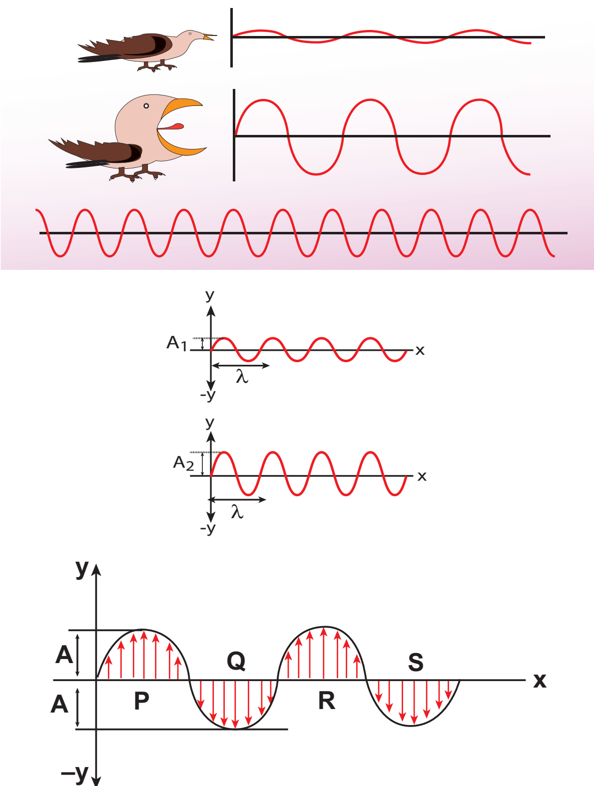
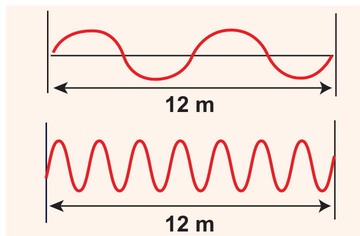


Figure 11.14 Waves of different amplitude

The waves shown in the Figure 11.14 have same wavelength, same frequency and same time period and also move with same velocity. The only difference between two waves is the height of either crest or trough. This means, the height of the crest or trough also signifies a wave character. So we define a quantity called an amplitude of the wave, as the maximum displacement of the medium with respect to a reference axis (for example in this case x-axis). Here, it is denoted by A.

EXAMPLE 11.5

Consider a string whose one end is attached to a wall. Then compute the following in both situations given in figure (assume waves crosses the distance in one second)



(a) Wavelength, (b) Frequency and (c) Velocity

Solution

	First case	Second case
(a) Wavelength	$\lambda = 6$ m	$\lambda = 2$ m
(b) Frequency	$f = 2$ Hz	$f = 6$ Hz
(c) Velocity	$v = 6 \times 2$ $= 12\text{ m s}^{-1}$	$v = 2 \times 6$ $= 12\text{ m s}^{-1}$

This means that the speed of the wave along a string is a constant. Higher the frequency, shorter the wavelength and vice versa, and their product is velocity which remains the same.

11.3 VELOCITY OF WAVES IN DIFFERENT MEDIA

Suppose a hammer is stroked on long rails at a distance and when a person keeps his ear near the rails at the other end he/she will hear two sounds, at different instants. The sound that is heard through the rails (solid medium)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

is faster than the sound we hear through the air (gaseous medium). This implies the velocity of sound is different in different media.

In this section, we shall derive the velocity of waves in two different cases:

1. The velocity of a transverse waves along a stretched string.
2. The velocity of a longitudinal waves in an elastic medium.

11.3.1 Velocity of transverse waves in a stretched string

Let us compute the velocity of transverse travelling waves on a string. When a jerk is given at one end (left end) of the rope, the wave pulses move towards right end with a velocity v as shown in the Figure 11.15 (a). This means that the pulses move with a velocity v with respect to an observer who is at rest frame. Suppose an observer also moves with same velocity v in the direction of motion of the wave pulse, then that observer will notice that the wave pulse is stationary and the rope is moving with pulse with the same velocity v.

Consider an elemental segment in the string as shown in the Figure 11.15 (b). Let A and B be two points on the string at an instant of time. Let dl and dm be the length and mass of the elemental string, respectively. By definition, linear mass density, μ is

$$\mu = \frac{dm}{dl} \qquad (11.5)$$

$$dm = \mu\, dl \qquad (11.6)$$

The elemental string AB has a curvature which looks like an arc of a circle with centre at O, radius R and the arc subtending an angle θ at the origin O as shown in Figure 11.15(b). The angle θ can be written in terms of arc length and radius as $\theta = \frac{dl}{R}$. The centripetal acceleration supplied by the tension in the string is

$$a_{cp} = \frac{v^2}{R} \qquad (11.7)$$

Then, centripetal force can be obtained when mass of the string (dm) is included in equation (11.7)

$$F_{cp} = \frac{(dm)v^2}{R} \qquad (11.8)$$

The centripetal force experienced by elemental string can be calculated by substituting equation (11.6) in equation (11.8) we get

$$\frac{(dm)v^2}{R} = \frac{\mu v^2\, dl}{R} \qquad (11.9)$$

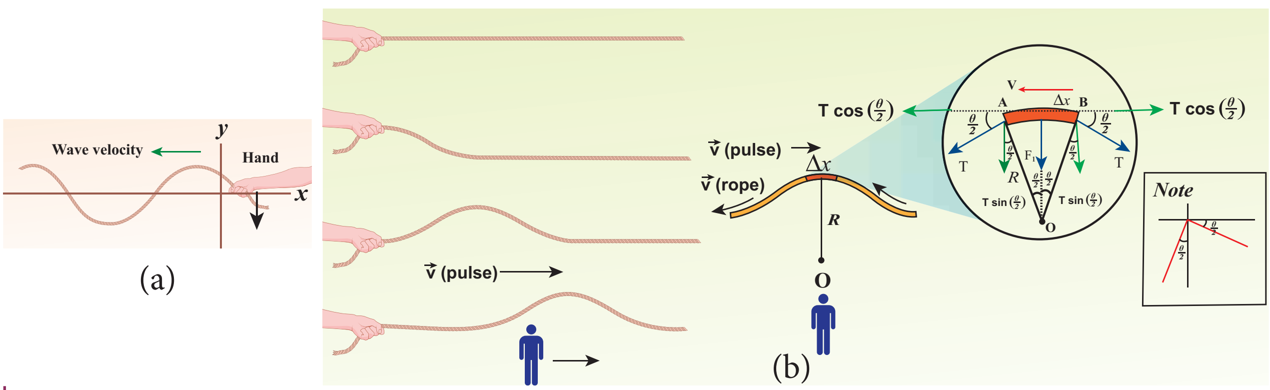


Figure 11.15 (a) Transverse waves in a stretched string. (b) Elemental segment in a stretched string is zoomed and the pulse seen from an observer frame who moves with velocity v.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The tension T acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation in the tension force can be ignored. We can resolve T into horizontal component $T\cos\left(\frac{\theta}{2}\right)$ and vertical component $T\sin\left(\frac{\theta}{2}\right)$. The horizontal components at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small, the vertical components at A and B appears to acts vertical towards the centre of the arc and hence, they add up. The net radial force F_r is

$$F_r = 2T\sin\left(\frac{\theta}{2}\right) \qquad (11.10)$$

Since the amplitude of the wave is very small when it is compared with the length of the string, the sine of small angle is approximated as $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$. Hence, equation (11.10) can be written as

$$F_r = 2T \times \frac{\theta}{2} = T\theta \qquad (11.11)$$

But $\theta = \frac{dl}{R}$, therefore substituting in equation (11.11), we get

$$F_r = T\frac{dl}{R} \qquad (11.12)$$

Applying Newton's second law to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force. Hence equating equation (11.9) and equation (11.12), we have

$$T\frac{dl}{R} = \mu v^2 \frac{dl}{R}$$

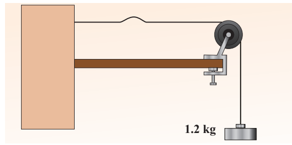
$$v = \sqrt{\frac{T}{\mu}} \text{ measured in m s}^{-1} \qquad (11.13)$$

Observations:

- The velocity of the string is
  a. directly proportional to the square root of the tension force
  b. inversely proportional to the square root of linear mass density
  c. independent of shape of the waves.

EXAMPLE 11.6

Calculate the velocity of the travelling pulse as shown in the figure below. The linear mass density of pulse is 0.25 kg m^{-1}. Further, compute the time taken by the travelling pulse to cover a distance of 30 cm on the string.



Solution

The tension in the string is $T = mg = 1.2 \times 9.8 = 11.76$ N

The mass per unit length is $\mu = 0.25$ kg m^{-1}

Therefore, velocity of the wave pulse is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{11.76}{0.25}} = 6.858\ m\,s^{-1} = 6.8\ m\,s^{-1}$$

The time taken by the pulse to cover the distance of 30 cm is

$$t = \frac{d}{v} = \frac{30\times10^{-2}}{6.8} = 0.044\,s = 44\ ms$$ where,

ms = milli second.

11.3.2 Velocity of longitudinal waves in an elastic medium

Consider an elastic medium (here we assume air) having a fixed mass contained in a long tube (cylinder) whose cross sectional area is A and maintained under a pressure P. One can generate longitudinal waves in the fluid either by displacing the fluid using a piston or by keeping a vibrating tuning fork at one end of the tube. Let us assume that the direction of propagation of waves coincides with the

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

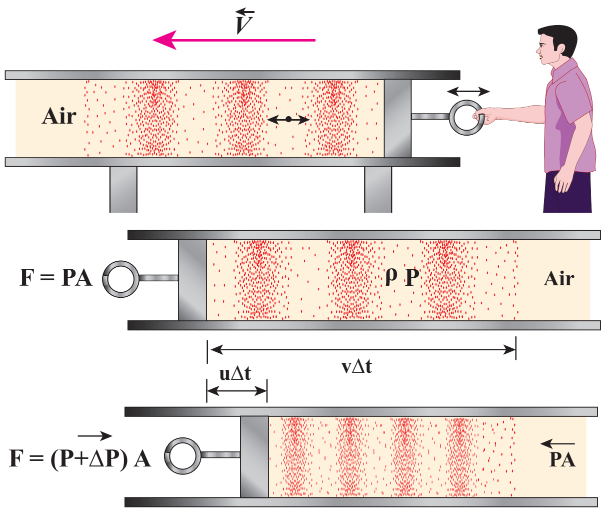


Figure 11.16 Longitudinal waves in the fluid by displacing the fluid using a piston

axis of the cylinder. Let ρ be the density of the fluid which is initially at rest. At $t = 0$, the piston at left end of the tube is set in motion toward the right with a speed u.

Let u be the velocity of the piston and v be the velocity of the elastic wave. In time interval Δt, the distance moved by the piston $\Delta d = u\ \Delta t$. Now, the distance moved by the elastic disturbance is $\Delta x = v\Delta t$. Let Δm be the mass of the air that has attained a velocity v in a time Δt. Therefore,

$$\Delta m = \rho\ A\ \Delta x = \rho\ A\ (v\ \Delta t)$$

Then, the momentum imparted due to motion of piston with velocity u is

$$\Delta p = [\rho\ A\ (v\ \Delta t)]u$$

But the change in momentum is impulse.

The net impulse is

$$I = (\Delta P\ A)\Delta t$$

Or $$(\Delta P\ A)\Delta t = [\rho\ A\ (v\ \Delta t)]u$$

$$\Delta P = \rho\ v\ u \qquad (11.14)$$

When the sound wave passes through air, the small volume element (ΔV) of the air undergoes regular compressions and rarefactions. So, the change in pressure can also be written as

$$\Delta P = B\frac{\Delta V}{V}$$

where, V is original volume and B is known as bulk modulus of the elastic medium.

But $V = A\ \Delta x = A\ v\ \Delta t$ and

$\Delta V = A\ \Delta d = A\ u\ \Delta t$

Therefore,

$$\Delta P = B\frac{A u \Delta t}{A v \Delta t} = B\frac{u}{v} \qquad (11.15)$$

Comparing equation (11.14) and equation (11.15), we get

$$\rho v u = B\frac{u}{v} \text{ or } v^2 = \frac{B}{\rho}$$

$$\Rightarrow v = \sqrt{\frac{B}{\rho}} \qquad (11.16)$$

In general, the velocity of a longitudinal wave in elastic medium is $v = \sqrt{\frac{E}{\rho}}$, where E is the modulus of elasticity of the medium.

Cases: For a solid :

(i) one dimension rod (1D)

$$v = \sqrt{\frac{Y}{\rho}} \qquad (11.17)$$

where Y is the Young's modulus of the material of the rod and ρ is the density of the rod. The 1D rod will have only Young's modulus.

(ii) Three dimension rod (3D) The speed of longitudinal wave in a solid is

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}} \qquad (11.18)$$

where η is the modulus of rigidity, K is the bulk modulus and ρ is the density of the rod.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Cases: For liquids:

$$v = \sqrt{\frac{K}{\rho}} \qquad (11.19)$$

where, K is the bulk modulus and ρ is the density of the rod.

EXAMPLE 11.7

Calculate the speed of sound in a steel rod whose Young's modulus $Y = 2 \times 10^{11}$ N m^{-2} and $\rho = 7800\ kg\ m^{-3}$.

Solution

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times10^{11}}{7800}} = \sqrt{0.2564\times10^{8}}$$

$$= 0.506\times10^{4} ms^{-1} = 5\times10^{3} ms^{-1}$$

Therefore, longitudinal waves travel faster in a solid than in a liquid or a gas. Now you may understand why a shepherd checks before crossing railway track by keeping his ears on the rails to safegaurd his cattle.

EXAMPLE 11.8

An increase in pressure of 100 kPa causes a certain volume of water to decrease by 0.005% of its original volume.

(a) Calculate the bulk modulus of water?.

(b) Compute the speed of sound (compressional waves) in water?.

Solution

(a) Bulk modulus

$$B = V\left|\frac{\Delta P}{\Delta V}\right| = \frac{100\times10^{3}}{0.005\times10^{-2}} =$$

$$= \frac{100\times10^{3}}{5\times10^{-5}} = 2000\ MPa,$$ where MP_a mega pascal

(b) Speed of sound in water is

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2000\times10^{6}}{1000}} = 1414 ms^{-1}$$

Note The velocities of both transverse waves and longitudinal waves depend on elastic property (like string tension T or bulk modulus B) and inertial property (like density or mass per unit length) i.e., $v = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$

Table 11.2: Speed of sound in various media

S.No.	Medium	Speed in m s^{-1}
	Solids	
1.	Rubber	1600
2.	Gold	3240
3.	Brass	4700
4.	Copper	5010
5.	Iron	5950
6.	Aluminum	6420
	Liquids at 25°C	
1.	Kerosene	1324
2.	Mercury	1450
3.	Water	1493
4.	Sea Water	1533
	Gas (at 0°C)	
1.	Oxygen	317
2.	Air	331
3.	Helium	972
4.	Hydrogen	1286
	Gas (at 20°C)	
1.	Air	343

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

11.4 PROPAGATION OF SOUND WAVES

We know that sound waves are longitudinal waves, and when they propagate compressions and rarefactions are formed. In the following section, we compute the speed of sound in air by Newton's method and also discuss the Laplace correction and the factors affecting sound in air.

11.4.1 Newton's formula for speed of sound waves in air

Sir Isaac Newton assumed that when sound propagates in air, the formation of compression and rarefaction takes place in a very slow manner so that the process is isothermal in nature. That is, the heat produced during compression (pressure increases, volume decreases), and heat lost during rarefaction (pressure decreases, volume increases) occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law, Mathematically

$$PV = \text{Constant} \qquad (11.20)$$

Differentiating equation (11.20), we get

$$PdV + VdP = 0$$

$$\text{or,} \quad P = -V\frac{dP}{dV} = B_T \qquad (11.21)$$

where, B_T is an isothermal bulk modulus of air. Substituting equation (11.21) in equation (11.16), the speed of sound in air is

$$v_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad (11.22)$$

Since P is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8)\ \text{N m}^{-2}$$

$\rho = 1.293\ kg\ m^{-3}$. here ρ is density of air

Then the speed of sound in air at Normal Temperature and Pressure (NTP) is

$$v_T = \sqrt{\frac{(0.76 \times 13.6 \times 10^3 \times 9.8)}{1.293}}$$

$= 279.80\ \text{m s}^{-1} \approx 280\ \text{ms}^{-1}$ (theoretical value)

But the speed of sound in air at 0°C is experimentally observed as $332\ \text{m s}^{-1}$ which is close upto 16% more than theoretical value (Percentage error is $\frac{(332-280)}{332} \times 100\% = 15.6\%$). This error is not small

11.4.2 Laplace's correction

In 1816, Laplace satisfactorily corrected this discrepancy by assuming that when the sound propagates through a medium, the particles oscillate very rapidly such that the compression and rarefaction occur very fast. Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat. Since, temperature is no longer considered as a constant here, sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law (not Boyle's law as Newton assumed), which is

$$PV^{\gamma} = \text{constant} \qquad (11.23)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

where, $\gamma = \frac{C_P}{C_v}$, which is the ratio between specific heat at constant pressure and specific heat at constant volume.

Differentiating equation (11.23) on both the sides, we get

$$V^{\gamma} dP + P(\gamma V^{\gamma-1} dV) = 0$$

$$\text{or,} \quad \gamma P = -V\frac{dp}{dV} = B_A \qquad (11.24)$$

where, B_A is the adiabatic bulk modulus of air. Now, substituting equation (11.24) in equation (11.16), the speed of sound in air is

$$v_A = \sqrt{\frac{B_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma}\, v_T \qquad (11.25)$$

Since air contains mainly, nitrogen, oxygen, hydrogen etc, (diatomic gas), we take γ = 1.47. Hence, speed of sound in air is $v_A = (\sqrt{1.4})(280 \text{ m s}^{-1}) = 331.30 \text{ m s}^{-1}$, which is very much closer to experimental data.

11.4.3 Factors affecting speed of sound in gases

Let us consider an ideal gas whose equation of state is

$$PV = nRT \qquad (11.26)$$

where, P is pressure, V is volume, T is temperature, n is number of mole and R is universal gas constant. For a given mass of a molecule, equation (11.26) can be written as

$$\frac{PV}{T} = \text{Constant} \qquad (11.27)$$

For a fixed mass m, density of the gas inversely varies with volume. i.e.,

$$\rho \propto \frac{1}{V}, \quad V = \frac{m}{\rho} \qquad (11.28)$$

Substituting equation (11.28) in equation (11.27), we get

$$\frac{P}{\rho} = cT \qquad (11.29)$$

where c is constant.

The speed of sound in air given in equation (11.25) can be written as

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma c T} \qquad (11.30)$$

From the above relation we observe the following

(a) Effect of pressure :

For a fixed temperature, when the pressure varies, correspondingly density also varies such that the ratio $\left(\frac{P}{\rho}\right)$ becomes constant.

This means that the speed of sound is independent of pressure for a fixed temperature. If the temperature remains same at the top and the bottom of a mountain then the speed of sound will remain same at these two points. But, in practice, the temperatures are not same at top and bottom of a mountain; hence, the speed of sound is different at different points.

(b) Effect of temperature :

Since $v \propto \sqrt{T}$,

the speed of sound varies directly to the square root of temperature in kelvin.

Let v_0 be the speed of sound at temperature at 0° C or 273 K and v be the speed of sound at any arbitrary temperature T (in kelvin), then

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273+t}{273}}$$

$$v = v_0\sqrt{1+\frac{t}{273}} \cong v_0\left(1+\frac{t}{546}\right)$$

(using binomial expansion)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Since $v_0 = 331\, m\, s^{-1}$ at 0^0C, v at any temperature in t^0C is

$$v = (331 + 0.60t)\ m\, s^{-1}$$

Thus the speed of sound in air increases by 0.61 $m\, s^{-1}$ per degree celcius rise in temperature. Note that when the temperature is increased, the molecules will vibrate faster due to gain in thermal energy and hence, speed of sound increases.

(c) Effect of density :

Let us consider two gases with different densities having same temperature and pressure. Then the speed of sound in the two gases are

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \quad (11.31)$$

and

$$v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}} \quad (11.32)$$

Taking ratio of equation (11.31) and equation (11.32), we get

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

For gases having same value of γ,

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} \quad (11.33)$$

Thus the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

(d) Effect of moisture (humidity):

We know that density of moist air is 0.625 of that of dry air, which means the presence of moisture in air (increase in humidity) decreases its density. Therefore, speed of sound increases with rise in humidity. From equation (11.30)

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

Let ρ_1, v_1 and ρ_2, v_2 be the density and speeds of sound in dry air and moist air, respectively. Then

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\rho_2}{\rho_1}} \quad \text{if } \gamma_1 = \gamma_2$$

Since P is the total atmospheric pressure, it can be shown that

$$\frac{\rho_2}{\rho_1} = \frac{P}{p_1 + 0.625 p_2}$$

where p_1 and p_2 are the partial pressures of dry air and water vapour respectively. Then

$$v_1 = v_2 \sqrt{\frac{P}{p_1 + 0.625 p_2}} \quad (11.34)$$

(e) Effect of wind:

The speed of sound is also affected by blowing of wind. In the direction along the wind blowing, the speed of sound increases whereas in the direction opposite to wind blowing, the speed of sound decreases.

EXAMPLE 11.9

The ratio of the densities of oxygen and nitrogen is 16:14. Calculate the temperature when the speed of sound in nitrogen gas at 17°C is equal to the speed of sound in oxygen gas.

Solution

From equation (11.25), we have

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

But $\rho = \frac{M}{V}$

Therefore,

$$v = \sqrt{\frac{\gamma PV}{M}}$$

Using equation (11.26)

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Where, R is the universal gas constant and M is the molecular mass of the gas. The speed of sound in nitrogen gas at 17°C is

$$v_N = \sqrt{\frac{\gamma R(273K + 17K)}{M_N}}$$

$$= \sqrt{\frac{\gamma R(290K)}{M_N}} \quad (1)$$

Similarly, the speed of sound in oxygen gas at t in K is

$$v_0 = \sqrt{\frac{\gamma R(273K + t)}{M_0}} \quad (2)$$

Given that the value of γ is same for both the gases, the two speeds must be equal. Hence, equating equation (1) and (2), we get

$$v_O = v_N$$

$$\sqrt{\frac{\gamma R(273 + t)}{M_0}} = \sqrt{\frac{\gamma R(290)}{M_N}}$$

Squaring on both sides and cancelling γ R term and rearranging, we get

$$\frac{M_0}{M_N} = \frac{273 + t}{290} \quad (3)$$

Since the densities of oxygen and nitrogen is 16:14,

$$\frac{\rho_0}{\rho_N} = \frac{16}{14} \quad (4)$$

$$\frac{\rho_0}{\rho_N} = \frac{\frac{M_O}{V}}{\frac{M_N}{V}} = \frac{M_0}{M_N} \Rightarrow \frac{M_0}{M_N} = \frac{16}{14} \quad (5)$$

Substituting equation (5) in equation (3), we get

$$\frac{273 + t}{290} = \frac{16}{14} \Rightarrow 3822 + 14t = 4640$$

$$\Rightarrow t = 58.4\text{ K}$$

11.5 REFLECTION OF SOUND WAVES

When sound wave passes from one medium to another medium, the following things can happen

(a) **Reflection of sound:** If the medium is highly dense (highly rigid), the sound can be reflected completely (bounced back) to the original medium.

(b) **Refraction of sound:** When the sound waves propagate from one medium to another medium such that there can be some energy loss due to absorption by the second medium.

In this section, we will consider only the reflection of sound waves in a medium when it experiences a harder surface. Similar to light, sound can also obey the *laws of reflection*, which states that

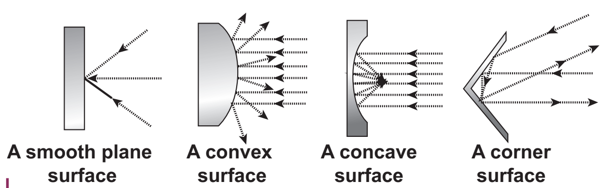


Figure 11.17 Reflection of sound in different surfaces

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(i) The angle of incidence of sound is equal to the angle of reflection.

(ii) When the sound wave is reflected by a surface then the incident wave, reflected wave and the normal at the point of incidence all lie in the same plane.

Similar to reflection of light from a mirror, sound also reflects from a harder flat surface, This is called as **specular reflection.**

Specular reflection is observed only when the wavelength of the source is smaller than dimensions of the reflecting surface, as well as smaller than surface irregularities.

11.5.1 Reflection of sound through the plane surface

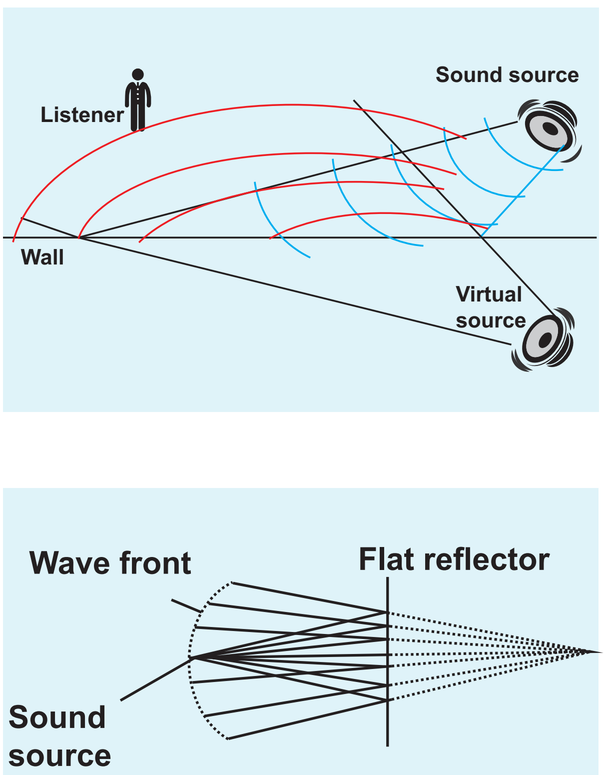


Figure 11.18 Reflection of sound through the plane surface

When the sound waves hit the plane wall, they bounce off in a manner similar to that of light. Suppose a loudspeaker is kept at an angle with respect to a wall (plane surface), then the waves coming from the source (assumed to be a point source) can be treated as spherical wave fronts (say, compressions moving like a spherical wave front). Therefore, the reflected wave front on the plane surface is also spherical, such that its centre of curvature (which lies on the other side of plane surface) can be treated as the image of the sound source (virtual or imaginary loud speaker) which can be assumed to be at a position behind the plane surface. These are shown in Figures 11.18, 11.19.

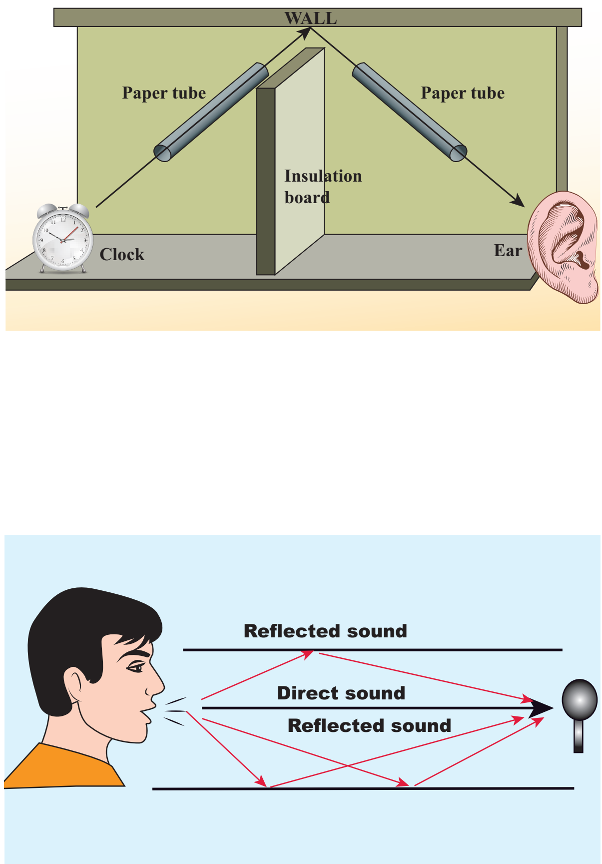


Figure 11.19 Common examples for reflection of sound in real situation

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

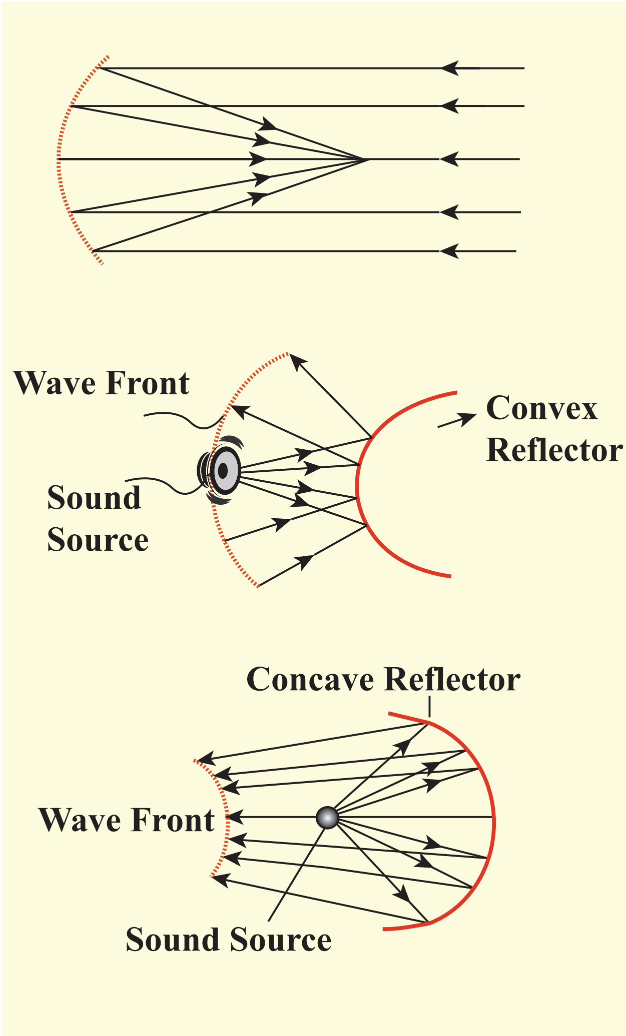


Figure 11.20 Reflection of sound through the curved surface

11.5.2 Reflection of sound through the curved surface

The behaviour of sound is different when it is reflected from different surfaces-convex or concave or plane. The sound reflected from a convex surface is spread out and so it is easily attenuated and weakened. Whereas, if it is reflected from the concave surface it will converge at a point and this can be easily amplified. The parabolic reflector (curved reflector) which is used to focus the sound precisely to a point is used in designing the parabolic mics which are known as high directional microphones.

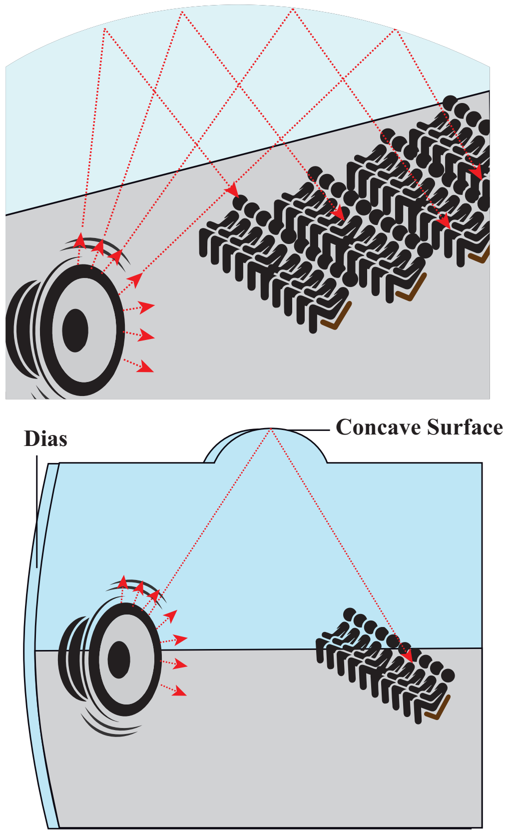


Figure 11.21 Sound in a big auditorium

We know that any surface (smooth or rough) can absorb sound. For example, the sound produced in a big hall or auditorium or theatre is absorbed by the walls, ceilings, floor, seats etc. To avoid such losses, a curved sound board (concave board) is kept in front of the speaker, so that the board reflects the sound waves of the speaker towards the audience. This method will minimize the spreading of sound waves in all possible direction in that hall and also enhances the uniform distribution of sound throughout the hall. That is why a person sitting at any position in that hall can hear the sound without any disturbance.

11.5.3 Applications of reflection of sound waves

(a) **Stethoscope:** It works on the principle of multiple reflections.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

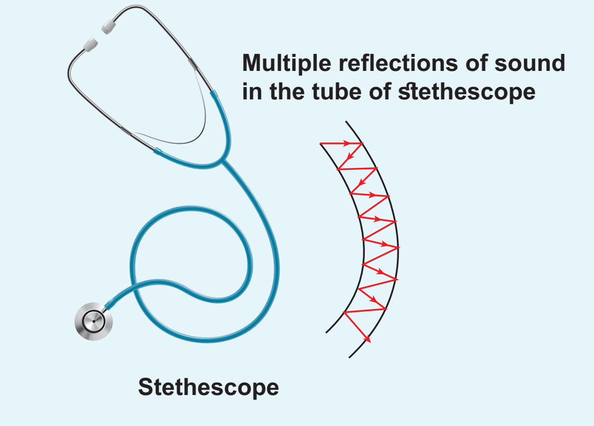


Figure 11.22 Stethoscope and multiple reflection of signal in a rubber tube

It consists of three main parts:

(i) Chest piece
(ii) Ear piece
(iii) Rubber tube

(i) Chest piece: It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

(ii) Ear piece: It is made up of metal tubes which are used to hear sounds detected by the chest piece.

(iii) Rubber tube: This tube connects both chest piece and ear piece. It is used to transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected, and it reaches the ear piece through this tube by multiple reflections.

(b) Echo: An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 $m\,s^{-1}$. If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall. After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds.

Scientists have estimated that *we can hear two sounds properly if the time gap or time interval between each sound is* $\left(\frac{1}{10}\right)^{th}$ *of a second (persistence of hearing)* i.e., 0.1 s. Then,

$$\text{velocity} = \frac{\text{Distance travelled}}{\text{time taken}} = \frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.4 \text{ m}$$

$$d = 17.2 \text{ m}$$

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

(c) SONAR: SO*und* **NA***vigation* and **R***anging*. Sonar systems make use of reflections of sound waves in water to locate the position or motion of an object. Similarly, dolphins and bats use the sonar principle to find their way in the darkness.

(d) Reverberation: In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function. The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation. The duration for which the sound persists is called reverberation time. It should be noted that the reverberation time greatly affects the quality of sound heard in a hall. Therefore, halls are constructed with some optimum reverberation time.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 11.10

Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 m s^{-1}.

Solution

The time taken by the sound to come back as echo is $2t = 4 \Rightarrow t = 2$ s

$\therefore$ The distance is $d = vt = (343 \text{ m s}^{-1})(2 \text{ s}) = 686$ m.

Note: Classification of sound waves: Sound waves can be classified in three groups according to their range of frequencies:

(1) ***Infrasonic waves:***
Sound waves having frequencies below 20 Hz are called infrasonic waves. These waves are produced during earthquakes. Human beings cannot hear these frequencies. Snakes can hear these frequencies.

(2) ***Audible waves:***
Sound waves having frequencies between 20 Hz to 20,000 Hz (20kHz) are called audible waves. Human beings can hear these frequencies.

(3) ***Ultrasonic waves:***
Sound waves having frequencies greater than 20 kHz are known as ultrasonic waves. Human beings cannot hear these frequencies. Bats can produce and hear these frequencies.

Note

(1.) Supersonic speed:
An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

(2.) Mach number:
It is the ratio of the velocity of source to the velocity of sound.

$$\text{Mach number} = \frac{\text{velocity of source}}{\text{velocity of sound}}$$

11.6 PROGRESSIVE WAVES (OR) TRAVELLING WAVES

If a wave that propagates in a medium is continuous then it is known as progressive wave or travelling wave.

11.6.1 Characteristics of progressive waves

1. Particles in the medium vibrate about their mean positions with the same amplitude.
2. The phase of every particle ranges from 0 to 2π.
3. No particle remains at rest permanently. During wave propagation, particles come to the rest position only twice at the extreme points.
4. Transverse progressive waves are characterized by crests and troughs whereas longitudinal progressive waves are characterized by compressions and rarefactions.
5. When the particles pass through the mean position they always move with the same maximum velocity.
6. The displacement, velocity and acceleration of particles separated from each other by $n\lambda$ are the same, where n is an integer, and λ is the wavelength.

11.6.2 Equation of a plane progressive wave

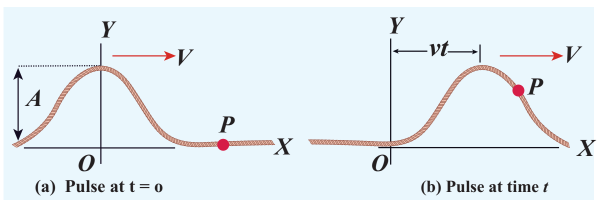


Figure 11.23 Wave pulse moving with velocity v at two instants at $t = 0$ and at time t

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Suppose we give a jerk on a stretched string at time $t = 0$ s. Let us assume that the wave pulse created during this disturbance moves along positive x direction with constant speed v as shown in Figure 11.23 (a). We can represent the shape of the wave pulse, mathematically as $y = y(x, 0) = f(x)$ at time $t = 0$ s. Assume that the shape of the wave pulse remains the same during the propagation. After some time t, the pulse moving towards the right and any point on it can be represented by x' (read it as x prime) as shown in Figure 11.23 (b). Then,

$$y(x, t) = f(x') = f(x - vt) \qquad (11.35)$$

Similarly, if the wave pulse moves towards left with constant speed v, then $y = f(x + vt)$. Both waves $y = f(x + vt)$ and $y = f(x - vt)$ will satisfy the following one dimensional differential equation known as the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad (11.36)$$

where the symbol ∂ represent partial derivative (read $\frac{\partial y}{\partial x}$ as partial y by partial x). Not all the solutions satisfying this differential equation can represent waves, because any physical acceptable wave must take finite values for all values of x and t. But if the function represents a wave then it must satisfy the differential equation. Since, in one dimension (one independent variable), the partial derivative with respect to x is the same as total derivative in coordinate x, we write

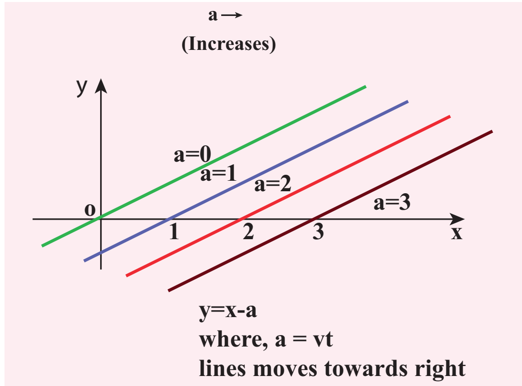
$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2} \qquad (11.37)$$

This can be extended to more than one dimension (two, three, etc.). Here, for simplicity, we focus only on the one dimensional wave equation.

EXAMPLE 11.11

Sketch $y = x - a$ for different values of a.

Solution

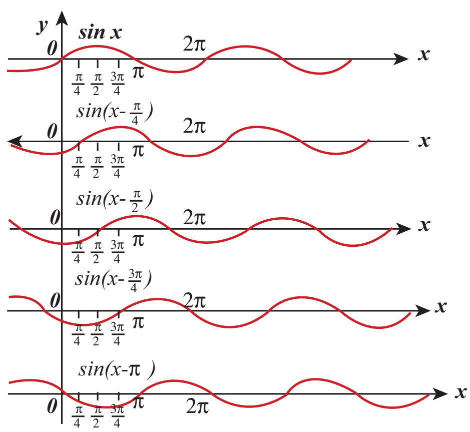


This implies, when increasing the value of a, the line shifts towards right side. For $a = vt$, $y = x - vt$ satisfies the differential equation. Though this function satisfies the differential equation, it is not finite for all values of x and t. Hence, it does not represent a wave.

EXAMPLE 11.12

How does the wave $y = \sin(x - a)$ for $a = 0$, $a = \frac{\pi}{4}$, $a = \frac{\pi}{2}$, $a = \frac{3\pi}{2}$ and $a = \pi$ look like?. Sketch this wave.

Solution



From the above picture we observe that $y = \sin(x-a)$ for $a = 0$, $a = \frac{\pi}{4}$, $a = \frac{\pi}{2}$,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

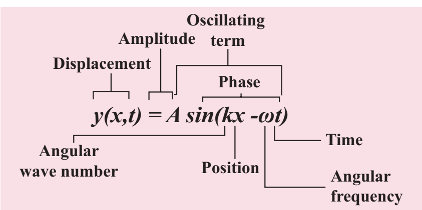
$a = \frac{3\pi}{2}$ and $a = \pi$, the function $y = \sin(x-a)$ shifts towards right. Further, we can take $a = vt$ and $v = \frac{\pi}{4}$, and sketching for different times $t = 0s$, $t = 1s$, $t = 2s$ etc., we once again observe that $y = \sin(x-vt)$ moves towards the right. Hence, $y = \sin(x-vt)$ is a travelling (or progressive) wave moving towards the right. If $y = \sin(x+vt)$ then the travelling (or progressive) wave moves towards the left. Thus, any arbitrary function of type $y = f(x-vt)$ characterising the wave must move towards right and similarly, any arbitrary function of type $y = f(x+vt)$ characterizing the wave must move towards left.

EXAMPLE 11.13

Check the dimensional of the wave $y = \sin(x-vt)$. If it is dimensionally wrong, write the above equation in the correct form.

Solution

Dimensionally it is not correct. we know that $y = \sin(x-vt)$ must be a dimensionless quantity but $x-vt$ has dimension. The correct equation is $y = \sin(kx-\omega t)$, where k and ω have the dimensions of inverse of length and inverse of time respectively. The sine functions and cosine functions are periodic functions with period 2π. Therefore, the correct expression is $y = \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$ where λ and T are wavelength and time period, respectively. In general, $y(x,t) = A\sin(kx-\omega t)$.



11.6.3 Graphical representation of the wave

Let us graphically represent the two forms of the wave variation

(a) Space (or Spatial) variation graph

(b) Time (or Temporal) variation graph

(a) Space variation graph

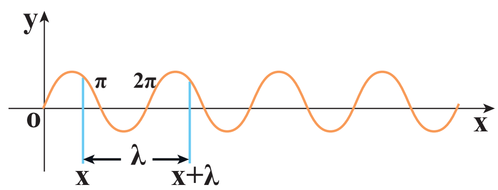


Figure 11.24 Graph of sinusoidal function $y = A\sin(kx)$

By keeping the time fixed, the change in displacement with respect to x is plotted. Let us consider a sinusoidal graph, $y = A\sin(kx)$ as shown in the Figure 11.24, where k is a constant. Since the wavelength λ denotes the distance between any two points in the same state of motion, the displacement y is the same at both the ends $y = x$ and $y = x + \lambda$, i.e.,

$$y = A\sin(kx) = A\sin(k(x+\lambda)) = A\sin(kx + k\lambda) \quad (11.38)$$

The sine function is a periodic function with period 2π. Hence,

$$y = A\sin(kx + 2\pi) = A\sin(kx) \quad (11.39)$$

Comparing equation (11.38) and equation (11.39), we get

$$kx + k\lambda = kx + 2\pi$$

This implies

$$k = \frac{2\pi}{\lambda} \text{ rad m}^{-1} \quad (11.40)$$

where k is called wave number. This measures how many wavelengths are present in 2π radians.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The spatial periodicity of the wave is

$$\lambda = \frac{2\pi}{k} \text{ in m}$$

Then,

At $t = 0$ s $\quad y(x, 0) = y(x + \lambda, 0)$ and

At any time t, $y(x, t) = y(x + \lambda, t)$

EXAMPLE 11.14

The wavelength of two sine waves are λ_1 = 1m and λ_2 = 6m. Calculate the corresponding wave numbers.

Solution

$$k_1 = \frac{2\pi}{1} = 6.28 \text{ rad m}^{-1}$$

$$k_2 = \frac{2\pi}{6} = 1.05 \text{ rad m}^{-1}$$

(b) Time variation graph

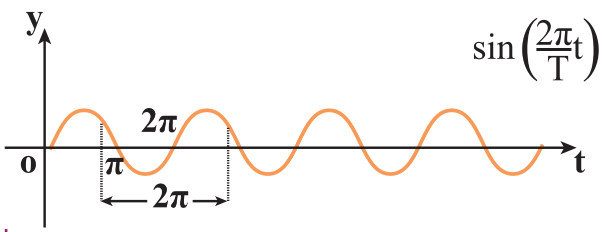


Figure 11.25 Graph of sinusoidal function $y = A\sin(\omega t)$

By keeping the position fixed, the change in displacement with respect to time is plotted. Let us consider a sinusoidal graph, $y = A\sin(\omega t)$ as shown in the Figure 11.25, where ω is angular frequency of the wave which measures how quickly wave oscillates in time or number of cycles per second.

The temporal periodicity or time period is

$$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$$

The angular frequency is related to frequency f by the expression $\omega = 2\pi f$, where the frequency f is defined as the number of oscillations made by the medium particle per second. Since inverse of frequency is time period, we have,

$$T = \frac{1}{f} \text{ in seconds}$$

This is the time taken by a medium particle to complete one oscillation. Hence, we can define the speed of a wave (wave speed, v) as the distance traversed by the wave per second

$$v = \frac{\lambda}{T} = \lambda f \text{ in m s}^{-1}$$

which is the same relation as we obtained in equation (11.4).

11.6.4 Particle velocity and wave velocity

In a plane progressive harmonic wave, the constituent particles in the medium oscillate simple harmonically about their equilibrium positions. When a particle is in motion, the rate of change of displacement at any instant of time is defined as velocity of the particle at that instant of time. This is known as particle velocity.

$$v_P = \frac{dy}{dt} \text{ m s}^{-1} \qquad (11.41)$$

But $\quad y(x, t) = A\sin(kx - \omega t) \qquad (11.42)$

Therefore, $\frac{dy}{dt} = -\omega A\cos(kx - \omega t) \quad (11.43)$

Similarly, we can define velocity (here speed) for the travelling wave (or progressive wave). In order to determine the velocity of a progressive wave, let us consider a progressive wave (shown in Figure 11.23) moving towards right. This can be mathematically represented as a sinusoidal wave. Let P be any point on the phase of the wave and y_P be its displacement with respect

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

to the mean position. The displacement of the wave at an instant *t* is

$$y = y(x,t) = A \sin(kx - \omega t)$$

At the next instant of time $t' = t + \Delta t$ the position of the point *P* is $x' = x + \Delta x$. Hence, the displacement of the wave at this instant is

$$y = y(x', t') = y(x + \Delta x, t + \Delta t)$$
$$= A \sin[k(x + \Delta x) - \omega(t + \Delta t)] \quad (11.44)$$

Since the shape of the wave remains the same, this means that the phase of the wave remains constant (i.e., the *y*- displacement of the point is a constant). Therefore, equating equation (11.42) and equation (11.44), we get

$y(x',t') = y(x,t)$, which implies

$$A \sin[k(x + \Delta x) - \omega(t + \Delta t)] = A \sin(kx - \omega t)$$

Or

$$k(x + \Delta x) - \omega(t + \Delta t) = kx - \omega t = \text{constant} \quad (11.45)$$

On simplification of equation (11.45), we get

$$v = \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v_p \quad (11.46)$$

where v_p is called wave velocity or phase velocity.

By expressing the angular frequency and wave number in terms of frequency and wave length, we obtain

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} = \lambda f$$

EXAMPLE 11.15

A mobile phone tower transmits a wave signal of frequency 900MHz. Calculate the length of the waves transmitted from the mobile phone tower.

Solution

Frequency, $f = 900\ MHz = 900 \times 10^6\ Hz$

The speed of wave is $c = 3 \times 10^8 \text{m s}^{-1}$

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.33m$$

11.7 SUPERPOSITION PRINCIPLE

When a jerk is given to a stretched string which is tied at one end, a wave pulse is produced and the pulse travels along the string. Suppose two persons holding the stretched string on either side give a jerk simultaneously, then these two wave pulses move towards each other, meet at some point and move away from each other with their original identity. Their behaviour is very different only at the crossing/meeting points; this behaviour depends on whether the two pulses have the same or different shape as shown in Figure 11.26.

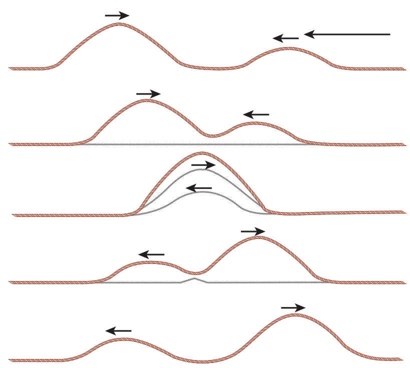


Figure 11.26 Superposition of two waves

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

When the pulses have the same shape, at the crossing, the total displacement is the algebraic sum of their individual displacements and hence its net amplitude is higher than the amplitudes of the individual pulses. Whereas, if the two pulses have same amplitude but shapes are 180° out of phase at the crossing point, the net amplitude vanishes at that point and the pulses will recover their identities after crossing. Only waves can possess such a peculiar property and it is called *superposition of waves*. This means that the principle of superposition explains the net behaviour of the waves when they overlap. Generalizing to any number of waves i.e, if two are more waves in a medium move simultaneously, when they overlap, their total displacement is the vector sum of the individual displacements. We know that the waves satisfy the wave equation which is a linear second order homogeneous partial differential equation in both space coordinates and time. Hence, their linear combination (often called as linear superposition of waves) will also satisfy the same differential equation.

To understand mathematically, let us consider two functions which characterize the displacement of the waves, for example,

$$y_1 = A_1 \sin(kx - \omega t)$$

and

$$y_2 = A_2 \cos(kx - \omega t)$$

Since, both y_1 and y_2 satisfy the wave equation (solutions of wave equation) then their algebraic sum

$$y = y_1 + y_2$$

also satisfies the wave equation. This means, the displacements are additive. Suppose we multiply y_1 and y_2 with some constant then their amplitude is scaled by that constant

Further, if C_1 and C_2 are used to multiply the displacements y_1 and y_2, respectively, then, their net displacement y is

$$y = C_1 y_1 + C_2 y_2$$

This can be generalized to any number of waves. In the case of n such waves in more than one dimension the displacements are written using vector notation.

Here, the net displacement $\vec{y}$ is

$$\vec{y} = \sum_{i=1}^{n} C_i \vec{y_i}$$

The principle of superposition can explain the following :

(a) Space (or spatial) Interference (also known as Interference)

(b) Time (or Temporal) Interference (also known as Beats)

(c) Concept of stationary waves

Waves that obey principle of superposition are called linear waves (amplitude is much smaller than their wavelengths). In general, if the amplitude of the wave is not small then they are called non-linear waves. These violate the linear superposition principle, e.g. laser. In this chapter, we will focus our attention only on linear waves.

We will discuss the following in different subsections:

11.7.1 Interference of waves

Figure 11.27 Interference of waves

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Interference is a phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

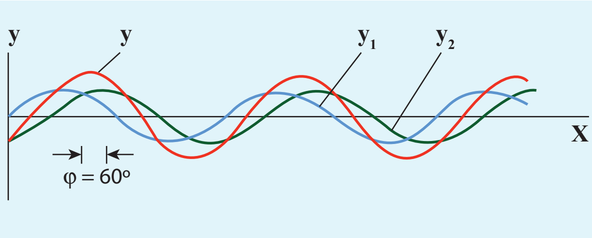


Figure 11.28 Interference of two sinusoidal waves

Consider two harmonic waves having identical frequencies, constant phase difference φ and same wave form (can be treated as coherent source), but having amplitudes A_1 and A_2, then

$$y_1 = A_1 \sin(kx - \omega t) \qquad (11.47)$$

$$y_2 = A_2 \sin(kx - \omega t + \varphi) \qquad (11.48)$$

Suppose they move simultaneously in a particular direction, then interference occurs (i.e., overlap of these two waves). Mathematically

$$y = y_1 + y_2 \qquad (11.49)$$

Therefore, substituting equation (11.47) and equation (11.48) in equation (11.49), we get

$$y = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \varphi)$$

Using trigonometric identity $\sin(\alpha+\beta) = (\sin\alpha\cos\beta + \cos\alpha\sin\beta)$, we get

$$y = A_1 \sin(kx - \omega t) + A_2 [\sin(kx - \omega t)\cos\varphi + \cos(kx - \omega t)\sin\varphi]$$

$$y = \sin(kx - \omega t)(A_1 + A_2\cos\varphi) + A_2 \sin\varphi\cos(kx - \omega t) \qquad (11.50)$$

Let us re-define

$$A\cos\theta = (A_1 + A_2\cos\varphi) \qquad (11.51)$$

and $$A\sin\theta = A_2\sin\varphi \qquad (11.52)$$

then equation (11.50) can be rewritten as

$$y = A\sin(kx-\omega t)\cos\theta + A\cos(kx-\omega t)\sin\theta$$

$$y = A(\sin(kx-\omega t)\cos\theta + \sin\theta\cos(kx-\omega t))$$

$$y = A\sin(kx-\omega t+\theta) \qquad (11.53)$$

By squaring and adding equation (11.51) and equation (11.52), we get

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\varphi \qquad (11.54)$$

Since, intensity is square of the amplitude ($I = A^2$), we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi \qquad (11.55)$$

This means the resultant intensity at any point depends on the phase difference at that point.

(a) For constructive interference:

When crests of one wave overlap with crests of another wave, their amplitudes will add up and we get constructive interference. The resultant wave has a larger amplitude than the individual waves as shown in Figure 11.29 (a).

The constructive interference at a point occurs if there is maximum intensity at that point, which means that

$\cos\varphi = +1 \Rightarrow \varphi = 0, 2\pi, 4\pi, \ldots = 2n\pi$,
where $n = 0,1,2,\ldots$

This is the phase difference in which two waves overlap to give constructive interference.

Therefore, for this resultant wave,

$$I_{maximum} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2$$

Hence, the resultant amplitude

$$A = A_1 + A_2$$

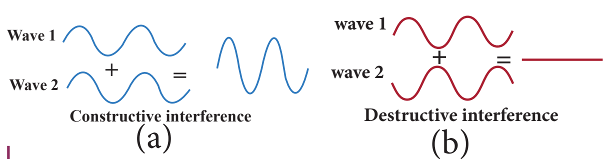


Figure 11.29 (a) Constructive interference (b) Destructive interference

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(b) For destructive interference:

When the trough of one wave overlaps with the crest of another wave, their amplitudes "cancel" each other and we get destructive interference as shown in Figure 11.29 (b). The resultant amplitude is nearly zero. The destructive interference occurs if there is minimum intensity at that point, which means $\cos\varphi = -1 \Rightarrow \varphi = \pi, 3\pi, 5\pi, \ldots = (2n-1)\pi$, where n = 0,1,2,…. i.e. This is the phase difference in which two waves overlap to give destructive interference. Therefore,

$$I_{minimum} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(A_1 - A_2\right)^2$$

Hence, the resultant amplitude

$$A = |A_1 - A_2|$$

Let us consider a simple instrument to demonstrate the interference of sound waves as shown in Figure 11.30.

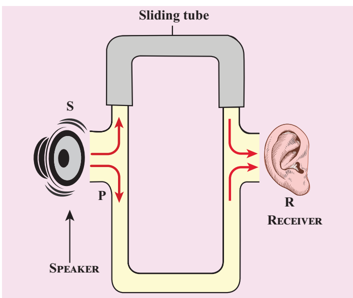


Figure 11.30 Simple instrument to demonstrate interference of sound waves

A sound wave from a loudspeaker S is sent through the tube P. This looks like a T-shaped junction. In this case, half of the sound energy is sent in one direction and the remaining half is sent in the opposite direction. Therefore, the sound waves that reach the receiver R can travel along either of two paths. The distance covered by the sound wave along any path from the speaker to receiver is called the path length . From the Figure 11.30, we notice that the lower path length is fixed but the upper path length can be varied by sliding the upper tube i.e., is varied. The difference in path length is known as path difference,

$$\Delta r = |r_2 - r_1|$$

Suppose the path difference is allowed to be either zero or some integer (or integral) multiple of wavelength λ. Mathematically, we have

$$\Delta r = n\lambda \qquad \text{where, n = 0, 1, 2, 3,....}$$

Then the two waves arriving from the paths r_1 and r_2 reach the receiver at any instant are in phase (the phase difference is 0° or 2π) and interfere constructively as shown in Figure 11.31.

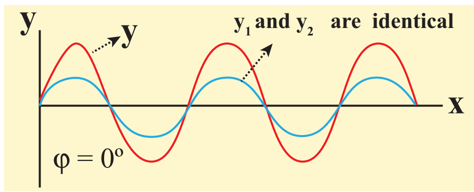


Figure 11.31 Maximum intensity when the phase difference is 0°

Therefore, in this case, maximum sound intensity is detected by the receiver. If the path difference is some half-odd-integer (or half-integral) multiple of wavelength λ, mathematically, $\Delta r = n\frac{\lambda}{2}$

where, n = 1,3,... (n is odd)

then the two waves arriving from the paths r_1 and r_2 and reaching the receiver at any instant are out of phase (phase difference of π or 180°). They interfere destructively as shown in Figure 11.32. They will cancel each other.

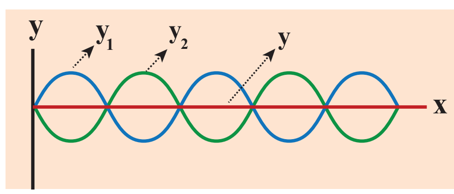


Figure 11.32 Minimum intensity when the phase difference is 180°

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

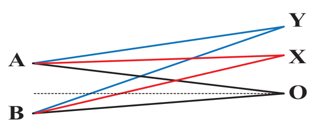
Therefore, the amplitude is minimum or zero amplitude which means no sound. No sound intensity is detected by the receiver in this case.
The relation between path difference and phase difference is

$$\text{phase difference} = \frac{2\pi}{\lambda}\ (\text{path difference}) \qquad (11.56)$$

i.e., $\Delta\varphi = \frac{2\pi}{\lambda}\Delta r$ or $\Delta r = \frac{\lambda}{2\pi}\Delta\varphi$

EXAMPLE 11.16

Consider two sources A and B as shown in the figure below. Let the two sources emit simple harmonic waves of same frequency but of different amplitudes, and both are in phase (same phase). Let O be any point equidistant from A and B as shown in the figure. Calculate the intensity at points O, Y and X. (X and Y are not equidistant from A & B)



Solution

The distance between OA and OB are the same and hence, the waves starting from A and B reach O after covering equal distances (equal path lengths). Thus, the path difference between two waves at O is zero.

$$OA - OB = 0$$

Since the waves are in the same phase, at the point O, the phase difference between two waves is also zero. Thus, the resultant intensity at the point O is maximum.
Consider a point Y, such that the path difference between two waves is λ. Then the phase difference at Y is

$$\Delta\varphi = \frac{2\pi}{\lambda} \times \Delta r = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

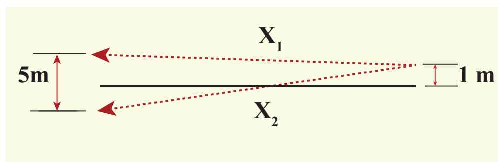
Therefore, at the point Y, the two waves from A and B are in phase, hence, the intensity will be maximum.
Consider a point X, and let the path difference the between two waves be $\frac{\lambda}{2}$. Then the phase difference at X is

$$\Delta\varphi = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

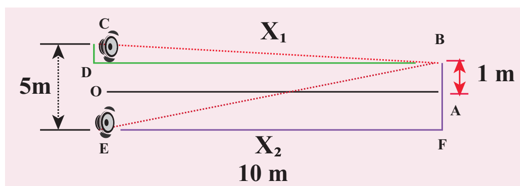
Therefore, at the point X, the waves meet and are in out of phase, Hence, due to destructive interference, the intensity will be minimum.

EXAMPLE 11.17

Two speakers C and E are placed 5 m apart and are driven by the same source. Let a man stand at A which is 10 m away from the mid point O of C and E. The man walks towards the point O which is at 1 m (parallel to OC) as shown in the figure. He receives the first minimum in sound intensity at B. Then calculate the frequency of the source. (Assume speed of sound = 343 m s^{-1})



Solution



The first minimum occurs when the two waves reaching the point B are 180° (out of phase). The path difference $\Delta x = \frac{\lambda}{2}$.

In order to calculate the path difference, we have to find the path lengths x_1 and x_2.
In a right triangle BDC,

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$DB = 10\text{m and } OC = \frac{1}{2}(5) = 2.5\text{m}$

$CD = OC - 1 = (2.5 \text{ m}) - 1 \text{ m} = 1.5 \text{ m}$

$x_1 = \sqrt{(10)^2 + (1.5)^2} = \sqrt{100 + 2.25}$

$= \sqrt{102.25} = 10.1m$

In a right triangle EFB,

$DB = 10\text{m and } OE = \frac{1}{2}(5) = 2.5\text{m} = FA$

$FB = FA + AB = (2.5 \text{ m}) + 1 \text{ m} = 3.5 \text{ m}$

$x_2 = \sqrt{(10)^2 + (3.5)^2} = \sqrt{100 + 12.25}$

$= \sqrt{112.25} = 10.6m$

The path difference $\Delta x = x_2 - x_1 = 10.6$ m$-$10.1 m = 0.5 m. Required that this path difference

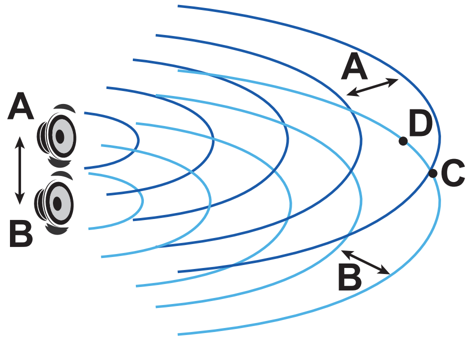
$$\Delta x = \frac{\lambda}{2} = 0.5 \Rightarrow \lambda = 1.0 \text{ m}$$

To obtain the frequency of source, we use

$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{343}{1} = 343 \text{ Hz}$$

$$= 0.3\ kHz$$

Note If the speakers were connected such that already the path difference is $\frac{\lambda}{2}$. Now, the path difference combines with a path difference of $\frac{\lambda}{2}$. This gives a total path difference of λ which means, the waves are in phase and there is a maximum intensity at point B.



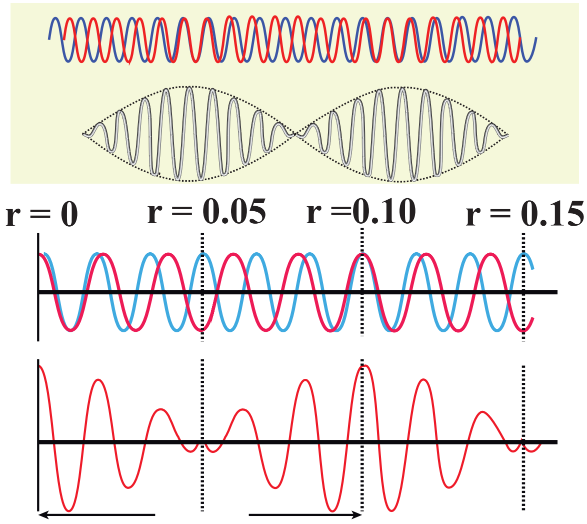


Figure 11.33: Two waves superimpose with different frequencies such that there is a time alternation in constructive and destructive interference i.e., they are periodically in and out of phase

11.7.2 Formation of beats

When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.

Number of beats per second

$$n = |f_1 - f_2| \text{ per second}$$

BUFH7E

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Additional information (Not for examination): Mathematical treatment of beats

For mathematical treatment, let us consider two sound waves having same amplitude and slightly different frequencies f_1 and f_2, superimposed on each other.

Since the sound wave (pressure wave) is a longitudinal wave, let us consider $y_1 = A\sin(\omega_1 t)$ and $y_2 = A\sin(\omega_2 t)$ to be displacements of the two waves at a point $x = 0$ with same amplitude (region having high pressures) and different angular frequencies ω_1 and ω_2, respectively. Then when they are allowed to superimpose we get the net displacement

$$y = y_1 + y_2$$

$$y = A\sin(\omega_1 t) + A\sin(\omega_2 t)$$

But

$$\omega_1 = 2\pi f_1 \text{ and } \omega_2 = 2\pi f_2$$

Then

$$y = A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t)$$

Using trigonometry formula

$$\sin C + \sin D = 2\cos\left(\frac{C-D}{2}\right)\sin\left(\frac{C+D}{2}\right)$$

$$y = 2A\cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right)\sin\left(2\pi\left(\frac{f_1 + f_2}{2}\right)t\right)$$

Let $$y_p = 2A\cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right) \qquad (11.57)$$

and if f_1 is slightly higher value than f_2 then, $\left(\frac{f_1 - f_2}{2}\right) \ll \left(\frac{f_1 + f_2}{2}\right)$ means y_p in equation (11.57) varies very slowly when compared to $\left(\frac{f_1 + f_2}{2}\right)$. Therefore

$$y = y_p \sin(2\pi f_{avg} t) \qquad (11.58)$$

This represents a simple harmonic wave of frequency which is an arithmetic average of frequencies of the individual waves, $f_{avg} = \left(\frac{f_1 + f_2}{2}\right)$ and amplitude y_p varies with time t.

Case (A):

The resultant amplitude is maximum when y_p is maximum. Since $y_p \propto \cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right)$, this means maximum amplitude occurs only when cosine takes ±1,

$$\cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right) = \pm 1$$

$$\Rightarrow \quad 2\pi\left(\frac{f_1 - f_2}{2}\right)t = n\pi,$$

$$\text{or, } (f_1 - f_2)t = n$$

$$\text{or, } t = \frac{n}{(f_1 - f_2)} \qquad n = 0,1,2,3, ...$$

Hence, the time interval between two successive maxima is

$$t_2 - t_1 = t_3 - t_2 = ... = \frac{1}{(f_1 - f_2)}; \quad n = |f_1 - f_2| = \frac{1}{|t_1 - t_2|}$$

Therefore, the number of beats produced per second is equal to the reciprocal of the time interval between two consecutive maxima i.e., $|f_1 - f_2|$.

Case (B):

The resultant amplitude is minimum i.e., it is equal to zero when y_p is minimum. Since $y_p \propto \cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right)$, this means, minimum occurs only when cosine takes 0,

$$\cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right) = 0,$$

$$\Rightarrow \quad 2\pi\left(\frac{f_1 - f_2}{2}\right)t = (2n+1)\frac{\pi}{2},$$

$$\Rightarrow \quad (f_1 - f_2)t = \frac{1}{2}(2n+1)$$

$$\text{or, } t = \frac{1}{2}\left(\frac{2n+1}{f_1 - f_2}\right), \text{ where } f_1 \neq f_2 \quad n = 0,1,2,3, ...$$

Hence, the time interval between two successive minima is

$$t_2 - t_1 = t_3 - t_2 = ... = \frac{1}{(f_1 - f_2)}; \quad n = |f_1 - f_2| = \frac{1}{|t_1 - t_2|}$$

Therefore, the number of beats produced per second is equal to the reciprocal of the time interval between two consecutive minima i.e., $|f_1 - f_2|$.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EXAMPLE 11.18

Consider two sound waves with wavelengths 5 m and 6 m. If these two waves propagate in a gas with velocity 330 ms^{-1}. Calculate the number of beats per second.

Solution

Given $\lambda_1 = 5m$ and $\lambda_2 = 6m$

Velocity of sound waves in a gas is $v = 330\ ms^{-1}$

The relation between wavelength and velocity is $v = \lambda f \Rightarrow f = \frac{v}{\lambda}$

The frequency corresponding to wavelength

λ_1 is $f_1 = \frac{v}{\lambda_1} = \frac{330}{5} = 66\,Hz$

The frequency corresponding to wavelength

λ_2 is $f_2 = \frac{v}{\lambda_2} = \frac{330}{6} = 55\,Hz$

The number of beats per second is

$|f_1 - f_2| = |66 - 55| = 11$ beats per sec

EXAMPLE 11.19

Two vibrating tuning forks produce waves whose equation is given by $y_1 = 5\sin(240\pi t)$ and $y_2 = 4\sin(244\pi t)$. Compute the number of beats per second.

Solution

Given $y_1 = 5\sin(240\pi t)$ and $y_2 = 4\sin(244\pi t)$

Comparing with $y = A\sin(2\pi f_1 t)$, we get

$$2\pi f_1 = 240\pi \Rightarrow f_1 = 120Hz$$

$$2\pi f_2 = 244\pi \Rightarrow f_2 = 122Hz$$

The number of beats produced is $|f_1 - f_2|$ $= |120 - 122| = |-2| = 2$ beats per sec

11.8 STANDING WAVES

11.8.1 Explanation of stationary waves

When the wave hits the rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern is formed, which are known as standing waves or stationary waves. Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is

$$y_1 = A\sin(kx - \omega t) \qquad (11.59)$$

(waves move toward right)

and the displacement of the second wave (reflected wave) is

$$y_2 = A\sin(kx + \omega t) \qquad (11.60)$$

(waves move toward left)

both will interfere with each other by the principle of superposition, the net displacement is

$$y = y_1 + y_2 \qquad (11.61)$$

Substituting equation (11.59) and equation (11.60) in equation (11.61), we get

$$y = A\sin(kx - \omega t) + A\sin(kx + \omega t) \qquad (11.62)$$

Using trigonometric identity, we rewrite equation (11.62) as

$$y(x, t) = 2A\cos(\omega t)\sin(kx) \qquad (11.63)$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward. Further, the displacement of the particle in equation (11.63) can be written in more compact form,

$$y(x,t) = A'\cos(\omega t)$$

where, $A' = 2A\sin(kx)$, implying that the particular element of the string executes simple harmonic motion with amplitude equals to A'. The maximum of this amplitude occurs at positions for which

$$\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = m\pi$$

where m takes half integer or half integral values. The position of maximum amplitude is known as *antinode*. Expressing wave number in terms of wavelength, we can represent the anti-nodal positions as

$$x_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2}, \text{ where, } m = 0,1,2\ldots \quad (11.64)$$

For $m = 0$ we have maximum at

$$x_0 = \frac{\lambda}{2}$$

For $m = 1$ we have maximum at

$$x_1 = \frac{3\lambda}{4}$$

For $m = 2$ we have maximum at

$$x_2 = \frac{5\lambda}{4}$$

and so on.

The distance between two successive anti-nodes can be computed by

$$x_m - x_{m-1} = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2} - \left(\frac{(2m+1)+1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{2}$$

Similarly, the minimum of the amplitude A' also occurs at some points in the space, and these points can be determined by setting

$$\sin(kx) = 0 \Rightarrow kx = 0, \pi, 2\pi, 3\pi, \ldots = n\pi$$

where n takes integer or integral values. Note that the elements at these points do not vibrate (not move), and the points are called *nodes*. The n^{th} nodal positions is given by,

$$x_n = n\frac{\lambda}{2} \text{ where, n = 0,1,2,...} \quad (11.65)$$

For $n = 0$ we have minimum at

$$x_0 = 0$$

For $n = 1$ we have minimum at

$$x_1 = \frac{\lambda}{2}$$

For $n = 2$ we have maximum at

$$x_2 = \lambda$$

and so on.

The distance between any two successive nodes can be calculated as

$$x_n - x_{n-1} = n\frac{\lambda}{2} - (n-1)\frac{\lambda}{2} = \frac{\lambda}{2}.$$

EXAMPLE 11.20

Compute the distance between anti-node and neighbouring node.

Solution

For n^{th} mode, the distance between anti-node and neighbouring node is

$$\Delta x_n = \left(\frac{2n+1}{2}\right)\frac{\lambda}{2} - n\frac{\lambda}{2} = \frac{\lambda}{4}$$

11.8.2 Characteristics of stationary waves

(1) Stationary waves are characterised by the confinement of a wave disturbance between two rigid boundaries. This means, the wave does not move forward or backward in a medium (does not advance), it remains steady at its place. Therefore, they are called "stationary waves or standing waves".

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Table 11.3: Comparison between progressive and stationary waves

S.No.	Progressive waves	Stationary waves
1.	Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.
2.	All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same.	Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at anti-nodes.
3.	These wave carry energy while propagating.	These waves do not transport energy.

(2) Certain points in the region in which the wave exists have *maximum amplitude, called as anti-nodes* and at certain points the *amplitude is minimum or zero, called as nodes*.

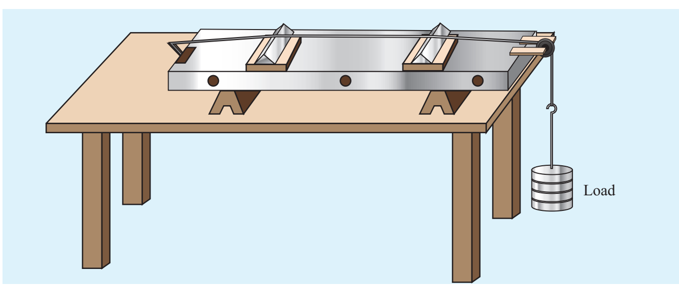
(3) The distance between two consecutive nodes (or) anti-nodes is $\frac{\lambda}{2}$.

(4) The distance between a node and its neighbouring anti-node is $\frac{\lambda}{4}$.

(5) The transfer of energy along the standing wave is zero.

11.8.3 Stationary waves in sonometer

Sono means *sound* related, and sonometer implies sound-related measurements. It is a device for demonstrating the relationship between the frequency of the sound produced in the transverse standing wave in a string, and the tension, length and mass per unit length of the string. Therefore, using this device, we can determine the following quantities:



BUPD92

Figure 11.34 Sonometer

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(a) the *frequency* of the tuning fork or frequency of alternating current

(b) the *tension* in the string

(c) the unknown hanging *mass*

Construction:

The sonometer is made up of a hollow box which is one meter long with a uniform metallic thin string attached to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley as shown in Figure 11.34. Since only one string is used, it is also known as monochord. The weights are added to the free end of the wire to increase the tension of the wire. Two adjustable wooden knives are put over the board, and their positions are adjusted to change the vibrating length of the stretched wire.

Working :

A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed.

If the length of the vibrating element is l then

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Let f be the frequency of the vibrating element, T the tension of in the string and μ the mass per unit length of the string. Then using equation (11.13), we get

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \text{ in Hertz} \qquad (11.66)$$

Let ρ be the density of the material of the string and d be the diameter of the string. Then the mass per unit length μ,

$$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}$$

$$\text{frequency } f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi d^2 \rho}{4}}}$$

$$\therefore \qquad f = \frac{1}{ld}\sqrt{\frac{T}{\pi\rho}} \qquad (11.67)$$

EXAMPLE 11.21

Let f be the fundamental frequency of the string. If the string is divided into three segments l_1, l_2 and l_3 such that the fundamental frequencies of each segments be f_1, f_2 and f_3, respectively. Show that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Solution

For a fixed tension T and mass density μ, frequency is inversely proportional to the string length i.e.

$$f \propto \frac{1}{l} \Rightarrow f = \frac{v}{2l} \Rightarrow l = \frac{v}{2f}$$

For the first length segment

$$f_1 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{2f_1}$$

For the second length segment

$$f_2 = \frac{v}{2l_2} \Rightarrow l_2 = \frac{v}{2f_2}$$

For the third length segment

$$f_3 = \frac{v}{2l_3} \Rightarrow l_3 = \frac{v}{2f_3}$$

Therefore, the total length

$$l = l_1 + l_2 + l_3$$

$$\frac{v}{2f} = \frac{v}{2f_1} + \frac{v}{2f_2} + \frac{v}{2f} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

11.8.4 Fundamental frequency and overtones

Let us now keep the rigid boundaries at x = 0 and x = L and produce a standing waves by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary must satisfy the following conditions

$$y(x = 0, t) = 0 \text{ and } y(x = L, t) = 0 \quad (11.68)$$

Since the nodes formed are at a distance $\frac{\lambda_n}{2}$ apart, we have $n\left(\frac{\lambda_n}{2}\right) = L$, where n is an integer, L is the length between the two boundaries and λ_n is the specific wavelength that satisfy the specified boundary conditions. Hence,

$$\lambda_n = \left(\frac{2L}{n}\right) \quad (11.69)$$

What will happen to wavelength if n is taken as zero? Why is this not permitted?

Therefore, not all wavelengths are allowed. The (allowed) wavelengths should fit with the specified boundary conditions, i.e., for n = 1, the first mode of vibration has specific wavelength λ_1 = 2L. Similarly for n = 2, the second mode of vibration has specific wavelength

$$\lambda_2 = \left(\frac{2L}{2}\right) = L$$

For n = 3, the third mode of vibration has specific wavelength

$$\lambda_3 = \left(\frac{2L}{3}\right)$$

and so on.

The frequency of each mode of vibration (called natural frequency) can be calculated. We have,

$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right) \quad (11.70)$$

The lowest natural frequency is called *the fundamental frequency*.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right) \quad (11.71)$$

The second natural frequency is called *the first over tone*.

$$f_2 = 2\left(\frac{v}{2L}\right) = \frac{1}{L}\sqrt{\frac{T}{\mu}}$$

The third natural frequency is called *the second over tone*.

$$f_3 = 3\left(\frac{v}{2L}\right) = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$$

and so on.

Therefore, the nth natural frequency can be computed as integral (or integer) multiple of fundamental frequency, i.e.,

$$f_n = nf_1, \quad \text{where } n \text{ is an integer} \quad (11.72)$$

If natural frequencies are written as integral multiple of fundamental frequencies, then the frequencies are called harmonics. Thus, the first harmonic is $f_1 = f_1$ (the fundamental frequency is called first harmonic), the second harmonic is $f_2 = 2f_1$, the third harmonic is $f_3 = 3f_1$ etc.

EXAMPLE 11.22

Consider a string in a guitar whose length is 80 cm and a mass of 0.32 g with tension 80 N is plucked. Compute the first four lowest frequencies produced when it is plucked.

Solution

The velocity of the wave

$$v = \sqrt{\frac{T}{\mu}}$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The length of the string, L = 80 cm=0.8 m

The mass of the string, m = 0.32 g =0.32 × 10^{-3}kg

Therefore, the linear mass density,

$$\mu = \frac{0.32\times10^{-3}}{0.8} = 0.4\times10^{-3}\,kg\,m^{-1}$$

The tension in the string, T = 80 N

$$v = \sqrt{\frac{80}{0.4\times10^{-3}}} = 447.2 \text{ m s}^{-1}$$

The wavelength corresponding to the fundamental frequency f_1 is $\lambda_1 = 2L = 2 \times 0.8 = 1.6$ m

The fundamental frequency f_1 corresponding to the wavelength λ_1

$$f_1 = \frac{v}{\lambda_1} = \frac{447.2}{1.6} = 279.5\,Hz$$

Similarly, the frequency corresponding to the second harmonics, third harmonics and fourth harmonics are

$$f_2 = 2f_1 = 559 \text{ Hz}$$

$$f_3 = 3f_1 = 838.5 \text{ Hz}$$

$$f_4 = 4f_1 = 1118 \text{ Hz}$$

11.8.5 Laws of transverse vibrations in stretched strings

There are three laws of transverse vibrations of stretched strings which are given as follows:

(i) The law of length :

For a given wire with tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length. Therefore,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

$\Rightarrow l \times f$ = C, where C is a constant

(ii) The law of tension:

For a given vibrating length l (fixed) and mass per unit length μ (fixed) the frequency varies directly with the square root of the tension T,

$$f \propto \sqrt{T}$$

$$\Rightarrow f = A\sqrt{T}, \text{ where A is a constant}$$

(iii) The law of mass:

For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ,

$$f \propto \frac{1}{\sqrt{\mu}}$$

$$\Rightarrow f = \frac{B}{\sqrt{\mu}}, \text{ where B is a constant}$$

11.9 INTENSITY AND LOUDNESS

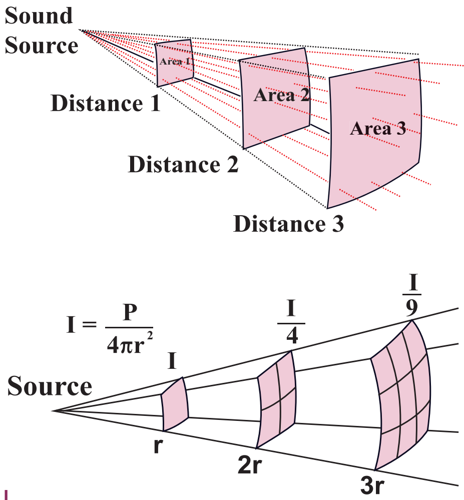
Consider a source and two observers (listeners). The source emits sound waves which carry energy. The sound energy emitted by the source is same regardless of whoever measures it, i.e., it is independent of any observer standing in that region. But the sound received by the two observers may be different; this is due to some factors like sensitivity of ears, etc. To quantify such thing, we define two different quantities known as intensity and loudness of sound.

11.9.1 Intensity of sound

When a sound wave is emitted by a source, the energy is carried to all possible surrounding points. The average sound energy emitted or

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

transmitted per unit time or per second is called sound power. Therefore, the *intensity of sound* is defined as *"the sound power transmitted per unit area taken normal to the propagation of the sound wave"*.



BUY9AP

Figure 11.35 Intensity of sound waves

For a particular source (fixed source), the sound intensity is inversely proportional to the square of the distance from the source.

$$I = \frac{power\ of\ the\ source}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

This is known as inverse square law of sound intensity.

EXAMPLE 11.23

A baby cries on seeing a dog and the cry is detected at a distance of 3.0 m such that the intensity of sound at this distance is $10^{-2}\ W\ m^{-2}$. Calculate the intensity of the baby's cry at a distance 6.0 *m*.

Solution

I_1 is the intensity of sound detected at a distance 3.0 m and it is given as $10^{-2}\ W\ m^{-2}$.

Let I_2 be the intensity of sound detected at a distance 6.0 m. Then,

$r_1 = 3.0$ m, $r_2 = 6.0$ m

and since, $I \propto \frac{1}{r^2}$

the power output does not depend on the observer and depends on the baby. Therefore,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$I_2 = I_1 \frac{r_1^2}{r_2^2}$$

$$I_2 = 0.25 \times 10^{-2}\ W\ m^{-2}$$

11.9.2 Loudness of sound

Two sounds with same intensities need not have the same loudness. For example, the sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market. Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased then loudness also increases. But additionally, not only does intensity matter, the internal and subjective experience of "how loud a sound is" i.e., the sensitivity of the listener also matters here. This is often called loudness. That is, loudness depends on both intensity of sound wave and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person) whereas the intensity of sound does not depend on the observer. The *loudness of sound* is defined as *"the degree of sensation of sound produced in the ear or the perception of sound by the listener"*.

11.9.3 Intensity and loudness of sound

Our ear can detect the sound with intensity level ranges from $10^{-2}\ Wm^{-2}$ to $20\ W\ m^{-2}$.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

According to Weber-Fechner's law, "loudness (L) is proportional to the logarithm of the actual intensity (I) measured with an accurate non-human instrument". This means that

$$L \propto \ln I$$
$$L = k \ln I$$

where k is a constant, which depends on the unit of measurement. The difference between two loudnesses, L_1 and L_0 measures the relative loudness between two precisely measured intensities and is called as sound intensity level. Mathematically, sound intensity level is

$$\Delta L = L_1 - L_0 = k \ln I_1 - k \ln I_0 = k \ln\left[\frac{I_1}{I_0}\right]$$

If $k = 1$, then sound intensity level is measured in bel, in honour of Alexander Graham Bell. Therefore,

$$\Delta L = \ln\left[\frac{I_1}{I_0}\right] \text{ bel}$$

However, this is practically a bigger unit, so we use a convenient smaller unit, called decibel. Thus, decibel $= \frac{1}{10}$ bel. Therefore, by multiplying and dividing by 10, we get

$$\Delta L = 10\left(\ln\left[\frac{I_1}{I_0}\right]\right)\frac{1}{10} \text{ bel}$$

$$\Delta L = 10 \ln\left[\frac{I_1}{I_0}\right] \text{ decibel with } k = 10$$

For practical purposes, we use logarithm to base 10 instead of natural logarithm,

$$\Delta L = 10 \log_{10}\left[\frac{I_1}{I_0}\right] \text{ decibel} \qquad (11.73)$$

EXAMPLE 11.24

The sound level from a musical instrument playing is 50 dB. If three identical musical instruments are played together then compute the total intensity. The intensity of the sound from each instrument is 10^{-12} W m^{-2}

Solution

$$\Delta L = 10 \log_{10}\left[\frac{I_1}{I_0}\right] = 50\ dB$$

$$\log_{10}\left[\frac{I_1}{I_0}\right] = 5\ dB$$

$\frac{I_1}{I_0} = 10^5 \Rightarrow I_1 = 10^5 I_0 = 10^5 \times 10^{-12}\ Wm^{-2}$

$I_1 = 10^{-7}\ Wm^{-2}$

Since three musical instruments are played, therefore, $I_{total} = 3I_1 = 3 \times 10^{-7}\ Wm^{-2}$.

11.10 VIBRATIONS OF AIR COLUMN

Musical instruments like flute, clarinet, nathaswaram, etc are known as wind instruments. They work on the principle of vibrations of air columns. The simplest form of a wind instrument is the organ pipe. It is made up of a wooden or metal pipe which produces the musical sound. For example, flute, clarinet and nathaswaram are organ pipe instruments. Organ pipe instruments are classified into two types:

(a) Closed organ pipes:

Figure 11.36: Clarinet is an example of a closed organ pipe

Look at the picture of a clarinet, shown in Figure 11.36. It is a pipe with one end closed and the other end open. If one end of a pipe is closed, the wave reflected at this closed end is 180° out of phase with the incoming wave. Thus there is no displacement of the

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

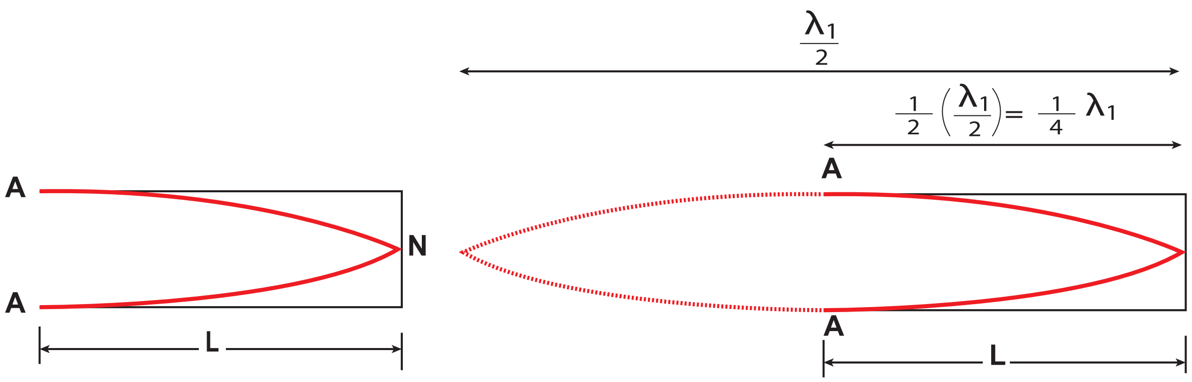


Figure 11.37 No motion of particles which leads to nodes at closed end and antinodes at open end (fundamental mode) (N-node, A-antinode)

particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.

Let us consider the simplest mode of vibration of the air column called the fundamental mode. Anti-node is formed at the open end and node at closed end. From the Figure 11.37, let L be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,

$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L \qquad (11.74)$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \qquad (11.75)$$

which is called the fundamental note.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.

The Figure 11.38 shows the second mode of vibration having two nodes and two anti-nodes, for which we have, from example 11.20.

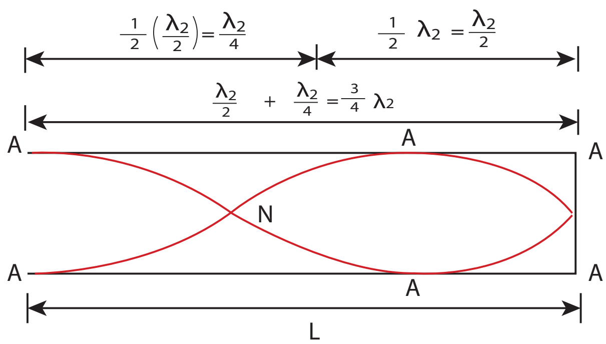


Figure 11.38 second mode of vibration having two nodes and two anti-nodes

$$4L = 3\lambda_2$$

$$L = \frac{3\lambda_2}{4} \text{ or } \lambda_2 = \frac{4L}{3}$$

The frequency for this,

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$$

is called *first over tone*, since here, the frequency is three times the fundamental frequency it is called *third harmonic.*

The Figure 11.39 shows third mode of vibration having three nodes and three anti-nodes.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

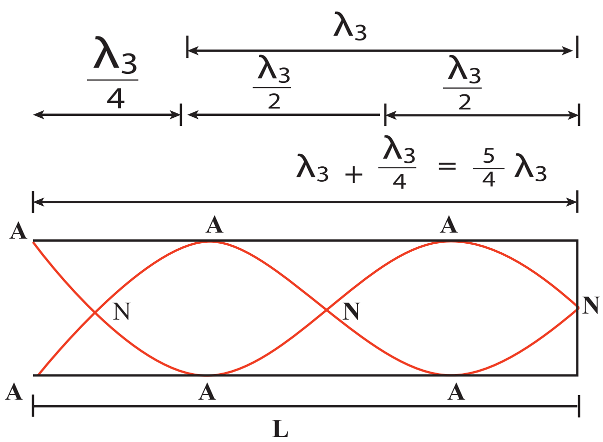


Figure 11.39 Third mode of vibration having three nodes and three anti-nodes

We have, $4L = 5\lambda_3$

$$L = \frac{5\lambda_3}{4} \text{ or } \lambda_3 = \frac{4L}{5}$$

The frequency

$$f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$$

is called *second over tone*, and since $n = 5$ here, this is called *fifth harmonic*. Hence, *the closed organ pipe has only odd harmonics and frequency of the* n^{th} *harmonic is* $f_n = (2n+1)f_1$. Therefore, *the frequencies of harmonics are in the ratio*

$$f_1 : f_2 : f_3 : f_4 : ... = 1 : 3 : 5 : 7 : ... \quad (11.76)$$

(b) Open organ pipes:

Figure 11.40 Flute is an example of open organ pipe

Consider the picture of a flute, shown in Figure 11.40. It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end, a node is formed at the mid-point of the pipe.

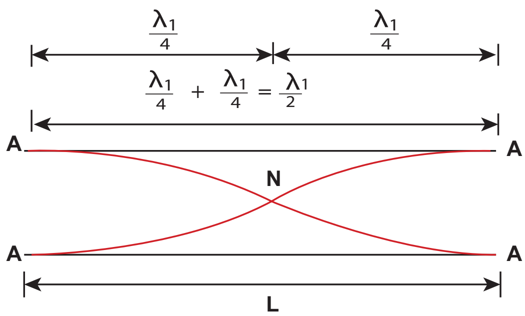


Figure 11.41 Antinodes are formed at the open end and a node is formed at the middle of the pipe.

From Figure 11.41, if L be the length of the tube, the wavelength of the wave produced is given by

$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L \quad (11.77)$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \quad (11.78)$$

which is called the *fundamental note*.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at one of the open ends. Such frequencies are called overtones.

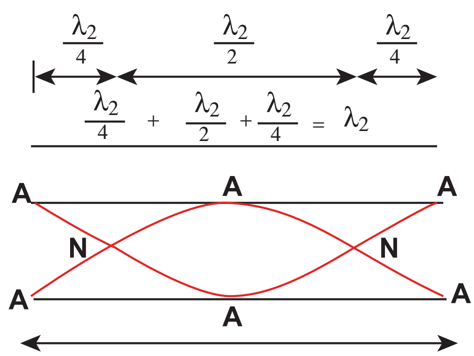


Figure 11.42 Second mode of vibration in open pipes having two nodes and three anti-nodes

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The Figure 11.42 shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore,

$$L = \lambda_2 \text{ or } \lambda_2 = L$$

The frequency

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \times \frac{v}{2L} = 2 f_1$$

is called ***first over tone***. Since $n = 2$ here, it is called the ***second harmonic***.

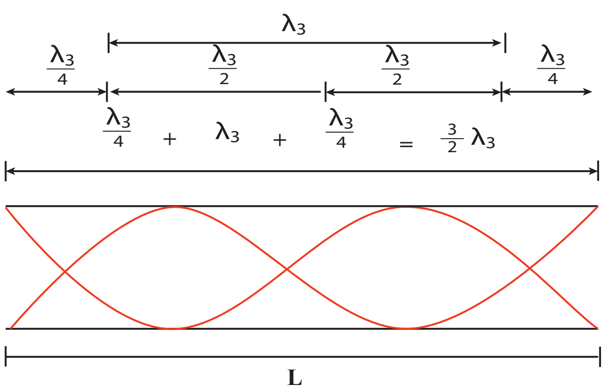


Figure 11.43 Third mode of vibration having three nodes and four anti-nodes

The Figure 11.43 above shows the third mode of vibration having three nodes and four anti-nodes

$$L = \frac{3}{2}\lambda_3 \ or \ \lambda_3 = \frac{2L}{3}$$

The frequency

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3 f_1$$

is called *second over tone*. Since $n = 3$ here, it is called the *third harmonic*.

Hence, the open organ pipe has all the harmonics and frequency of n^{th} harmonic is $f_n = nf_1$. Therefore, the frequencies of harmonics are in the ratio

$$f_1 : f_2 : f_3 : f_4 : ... = 1 : 2 : 3 : 4 : ... \quad (11.79)$$

EXAMPLE 11.25

If a flute sounds a note with 450*Hz*, what are the frequencies of the second, third, and fourth harmonics of this pitch?. If the clarinet sounds with a same note as 450*Hz*, then what are the frequencies of the lowest three harmonics produced ?.

Solution

For a flute which is an open pipe, we have

Second harmonics $f_2 = 2 f_1 = 900$ Hz

Third harmonics $f_3 = 3 f_1 = 1350$ Hz

Fourth harmonics $f_4 = 4 f_1 = 1800$ Hz

For a clarinet which is a closed pipe, we have

Second harmonics $f_2 = 3 f_1 = 1350$ Hz

Third harmonics $f_3 = 5 f_1 = 2250$ Hz

Fourth harmonics $f_4 = 7 f_1 = 3150$ Hz

EXAMPLE 11.26

If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.

Solution

Let l_2 be the length of the open organ pipe, with $l_1 = 30\,cm$ the length of the closed organ pipe.

It is given that the third harmonic of closed organ pipe is equal to the fundamental frequency of open organ pipe.

The third harmonic of a closed organ pipe is

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l_1} = 3 f_1$$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

The fundamental frequency of open organ pipe is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l_2}$

Therefore,

$$\frac{v}{2l_2} = \frac{3v}{4l_1} \Rightarrow l_2 = \frac{2l_1}{3} = 20\,cm$$

11.10.1 Resonance air column apparatus

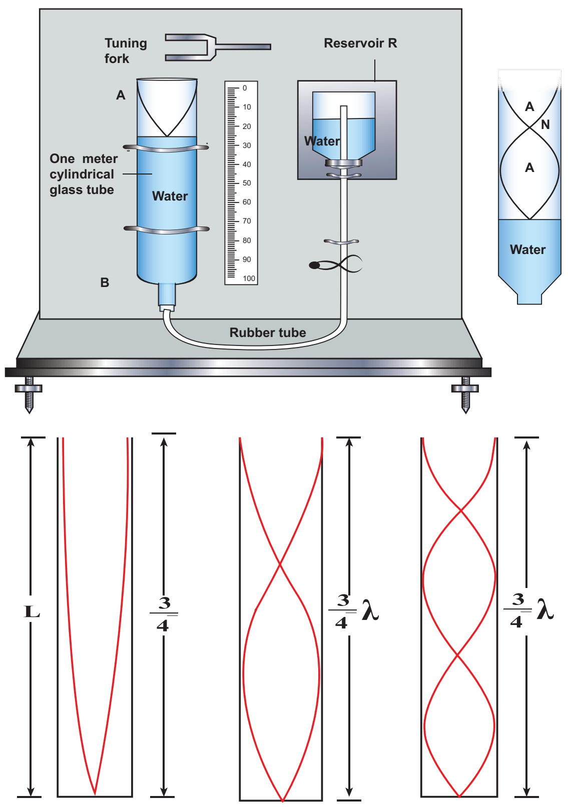


Figure 11.44: The resonance air column apparatus and first, second and third resonance

The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature. It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in Figure 11.44. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it. The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed end and other as the open end. Therefore, it behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the closed end. When a vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column. These waves move downward as shown in Figure 11.44, and reach the surfaces of water and get reflected and produce standing waves. The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork). At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced.

Let the first resonance occur at length L_1, then

$$\frac{1}{4}\lambda = L_1 \qquad (11.80)$$

But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction *e*, by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is

$$\frac{1}{4}\lambda = L_1 + e \qquad (11.81)$$

Now the length of the air column is increased to get the second resonance. Let L_2 be the length at which the second resonance occurs. Again taking end correction into account, we have

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$\frac{3}{4}\lambda = L_2 + e \quad (11.82)$$

In order to avoid end correction, let us take the difference of equation (11.80) and equation (11.79), we get

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e) - (L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L$$

$$\Rightarrow \lambda = 2\Delta L$$

The speed of the sound in air at room temperature can be computed by using the formula

$$v = f\lambda = 2f\Delta L$$

Further, to compute the end correction, we use equation (11.81) and equation (11.82), we get

$$e = \frac{L_2 - 3L_1}{2}$$

EXAMPLE 11.27

A frequency generator with fixed frequency of 343 Hz is allowed to vibrate above a 1.0 m high tube. A pump is switched on to fill the water slowly in the tube. In order to get resonance, what must be the minimum height of the water?. (speed of sound in air is 343 m s^{-1})

Solution

The wavelength, $\lambda = \frac{c}{f}$

$$\lambda = \frac{343\,ms^{-1}}{343\,Hz} = 1.0 \text{ m}$$

Let the length of the resonant columns be L_1, L_2 and L_3. The first resonance occurs at length L_1

$$L_1 = \frac{\lambda}{4} = \frac{1}{4} = 0.25 \text{ m}$$

The second resonance occurs at length L_2

$$L_2 = \frac{3\lambda}{4} = \frac{3}{4} = 0.75 \text{ m}$$

The third resonance occurs at length

$$L_3 = \frac{5\lambda}{4} = \frac{5}{4} = 1.25 \text{ m}$$

and so on.

Since total length of the tube is 1.0 m the third and other higher resonances do not occur. Therefore, the minimum height of water H_{min} for resonance is,

$$H_{min} = 1.0 \text{ m} - 0.75 \text{ m} = 0.25 \text{ m}$$

EXAMPLE 11.28

A student performed an experiment to determine the speed of sound in air using the resonance column method. The length of the air column that resonates in the fundamental mode with a tuning fork is 0.2 m. If the length is varied such that the same tuning fork resonates with the first overtone at 0.7 m. Calculate the end correction.

Solution

End correction

$$e = \frac{L_2 - 3L_1}{2} = \frac{0.7 - 3(0.2)}{2} = 0.05m$$

EXAMPLE 11.29

Consider a tuning fork which is used to produce resonance in an air column. A resonance air column is a glass tube whose length can be adjusted by a variable piston. At room temperature, the two successive resonances observed are at 20 cm and 85 cm of the column length. If the frequency of the length is 256 Hz, compute the velocity of the sound in air at room temperature.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Solution

Given two successive length (resonance) to be $L_1 = 20$ cm and $L_2 = 85$ cm

The frequency is $f = 256$ Hz

$$v = f\lambda = 2f\,\Delta L = 2f(L_2 - L_1)$$

$$= 2 \times 256 \times (85 - 20) \times 10^{-2}\ \text{m s}^{-1}$$

$$v = 332.8\ \text{cm}^{-1}$$

11.11 DOPPLER EFFECT

Often we have noticed that the siren sound coming from a police vehicle or ambulance increases when it comes closer to us and decreases when it moves away from us. When we stand near any passing train the train whistle initially increases and then it will decrease. This is known as Doppler Effect, named after Christian Doppler (1803 – 1853). Suppose a source produces sound with some frequency, we call it the as source frequency f_s. If the source and an observer are at a fixed distance then the observer observes the sound with frequency f_0. This is the same as the sound frequency produced by the source f_s, i.e., $f_0 = f_s$. Hence, there is no difference in frequency, implying no Doppler effect is observed.

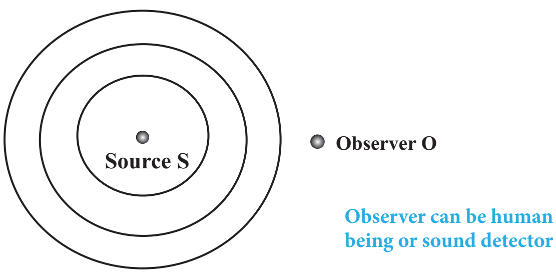


Figure 11.45 Both source and observer are stationary. No Doppler effect is observed.

What happens if either source or an observer or both move?. Certainly, $f_o \neq f_s$. That is, when the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect. The frequency perceived by the observer is known as apparent frequency. We can consider the following situations for the study of Doppler effect in sound waves

(a) **Source and Observer:** We can consider either the source or observer in motion or both are in motion. Further we can treat the motion to be along the line joining the source and the observer, or inclined at an angle θ to this line.

(b) **Medium:** We can treat the medium to be stationary or the direction of motion of the medium is along or opposite to the direction of propagation of sound.

(c) **Speed of Sound:** We can also consider the case where speed of the source or an observer is greater or lesser than the speed of sound.

In the following section, we make the following assumptions: the medium is stationary, and motion is along the line joining the source and the observer, and the speeds of the source and the observer are both less than the speed of sound in that medium.

We consider three cases:

(i) Source in motion and Observer is at rest.
   (a) Source moves towards observer
   (b) Source moves away from the observer

(ii) Observer in motion and Source is at rest.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(a) Observer moves towards Source
(b) Observer receding away from the Source

(iii) Both are in motion
(a) Source and Observer approach each other
(b) Source and Observer recede from each other
(c) Source chases Observer
(d) Observer chases Source

***Stationary observer* and *stationary source* means the observer and source are both at rest with respect to medium respectively**

11.11.1 Source in motion and the observer at rest

(a) Source moves towards the observer

Suppose a source S moves to the right (as shown in Figure 11.46) with a velocity v_s and let the frequency of the sound waves produced by the source be f_s. We assume the velocity of sound in a medium is v. The compression (sound wave front) produced by the source S at three successive instants of time are shown in the Figure 11.46. When S is at position x_1 the compression is at C_1. When S is at position x_2, the compression is at C_2 and similarly for x_3 and C_3. Assume that if C_1 reaches the observer's position A then at that instant C_2 reaches the point B and C_3 reaches the point C as shown in the Figure 11.46. It is obvious to see that the distance between compressions C_2 and C_3 is shorter than distance between C_1 and C_2. This means the wavelength decreases when the source S moves towards the observer O (since sound travels longitudinally and wavelength is the distance between two consecutive compressions). But frequency is inversely related to wavelength and therefore, frequency increases.

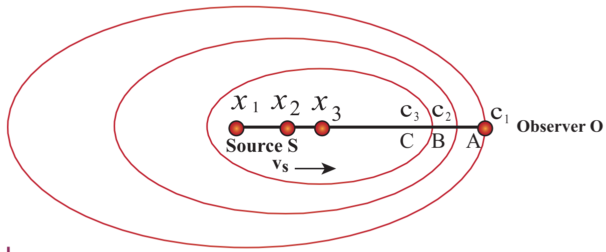


Figure 11.46 Source S moves towards an Observer O (right) with velocity

Let λ be the wavelength of the source S as measured by the observer when S is at position x_1 and λ' be wavelength of the source observed by the observer when S moves to position x_2. Then the change in wavelength is $\Delta\lambda = \lambda - \lambda' = v_s t$, where t is the time taken by the source to travel between x_1 and x_2. Therefore,

$$\lambda' = \lambda - v_s t \qquad (11.83)$$

But $$t = \frac{\lambda}{v} \qquad (11.84)$$

On substituting equation (11.84) in equation (11.83), we get

$$\lambda' = \lambda\left(1 - \frac{v_s}{v}\right)$$

Since frequency is inversely proportional to wavelength, we have

$$f' = \frac{v_s}{\lambda'} \text{ and } f = \frac{v_s}{\lambda}$$

Hence, $$f' = \frac{f}{\left(1 - \frac{v_s}{v}\right)} \qquad (11.85)$$

Since, $\frac{v_s}{v} \ll 1$, we use the binomial expansion and retaining only first order in $\frac{v_s}{v}$, we get

$$f' = f\left(1 + \frac{v_s}{v}\right)\text{v} \qquad (11.86)$$

(b) Source moves away from the observer: Since the velocity here of the source is opposite in direction when compared to case (a), therefore, changing the sign of the velocity of the source in the above case i.e, by substituting ($v_s \rightarrow -v_s$) in equation (11.83), we get

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$f' = \frac{f}{\left(1+\frac{v_s}{v}\right)} \quad (11.87)$$

Using binomial expansion again, we get,

$$f' = f\left(1-\frac{v_s}{v}\right) \quad (11.88)$$

11.11.2 Observer in motion and source at rest

(a) Observer moves towards Source

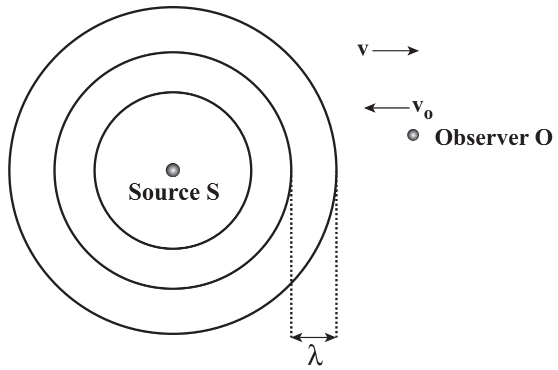


Figure 11.47 Observer moves towards Source

Let us assume that the observer O moves towards the source S with velocity v_o. The source S is at rest and the velocity of sound waves (with respect to the medium) produced by the source is v. From the Figure 11.47, we observe that both v_o and v are in opposite direction. Then, their relative velocity is $v_r = v + v_0$. The wavelength of the sound wave is $\lambda = \frac{v}{f}$, which means the frequency observed by the observer O is $f' = \frac{v_r}{\lambda}$. Then

$$f' = \frac{v_r}{\lambda} = \left(\frac{v+v_0}{v}\right)f = f\left(1+\frac{v_0}{v}\right) \quad (11.89)$$

(b) Observer recedes away from the Source

If the observer O is moving away (receding away) from the source S, then velocity v_0 and v moves in the same direction. Therefore, their relative velocity is $v_r = v - v_0$. Hence, the frequency observed by the observer O is

$$f' = \frac{v_r}{\lambda} = \left(\frac{v-v_0}{v}\right)f = f\left(1-\frac{v_0}{v}\right) \quad (11.90)$$

11.11.3 Both are in motion

(a) Source and observer approach each other

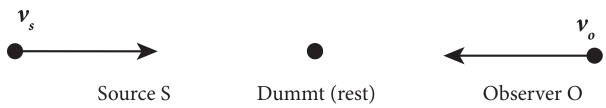


Figure 11.48 Source and Observer approach towards each other.

Let v_s and v_0 be the respective velocities of source and observer approaching each other as shown in Figure 11.48. In order to calculate the apparent frequency observed by the observer, as a simple calculation, let us have a dummy (behaving as observer or source) in between the source and observer. Since the dummy is at rest, the dummy (observer) observes the apparent frequency due to approaching source as given in equation (11.85) as

$$f_d = \frac{f}{\left(1-\frac{v_s}{v}\right)} \quad (11.91)$$

At that instant of time, the true observer approaches the dummy from the other side. Since the source (true source) comes in a direction opposite to true observer, the dummy (source) is treated as stationary source for the true observer at that instant. Hence, apparent frequency when the true observer approaches the stationary source (dummy source), from equation (11.89) is

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

$$f' = f_d\left(1+\frac{v_0}{v}\right) \Rightarrow f_d = \frac{f'}{\left(1+\frac{v_0}{v}\right)} \quad (11.92)$$

Since this is true for any arbitrary time, therefore, comparing equation (11.91) and equation (11.92), we get

$$\frac{f}{\left(1-\frac{v_s}{v}\right)} = \frac{f'}{\left(1+\frac{v_0}{v}\right)}$$

$$\Rightarrow \frac{v f'}{(v+v_0)} = \frac{v f}{(v-v_s)}$$

Hence, the apparent frequency as seen by the observer is

$$f' = \left(\frac{v+v_0}{v-v_s}\right) f \quad (11.93)$$

(b) Source and observer recede from each other

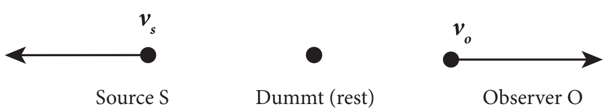


Figure 11.49 Source and Observer resides from each other

Here, we can derive the result as in the previous case. Instead of a detailed calculation, by inspection from Figure 11.49, we notice that the velocity of the source and the observer each point in opposite directions with respect to the case in (a) and hence, we substitute $(v_s \to -v_s)$ and $(v_0 \to -v_0)$ in equation (11.93), and therefore, the apparent frequency observed by the observer when the source and observer recede from each other is

$$f' = \left(\frac{v-v_0}{v+v_s}\right) f \quad (11.94)$$

(c) Source chases the observer

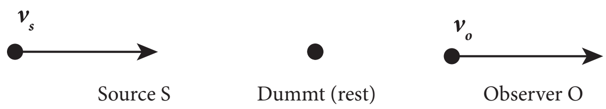


Figure 11.50 Source chases observer

Only the observer's velocity is oppositely directed when compared to case (a). Therefore, substituting $(v_0 \to -v_0)$ in equation (11.93), we get

$$f' = \left(\frac{v-v_0}{v-v_s}\right) f \quad (11.95)$$

(d) Observer chases the source

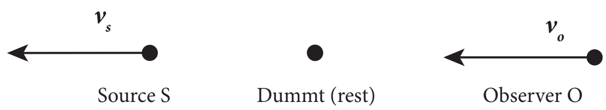


Figure 11.51 Observer chases Source

Only the source velocity is oppositely directed when compared to case (a). Therefore, substituting $v_s \to -v_s$ in equation (11.93), we get

$$f' = \left(\frac{v+v_0}{v+v_s}\right) f \quad (11.96)$$

Discuss with your teacher

"Doppler effect in light"

"Doppler effect in sound is asymmetrical where as Doppler effect in light is symmetrical"

11.11.4 Applications of Doppler effect

Doppler effect has many applications. Specifically Doppler effect in light has many applications in astronomy. As an example, while observing the spectra from distant

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

objects like stars or galaxies, it is possible to determine the velocities at which distant objects like stars or galaxies move towards or away from Earth. If the spectral lines of the star are found to shift towards red end of the spectrum (called as red shift) then the star is receding away from the Earth. Similarly, if the spectral lines of the star are found to shift towards the blue end of the spectrum (called as blue shift) then the star is approaching Earth.

Let $\Delta\lambda$ be the Doppler shift. Then $\Delta\lambda = \frac{v}{c}\lambda$, where v is the velocity of the star. It may be noted that Doppler shift measures only the radial component (along the line of sight) of the relative velocity v.

EXAMPLE 11.30

A sound of frequency 1500 Hz is emitted by a source which moves away from an observer and moves towards a cliff at a speed of 6 ms^{-1}.

(a) Calculate the frequency of the sound which is coming directly from the source.

(b) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 $m\,s^{-1}$

Solution

(a) Source is moving away and observer is stationary, therefore, the frequency of sound heard directly from source is

$$f' = \frac{f}{\left(1+\frac{v_s}{v}\right)} = \frac{1500}{\left(1+\frac{6}{330}\right)} = 1473\,Hz$$

(b) Sound is reflected from the cliff and reaches observer, therefore,

$$f' = \frac{f}{\left(1-\frac{v_s}{v}\right)} = \frac{1500}{\left(1-\frac{6}{330}\right)} = 1528\,Hz$$

EXAMPLE 11.31

An observer observes two moving trains, one reaching the station and other leaving the station with equal speeds of 8 $m\,s^{-1}$. If each train sounds its whistles with frequency 240 Hz, then calculate the number of beats heard by the observer.

Solution:

Observer is stationary

(i) Source (train) is moving towards an observer:

Apparent frequency due to train arriving station is

$$f_{in} = \frac{f}{\left(1-\frac{v_s}{v}\right)} = \frac{240}{\left(1-\frac{8}{330}\right)} = 246\,Hz$$

(ii) Source (train) is moving away form an observer:

Apparent frequency due to train leaving station is

$$f_{out} = \frac{f}{\left(1-\frac{v_s}{v}\right)} = \frac{240}{\left(1-\frac{8}{330}\right)} = 234\,Hz$$

So the number of beats = $|f_{in} - f_{out}|$ = (246-234) = 12

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SUMMARY

- A disturbance which carries energy and momentum from one point in space to another point in space without the transfer of medium is known as a wave.
- The waves which require medium for their propagation are known as mechanical waves.
- The waves which do not require medium for their propagation are known as non-mechanical waves.
- For a transverse wave, the vibration of particles in a medium is perpendicular to the direction of propagation of the wave.
- For a longitudinal wave, the vibration of particles in a medium is parallel to the direction of propagation of the wave.
- Elasticity and inertia are necessary properties of the medium for wave propagation.
- Waves formed in still water (ripples) are transverse and wave formed due to vibration of tuning fork is longitudinal.
- The distance between two consecutive crests or troughs is known as wavelength, λ.
- The number of waves which crossed a point per second is known as frequency, f.
- The time taken by one wave to cross a point is known as time period, T.
- velocity of the wave is $v = \lambda f$.
- Frequency is source dependent and wave velocity is medium dependent.
- The velocity of a transverse wave produce in a stretched string depends on tension in the string and mass per unit length. It does not depend on shape of the wave form.
- Velocity of transverse wave on a string is $v=\sqrt{\frac{T}{\mu}}\, ms^{-1}$.
- Velocity of longitudinal wave in an elastic medium is $v=\sqrt{\frac{E}{\rho}}\, ms^{-1}$.
- The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meters.
- The wave equation is $\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ in one space dimension.
- Wave number is given by $k=\frac{2\pi}{\lambda}\, rad\, m^{-1}$.
- During interference the resultant intensity is $I=I_1+I_2+2\sqrt{I_1 I_2}\cos\varphi$, where the intensity is square of the amplitude $I=A^2$.

  For constructive interference, $I_{maximum}=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2=\left(A_1+A_2\right)^2$.

  For destructive interference, $I_{minimum}=\left(\sqrt{I_1}-\sqrt{I_2}\right)^2=(A_1-A_2)^2$.
- When we superimpose two or more waves with slightly different frequencies then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SUMMARY (cont.)

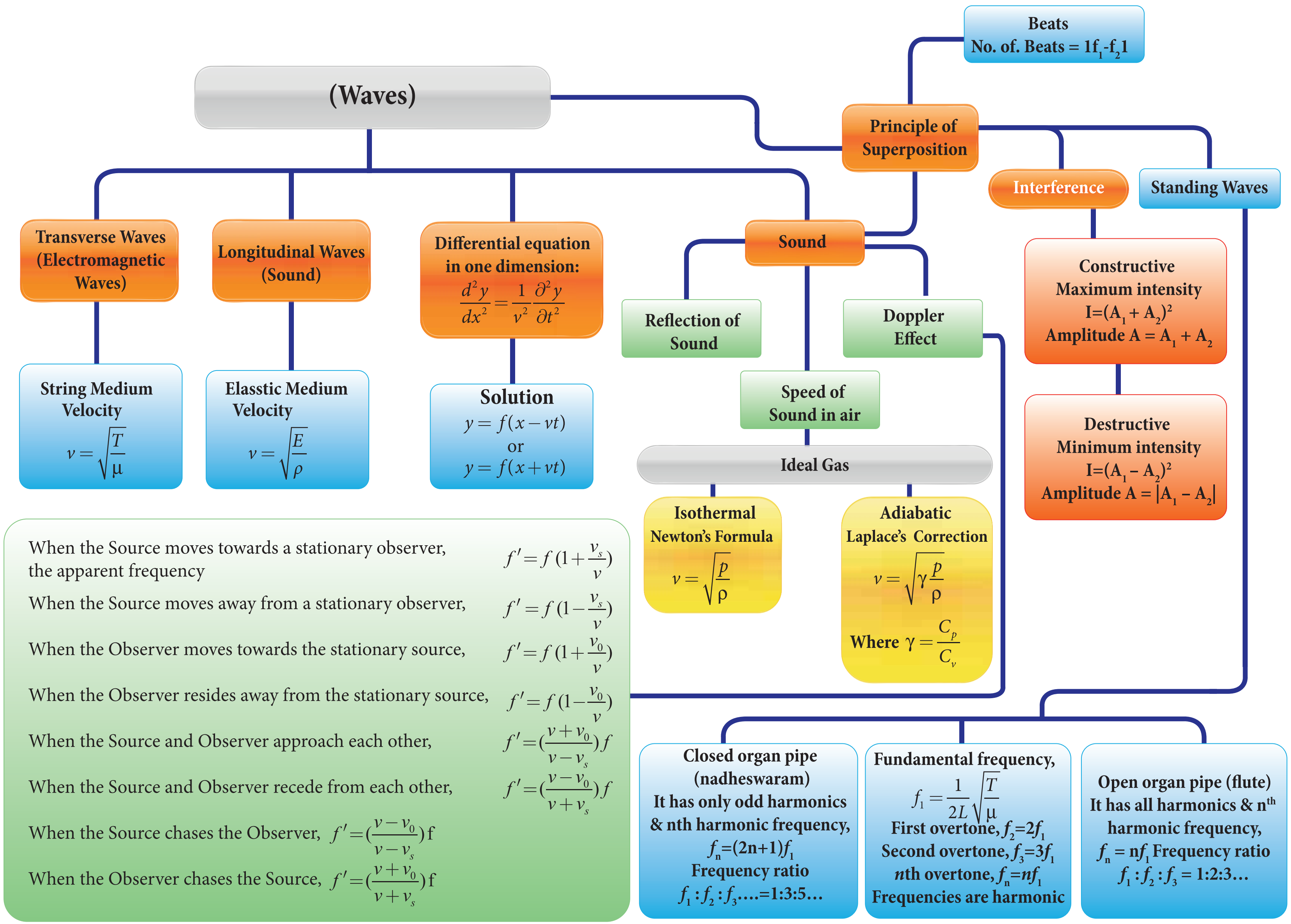
- If natural frequencies are written as integral multiples of fundamental frequency, then the frequencies are said to be in harmonics. Thus, the first harmonic is $v_1 = v_1$, (the fundamental frequency is called first harmonics), the second harmonics is $v_2 = 2\, v_1$, the third harmonics is $v_3 = 3\, v_1$, and so on.
- *Loudness of sound* is defined as *"the degree of sensation of sound produced in the ear or the perception of sound by the listener"*.
- *The intensity of sound* is defined as *"the sound power transmitted per unit area placed normal to the propagation of sound wave"*.
- Sound intensity level, $\Delta L = 10\log_{10}\frac{I_1}{I_0}\, decibel$.
- A closed organ pipe has only odd harmonics and the corresponding frequency of the n^{th} harmonic is $f_n = (2n+1) f_1$.
- In a closed organ pipe the frequencies of harmonics are in the ratio

  $f_1 : f_2 : f_3 : f_4 :... = 1 : 3 : 5 : 7 :...$
- The open organ pipe has all harmonics and frequency of the n^{th} harmonic is $f_n = n f_1$.
- In the open organ pipe the frequencies of harmonics are in the ratio

  $f_1 : f_2 : f_3 : f_4 :... = 1 : 2 : 3 : 4 :...$
- When the source and the observer are in relative motion with respect to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect. The frequency received by the observer is known as apparent frequency.
- When the Source moves towards a stationary observer, the apparent frequency $f' = f(1 + \frac{v_s}{v})$.
- When the Source moves away from a stationary observer, $f' = f(1 - \frac{v_s}{v})$.
- When the Observer moves towards the stationary source, $f' = f(1 + \frac{v_0}{v})$.
- When the Observer recedes away from the stationary source, $f' = f(1 - \frac{v_0}{v})$.
- When the Source and Observer approach each other, $f' = (\frac{v + v_0}{v - v_s}) f$.
- When the Source and Observer recede from each other, $f' = (\frac{v - v_0}{v + v_s}) f$.
- When the Source chases the Observer, $f' = (\frac{v - v_0}{v - v_s})\, f$.
- When the Observer chases the Source, $f' = (\frac{v + v_0}{v + v_s})\, f$.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CONCEPT MAP



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

EVALUATION

I. Multiple Choice Questions:

1. A student tunes his guitar by striking a 120 Hertz with a tuning fork, and simultaneously plays the 4th string on his guitar. By keen observation, he hears the amplitude of the combined sound oscillating thrice per second. Which of the following frequencies is the most likely the frequency of the 4th string on his guitar?.

   a) 130 b) 117
   c) 110 d) 120

2. A transverse wave moves from a medium A to a medium B. In medium A, the velocity of the transverse wave is 500 ms^{-1} and the wavelength is 5 m. The frequency and the wavelength of the wave in medium B when its velocity is 600 ms^{-1}, respectively are

   a) 120 Hz and 5 m
   b) 100 Hz and 5 m
   c) 120 Hz and 6 m
   d) 100 Hz and 6 m

3. For a particular tube, among six harmonic frequencies below 1000 Hz, only four harmonic frequencies are given : 300 Hz, 600 Hz, 750 Hz and 900 Hz. What are the two other frequencies missing from this list?.

   a) 100 Hz, 150 Hz
   b) 150 Hz, 450 Hz
   c) 450 Hz, 700 Hz
   d) 700 Hz, 800 Hz

   BV85CC

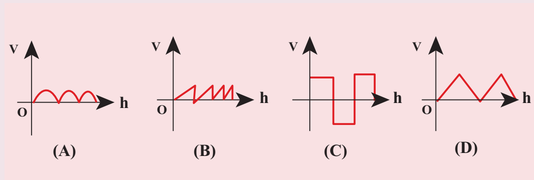
4. Which of the following options is correct?.

A	B
(1) Quality	(A) Intensity
(2) Pitch	(B) Waveform
(3) Loudness	(C) Frequency

Options for (1), (2) and (3), respectively are

   a) (B),(C) and (A)
   b) (C), (A) and (B)
   c) (A), (B) and (C)
   d) (B), (A) and (C)

5. Compare the velocities of the wave forms given below, and choose the correct option.



where, v_A, v_B, v_C and v_D are velocities given in (A), (B), (C) and (D), respectively.

   a) $v_A > v_B > v_D > v_C$
   b) $v_A < v_B < v_D < v_C$
   c) $v_A = v_B = v_D = v_C$
   d) $v_A > v_B = v_D > v_C$

6. A sound wave whose frequency is 5000 Hz travels in air and then hits the water surface. The ratio of its wavelengths in water and air is

   a) 4.30 b) 0.23
   c) 5.30 d) 1.23

7. A person standing between two parallel hills fires a gun and hears the first echo

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

after t_1 sec and the second echo after t_2 sec. The distance between the two hills is

a) $\frac{v(t_1 - t_2)}{2}$ b) $\frac{v(t_1 t_2)}{2(t_1 + t_2)}$

c) $v(t_1 + t_2)$ d) $\frac{v(t_1 + t_2)}{2}$

8. An air column in a pipe which is closed at one end, will be in resonance with the vibrating body of frequency 83Hz. Then the length of the air column is

a) 1.5 m (b) 0.5 m

(c) 1.0 m (d) 2.0 m

9. The displacement y of a wave travelling in the x direction is given by $y = (2 \times 10^{-3}) \sin(300t - 2x + \frac{\pi}{4})$, where x and y are measured in metres and t in second. The speed of the wave is

(a) 150 ms^{-1} (b) 300 ms^{-1}

(c) 450 ms^{-1} (d) 600 ms^{-1}

10. Consider two uniform wires vibrating simultaneously in their fundamental notes. The tensions, densities, lengths and diameter of the two wires are in the ratio 8 : 1, 1 : 2, $x : y$ and 4 : 1 respectively. If the note of the higher pitch has a frequency of 360 Hz and the number of beats produced per second is 10, then the value of $x : y$ is

(a) 36 : 35

(b) 35 : 36

(c) 1 : 1

(d) 1 : 2

11. Which of the following represents a wave

(a) $(x - vt)^3$ (b) $x(x+vt)$

(c) $\frac{1}{(x + vt)}$ (d) $\sin(x+vt)$

12. A man sitting on a swing which is moving to an angle of 60° from the vertical is blowing a whistle which has a frequency of 2.0 k Hz. The whistle is 2.0 m from the fixed support point of the swing. A sound detector which detects the whistle sound is kept in front of the swing. The maximum frequency the sound detector detected is

(a) 2.027 kHz (b) 1.974 kHz

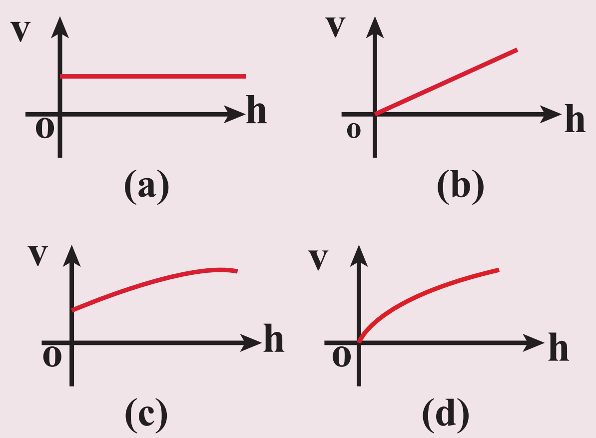
(c) 9.74 kHz (d) 1.011 kHz

13. Let $y = \frac{1}{1+x^2}$ at t = 0 s be the amplitude of the wave propagating in the positive x-direction. At $t = 2\,s$, the amplitude of the wave propagating becomes $y = \frac{1}{1+(x-2)^2}$. Assume that the shape of the wave does not change during propagation. The velocity of the wave is

(a) 0.5m s^{-1} (b) 1.0m s^{-1}

(c) 1.5m s^{-1} (d) 2.0m s^{-1}

14. A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. Which of the following plots shows the correct variation of speed v with height h from the lower end?



athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

15. An organ pipe A closed at one end is allowed to vibrate in its first harmonic and another pipe B open at both ends is allowed to vibrate in its third harmonic. Both A and B are in resonance with a given tuning fork. The ratio of the length of A and B is

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

Answers:

1) b	2) d	3) b	4) a
5) c	6) a	7) d	8) c
9) a	10) a	11) d	12) a
13) b	14) d	15) c	

II. Short Answer Questions

1. What is meant by waves?.
2. Write down the types of waves.
3. What are transverse waves?. Give one example.
4. What are longitudinal waves?. Give one example.
5. Define wavelength.
6. Write down the relation between frequency, wavelength and velocity of a wave.
7. What is meant by interference of waves?.
8. Explain the beat phenomenon.
9. Define intensity of sound and loudness of sound.
10. Explain Doppler Effect.
11. Explain red shift and blue shift in Doppler Effect.
12. What is meant by end correction in resonance air column apparatus?
13. Sketch the function $y = x + a$. Explain your sketch.
14. Write down the factors affecting velocity of sound in gases.
15. What is meant by an echo?. Explain.

III. Long Answer Questions

1. Discuss how ripples are formed in still water.
2. Briefly explain the difference between travelling waves and standing waves.
3. Show that the velocity of a travelling wave produced in a string is $v = \sqrt{\frac{T}{\mu}}$
4. Describe Newton's formula for velocity of sound waves in air and also discuss the Laplace's correction.
5. Write short notes on reflection of sound waves from plane and curved surfaces.
6. Briefly explain the concept of superposition principle.
7. Explain how the interference of waves is formed.
8. Describe the formation of beats.
9. What are stationary waves?. Explain the formation of stationary waves and also write down the characteristics of stationary waves.
10. Discuss the law of transverse vibrations in stretched strings.
11. Explain the concepts of fundamental frequency, harmonics and overtones in detail.
12. What is a sonometer?. Give its construction and working. Explain how to determine the frequency of tuning fork using sonometer.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

13. Write short notes on intensity and loudness.

14. Explain how overtones are produced in a

(a) Closed organ pipe

(b) Open organ pipe

15. How will you determine the velocity of sound using resonance air column apparatus?

16. What is meant by Doppler effect?. Discuss the following cases

(1) Source in motion and Observer at rest

(a) Source moves towards observer

(b) Source moves away from the observer

(2) Observer in motion and Source at rest.

(a) Observer moves towards Source

(b) Observer resides away from the Source

(3) Both are in motion

(a) Source and Observer approach each other

(b) Source and Observer resides from each other

(c) Source chases Observer

(d) Observer chases Source

IV. Numerical Problems

1. The speed of a wave in a certain medium is 900 m/s. If 3000 waves passes over a certain point of the medium in 2 minutes, then compute its wavelength?. **Answer** : $\lambda = 36$ m

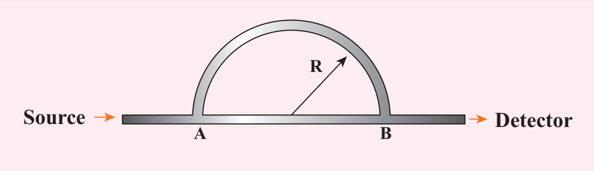
2. Consider a mixture of 2 mol of helium and 4 mol of oxygen. Compute the speed of sound in this gas mixture at 300 K. **Answer** : 400.9 ms^{-1}

3. A ship in a sea sends SONAR waves straight down into the seawater from the bottom of the ship. The signal reflects from the deep bottom bed rock and returns to the ship after 3.5 s. After the ship moves to 100 km it sends another signal which returns back after 2s. Calculate the depth of the sea in each case and also compute the difference in height between two cases.

Answer : $\Delta d = 1149.75$ m

4. A sound wave is transmitted into a tube as shown in figure. The sound wave splits into two waves at the point A which recombine at point B. Let R be the radius of the semi-circle which is varied until the first minimum. Calculate the radius of the semi-circle if the wavelength of the sound is 50.0 m.

Answer : $R = 21.9\ m$



5. N tuning forks are arranged in order of increasing frequency and any two successive tuning forks give n beats per second when sounded together. If the last fork gives double the frequency of the first (called as octave), Show that the frequency of the first tuning fork is $f = (N-1)n$.

6. Let the source propagate a sound wave whose intensity at a point (initially) be I. Suppose we consider a case when the amplitude of the sound wave is

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

doubled and the frequency is reduced to one-fourth. Calculate now the new intensity of sound at the same point ?.

Answer: $I_{new} \propto \frac{1}{4} I_{old}$.

7. Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.
Answer: 500Hz

8. A police in a siren car moving with a velocity 20 ms^{-1} chases a thief who is moving in a car with a velocity $v_0 ms^{-1}$. The police car sounds at frequency 300Hz, and both of them move towards a stationary siren of frequency 400Hz. Calculate the speed in which thief is moving.
Answer: $v_{thief} = 10\ m\ s^{-1}$

9. Consider the following function

(a) $y = x^2 + 2\ \alpha\ t\ x$

(b) $y = (x + vt)^2$

which among the above function can be characterized as a wave ?.

Answer: (a) function is not describing wave
(b) satisfies wave equation.

V. Conceptual Questions

1. Why is it that transverse waves cannot be produced in a gas?. Can the transverse waves can be produced in solids and liquids?
2. Why is the roar of our national animal different from the sound of a mosquito?
3. A sound source and listener are both stationary and a strong wind is blowing. Is there a Doppler effect?
4. In an empty room why is it that a tone sounds louder than in the room having things like furniture etc.
5. How do animals sense impending danger of hurricane?
6. Is it possible to realize whether a vessel kept under the tap is about to fill with water?

BOOKS FOR REFERENCE

1. Vibrations and Waves – A. P. French, CBS publisher and Distributors Pvt. Ltd.
2. Concepts of Physics – H. C. Verma, Volume 1 and Volume 2, Bharati Bhawan Publisher
3. Halliday, Resnick and Walker, Fundamentals of Physics, Wiley Publishers, 10th edition
4. Serway and Jewett, Physics for scientist and engineers with modern physics, Brook/Coole publishers, Eighth edition

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ICT CORNER

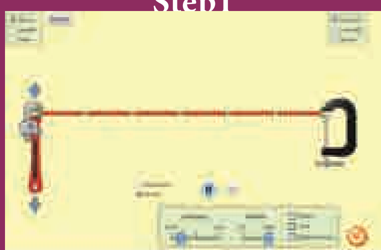
Waves

Through this activity you will be able to learn about the wave motion.

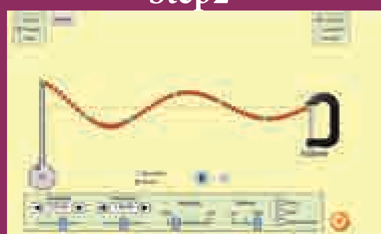
STEPS:

- Use the URL or scan the QR code to open 'PhET' simulation on 'waves on a string'. Click the play button.
- In the activity window a diagram of string is given. Click the play icon to see the motion of wave.
- We can see the 'oscillations' and 'pulse' by selecting on the table given in the left side window and by changing the 'amplitude' and 'frequency' is given below.
- By selecting the 'end types' on the right side window and repeat the same as before.

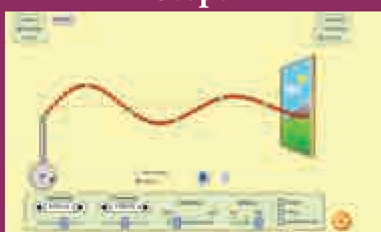
Step1



Step2



Step3



Step4

URL:

https://phet.colorado.edu/en/simulation/wave-on-a-string

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Higher Secondary First Year

PHYSICS

PRACTICAL

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

LIST OF PRACTICALS

1. Moment of Inertia of solid sphere of known mass using Vernier caliper
2. Non-uniform bending – verification of relation between the load and the depression using pin and microscope
3. Spring constant of a spring
4. Acceleration due to gravity using simple pendulum
5. Velocity of sound in air using resonance column
6. Viscosity of a liquid by Stoke's method
7. Surface tension by capillary rise method
8. Verification of Newton's law of cooling using calorimeter
9. Study of relation between the frequency and length of a given wire under constant tension using sonometer
10. Study of relation between length of a given wire and tension for constant frequency using sonometer
11. Verification of parallelogram law of forces (Demonstration only- not for examination)
12. Determination of density of a material of wire using screw gauge and physical balance (Demonstration only- Not for examination).

Note: Students should be instructed to perform the experiments given in ICT corner at the end of each unit of Volume 1. (Self study only- Not for examination)

BVH1DZ

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

1. MOMENT OF INERTIA OF A SOLID SPHERE OF KNOWN MASS USING VERNIER CALIPER

AIM To determine the moment of inertia of a solid sphere of known mass using Vernier caliper

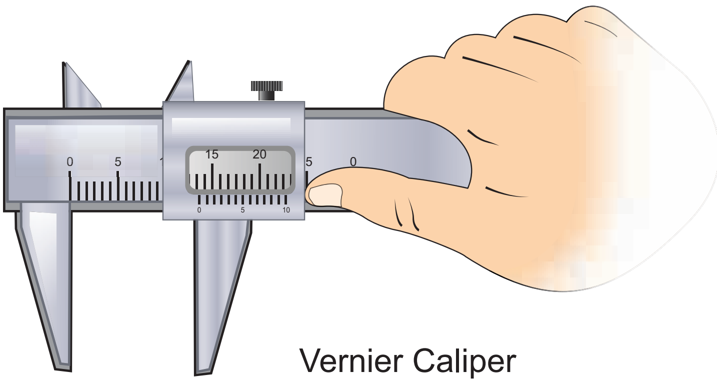
APPARATUS REQUIRED Vernier caliper, Solid sphere

FORMULA Moment of inertia of a solid sphere about its diameter $I_d = \frac{2}{5} MR^2$

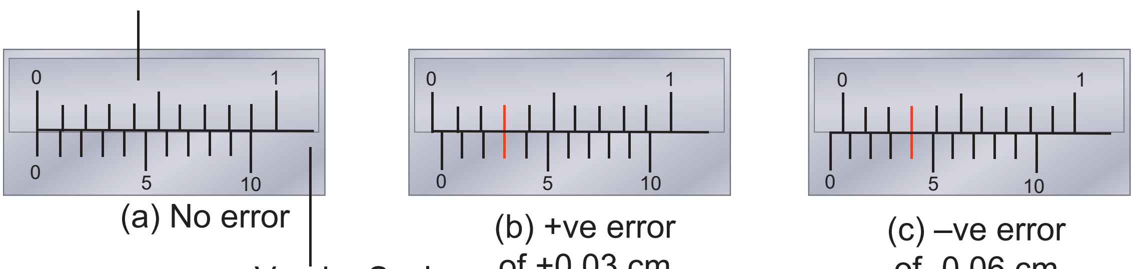
Where $M \rightarrow$ Mass of the sphere (known value to be given) in kg

$R \rightarrow$ Radius of the sphere in metre

DIAGRAM



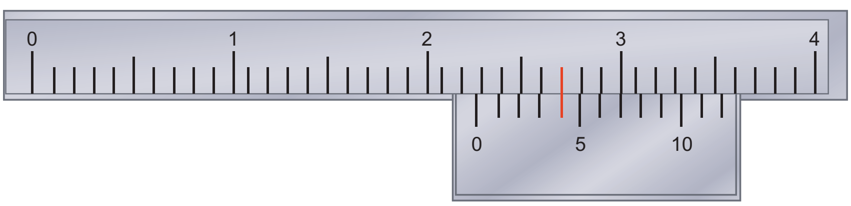
Vernier Caliper



(a) No error

(b) +ve error of +0.03 cm

(c) –ve error of -0.06 cm



(d) Vernier reading

A model reading
MSR = 2.2 cm ; VSC = 4 divisions;
Reading = [2.2 cm+(4x0.01cm)] = 2.24 cm

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

PROCEDURE

- The Vernier caliper is checked for zero errors and error if found is to be noted
- The sphere is kept in between the jaws of the Vernier caliper and the main scale reading (MSR)is noted.
- Vernier scale division which coincides with some main scale division (VSD) is noted. Zero correction made with this VSD gives Vernier scale reading (VSR).
- Multiply this VSR by Least Count (LC) and add it with MSR. This will be the diameter of the sphere.
- Observations are to be recorded for different positions of the sphere and the average value of the diameter is found. From this value radius of the sphere R is calculated.
- Using the known value of the mass of the sphere M and calculated radius of the sphere R the moment of inertia of the given sphere about its diameter can be calculated using the given formula.

LEAST COUNT (LC)

One main scale division (MSD) = ... cm

Number of Vernier scale divisions = ...

$$\text{Least Count (LC)} = \frac{\text{1 Main Scale Division (MSD)}}{\text{Total Vernier scale divisions}}$$

= .... cm

OBSERVATIONS

Zero error =

Zero correction =

Sl.No.	MSR cm	Vernier coincidence VSD	VSR = (VSD ± ZC)	Diameter of the sphere = 2R = (MSR +VSR x LC) cm
1				
2				
3				
4				
5				
6				

Mean diameter 2R = ... cm

Radius of the sphere R = ...cm

R = ... m

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CALCULATION

Mass of the sphere M = ... kg

(Known value is given)

Radius of the sphere R =. . . . . .metre

Moment of inertia of a solid sphere

about its diameter $I_d = \frac{2}{5} MR^2$ = ... kg m^2

RESULT

The moment of inertia of the given solid sphere about its diameter using Vernier caliper I_d = ………………….. kg m^2

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

2. NON – UNIFORM BENDING – VERIFICATION OF RELATION BETWEEN LOAD AND DEPRESSION USING PIN AND MICROSCOPE

AIM To verify the relation between the load and depression using non-uniform bending of a beam.

APPARATUS REQUIRED A long uniform beam (usually a metre scale), two knife – edge supports, mass hanger, slotted masses, pin, vernier microscope

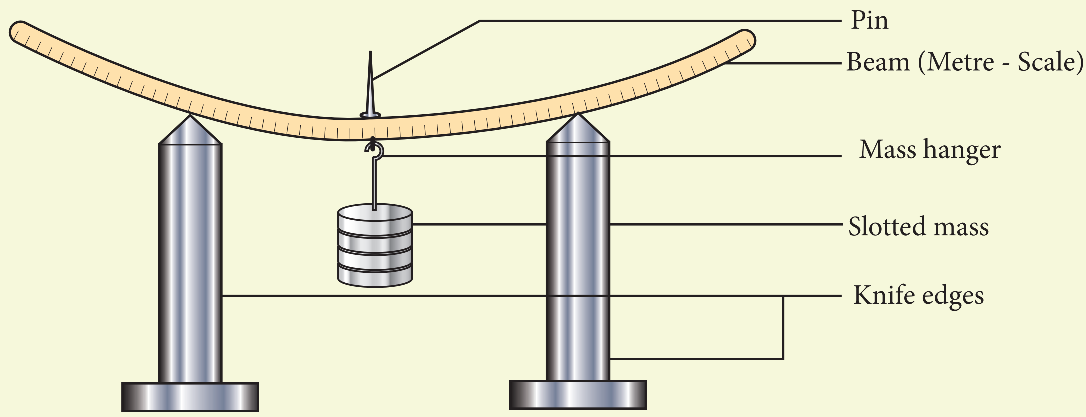
FORMULA

$$\frac{M}{s} = \text{a constant}$$

where M $\rightarrow$ Load applied (mass) (kg)

s $\rightarrow$ depression for the applied load(metre)

DIAGRAM



EXPERIMENTAL SETUP OF NON - UNIFORM BENDING PIN AND MICROSCOPE

PROCEDURE

- Place the two knife – edges on the table.
- Place the uniform beam (metre scale) on top of the knife edges.
- Suspend the mass hanger at the centre. A pin is attached at the centre of the scale where the hanger is hung.
- Place a vernier microscope in front of this arrangement
- Adjust the microscope to get a clear view of the pin

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- Make the horizontal cross-wire on the microscope to coincide with the tip of the pin. (Here mass hanger is the dead load M).
- Note the vertical scale reading of the vernier microscope
- Add the slotted masses one by one in steps of 0.05 kg (50 g) and take down the readings.
- Then start unloading by removing masses one by one and note the readings.
- Subtract the mean reading of each load from dead load reading. This gives the depressions for the corresponding load M.

OBSERVATIONS

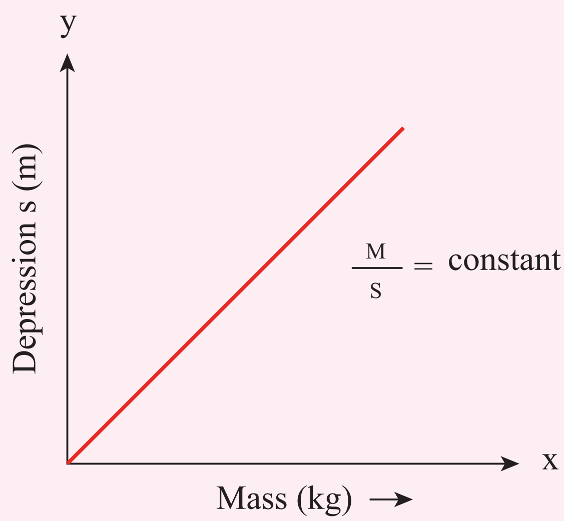
To find M/s

LOAD (kg)	MICROSCOPE READINGS (m)			DEPRESSION FOR M (kg) (s)	M/s kg m^{-1}
	INCREASING LOAD	DECREASING LOAD	MEAN		
M					
M + 0.05					
M + 0.10					
M + 0.15					
M + 0.20					
M + 0.25					
				Mean	

MODEL GRAPH

Load (M) vs Depression (s)

A graph between M and s can be drawn by taking M along X- axis and s along Y – axis. This is a straight line.



Relation between Mass and depression

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

CALCULATION

(i) $\frac{M}{s} =$

(ii) $\frac{M}{s} =$

(iii) $\frac{M}{s} =$

(iv) $\frac{M}{s} =$

(v) $\frac{M}{s} =$

RESULT

- The ratio between mass and depression for each load is calculated. This is found to be constant.
- Thus the relation between load and depression is verified by the method of non-uniform bending of a beam.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

3. SPRING CONSTANT OF A SPRING

AIM

To determine the spring constant of a spring by using the method of vertical oscillations

APPARATUS REQUIRED

Spring, rigid support, hook, 50 g mass hanger, 50 g slotted masses, stop clock, metre scale, pointer

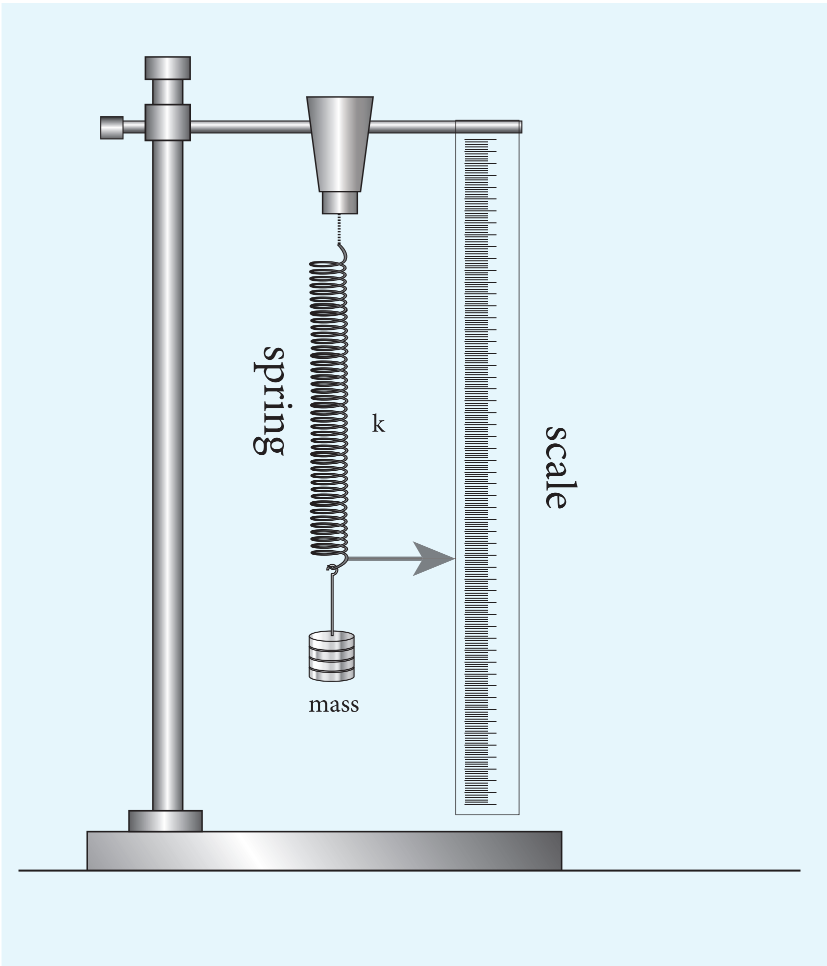
FORMULA

Spring constant of the spring $k = 4\pi^2 \left(\frac{M_2 - M_1}{T_2^2 - T_1^2}\right)$

where $M_1, M_2 \rightarrow$ selected loads in kg

$T_1, T_2 \rightarrow$ time period corresponding to masses M_1 and M_2 respectively in second

DIAGRAM



PROCEDURE

➢ A spring is firmly suspended vertically from a rigid clamp of a wooden stand at its upper end with a mass hanger attached to its lower end. A pointer fixed at the lower end of the spring moves over a vertical scale fixed.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- A suitable load M (eg; 100 g) is added to the mass hanger and the reading on the scale at which the pointer comes to rest is noted. This is the equilibrium position.
- The mass in the hanger is pulled downward and released so that the spring oscillates vertically on either side of the equilibrium position.
- When the pointer crosses the equilibrium position a stop clock is started and the time taken for 10 vertical oscillations is noted. Then the period of oscillation T is calculated.
- The experiment is repeated by adding masses in steps of 50 g to the mass hanger and period of oscillation at each time is calculated.
- For the masses M_1 and M_2 (with a difference of 50 g), if T_1 and T_2 are the corresponding periods, then the value $M_2 - M_1 / T_2^2 - T_1^2$ is calculated and its average is found.
- Using the given formula the spring constant of the given spring is calculated.

OBSERVATIONS

Sl. No.	Load M (g)	Time taken for 10 oscillations (s)			Period of oscillation T (s)	T^2 (s^2)	$\frac{M_2 - M_1}{T_2^2 - T_1^2}$ g s^{-2}
		Trial 1	Trial 2	Mean			
1	100						
2	150						
3	200						
4	250						
5	300						

Mean = ... g s^{-2}

= . . . . . kg s^{-2}

CALCULATION

Spring constant of the spring $k = 4\pi^2 \left(\frac{M_2 - M_1}{T_2^2 - T_1^2} \right)$

k = ... kg s^{-2}

RESULT

The spring constant of the given spring k = ... kg s^{-2}

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

4 ACCELERATION DUE TO GRAVITY USING SIMPLE PENDULUM

AIM To measure the acceleration due to gravity using a simple pendulum

APPARATUS REQUIRED Retort stand, pendulum bob, thread, meter scale, stop watch.

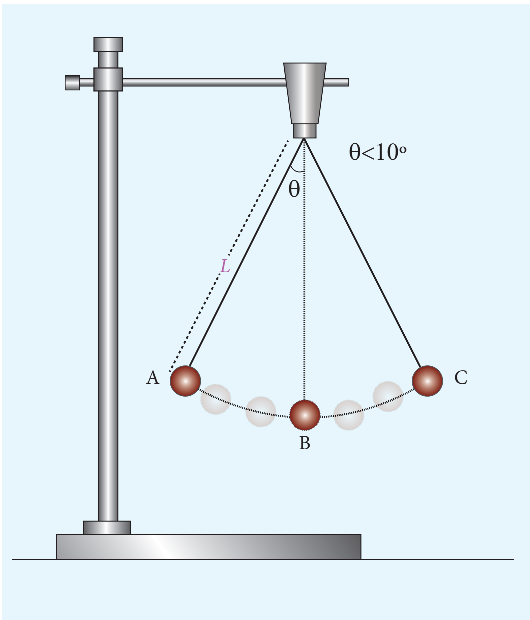
FORMULA Acceleration due to gravity $g = 4\pi^2 \left(\frac{L}{T^2}\right)$ (m s^{-2})

where T $\rightarrow$ Time period of simple pendulum (second)

g $\rightarrow$ Acceleration due to gravity (metre sec^{-2})

L $\rightarrow$ Length of the pendulum (metre)

DIAGRAM



PROCEDURE

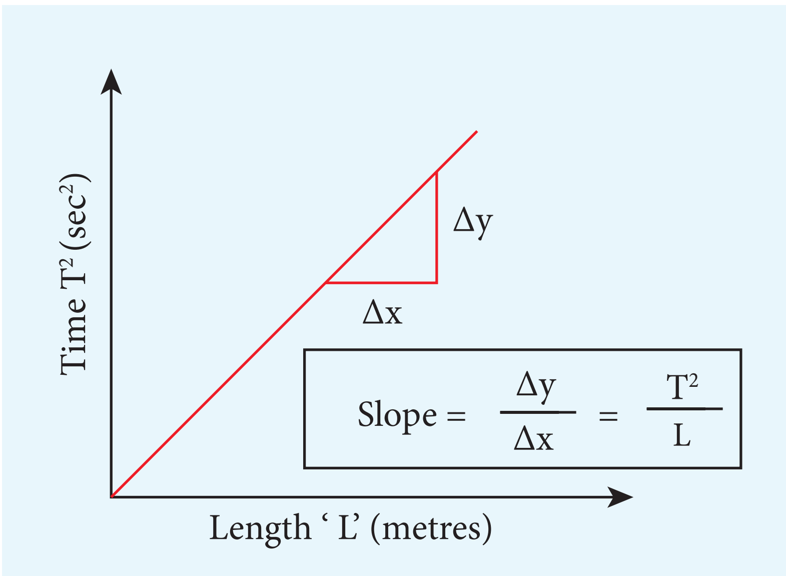
- Attach a small brass bob to the thread
- Fix this thread on to the stand
- Measure the length of the pendulum from top to the middle of the bob of the pendulum. Record the length of the pendulum in the table below.
- Note the time (t) for 10 oscillations using stop watch
- The period of oscillation $T = \frac{t}{10}$
- Repeat the experiment for different lengths of the pendulum 'L'. Find acceleration due to gravity g using the given formula.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

OBSERVATIONS To find the acceleration due to gravity 'g'

Length of the pendulum L (metre)	Time taken for 10 oscillations t (s)			Period of oscillation $T = \frac{t}{10}$ (s)	T^2 (s^2)	$g = \frac{4\pi^2 L}{T^2}$ $m\ s^{-2}$
	Trial 1	Trial 2	Average			
					Mean g =	

MODEL GRAPH



$slope = \frac{\Delta y}{\Delta x} = \frac{T^2}{L}$; 1/slope = L/T^2

RESULT

The acceleration due to gravity 'g' determined using simple pendulum is

i) By calculation = ... $m\ s^{-2}$

ii) By graph = ... $m\ s^{-2}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

5. VELOCITY OF SOUND IN AIR USING RESONANCE COLUMN

AIM

To determine the velocity of sound in air at room temperature using the resonance column.

APPARATUS REQUIRED

Resonance tube, three tuning forks of known frequencies, a rubber hammer, one thermometer, plumb line, set squares, water in a beaker.

FORMULA

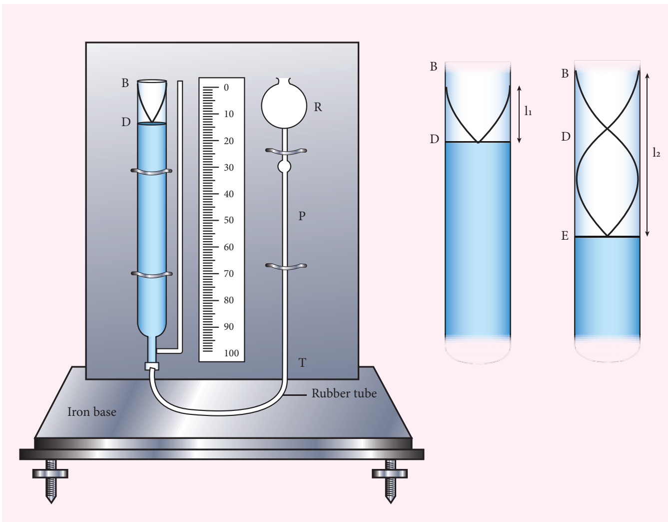
$V = 2\nu\,(l_2 - l_1)$ m s^{-1}

where V ⟶ Speed of sound in air (m s^{-1})

l_1 and l_2 ⟶ The length of the air column for the first and second resonance respectively (m)

ν ⟶ Frequency of the tuning fork (Hz)

DIAGRAM



PROCEDURE

- The inner tube of the resonance column is lowered so that the length of air column inside the tube is very small.
- Take a tuning fork of known frequency and strike it with a rubber hammer. The tuning fork now produces longitudinal waves with a frequency equal to the natural frequency of the tuning fork.
- Place the vibrating tuning fork horizontally above the tube. Sound waves pass down the total tube and reflect back at the water surface.
- Now, raise the tube and the tuning fork until a maximum sound is heard.
- Measure the length of air column at this position. This is taken as the first resonating length, l_1

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- Then raise the tube approximately about two times the first resonating length. Excite the tuning fork again and place it on the mouth of the tube.
- Change the height of the tube until the maximum sound is heard.
- Measure the length of air column at this position. This is taken as the second resonating length l_2
- We can now calculate the velocity of sound in air at room temperature by using the relation.

  $V = 2v(l_2 - l_1)$
- Repeat the experiment with forks of different frequency and calculate the velocity.
- The mean of the calculated values will give the velocity of sound in air at room temperature.

OBSERVATIONS

Sl. No.	Frequency of tuning fork v (Hz)	First resonating length $l_1 (cm)$			Second resonating length $l_2 (cm)$			$l_2 - l_1$ $(\times 10^{-2} m)$	Velocity of sound $V = 2v(l_2 - l_1)$ (m s^{-1})
		Trial 1	Trial 2	Mean	Trial 1	Trial 2	Mean		
1									
2									
3									
								Mean V =	

CALCULATION

Room temperature, t = _____________ ° C

Velocity sound in air at room temperature, $V = 2v(l_2 - l_1)$ = _____________ m s^{-1}

RESULT

Velocity of sound in air at room temperature, (V) = _____________ m s^{-1}

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

6. VISCOSITY OF A LIQUID BY STOKE'S METHOD

AIM

To determine the co-efficient of viscosity of the given liquid by stoke's method

APPARATUS REQUIRED

A long cylindrical glass jar, highly viscous liquid, metre scale, spherical ball, stop clock, thread.

FORMULA

$$\eta = \frac{2r^2(\delta - \sigma)g}{9V} \text{ N s m}^{-2}$$

where η – Coefficient of viscosity of liquid (N s m^{-2})

$r \rightarrow$ radius of spherical ball (m)

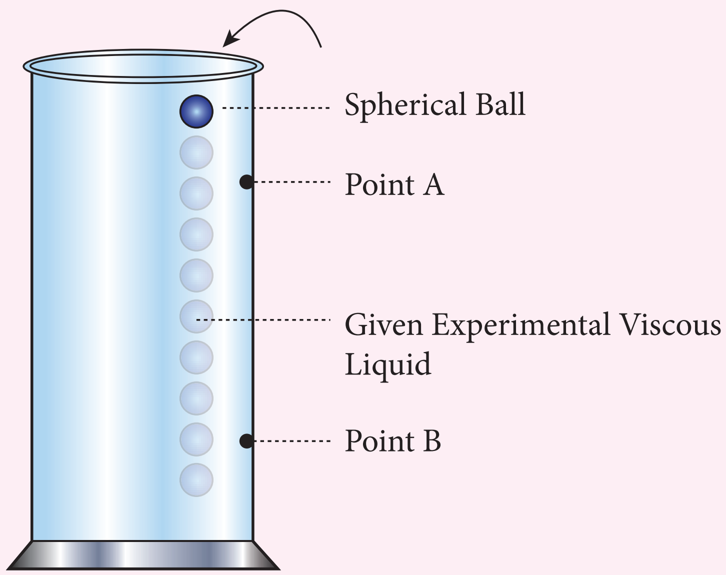
$\delta \rightarrow$ density of the steel sphere ($kg\ m^{-3}$)

$\sigma \rightarrow$ density of the liquid (kg m^{-3})

$g \rightarrow$ acceleration due to gravity (9.8 m s^{-2})

$V \rightarrow$ mean terminal velocity (m s^{-1})

DIAGRAM



EXPERIMENTAL SET UP OF MEASURING VISCOSITY BY STOKE'S METHOD

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

PROCEDURE

- A long cylindrical glass jar with markings is taken.
- Fill the glass jar with the given experimental liquid.
- Two points A and B are marked on the jar. The mark A is made well below the surface of the liquid so that when the ball reaches A it would have acquired terminal velocity V.
- The radius of the metal spherical ball is determined using screw gauge.
- The spherical ball is dropped gently into the liquid.
- Start the stop clock when the ball crosses the point A. Stop the clock when the ball reaches B.
- Note the distance between A and B and use it to calculate terminal velocity.
- Now repeat the experiment for different distances between A and B. Make sure that the point A is below the terminal stage.

OBSERVATIONS

To find Terminal Velocity:

S.No.	Distance covered by the spherical ball (d) (m)	Time taken (t) (s)	Terminal Velocity (V) d/t (m s^{-1})
		MEAN	

CALCULATION

Density of the spherical ball δ = ________ $kg\ m^{-3}$

Density of the given liquid σ = ________ $kg\ m^{-3}$

Coefficient of viscosity of the liquid $\eta = \dfrac{2r^2 g(\delta - \sigma)}{9V}$ = ________ $N\ s\ m^{-2}$

RESULT

The coefficient of viscosity of the given liquid by stoke's method η = ________ $N\ s\ m^{-2}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

7. SURFACE TENSION BY CAPILLARY RISE METHOD

AIM To determine surface tension of a liquid by capillary rise method.

APPARATUS REQUIRED A beaker of Water, capillary tube, vernier microscope, two holed rubber stopper, a knitting needle, a short rubber tubing and retort clamp.

FORMULA The surface tension of the liquid $T = \frac{hr\sigma g}{2}$ $N\,m^{-1}$

where $T \rightarrow$ Surface tension of the liquid ($N\,m^{-1}$)

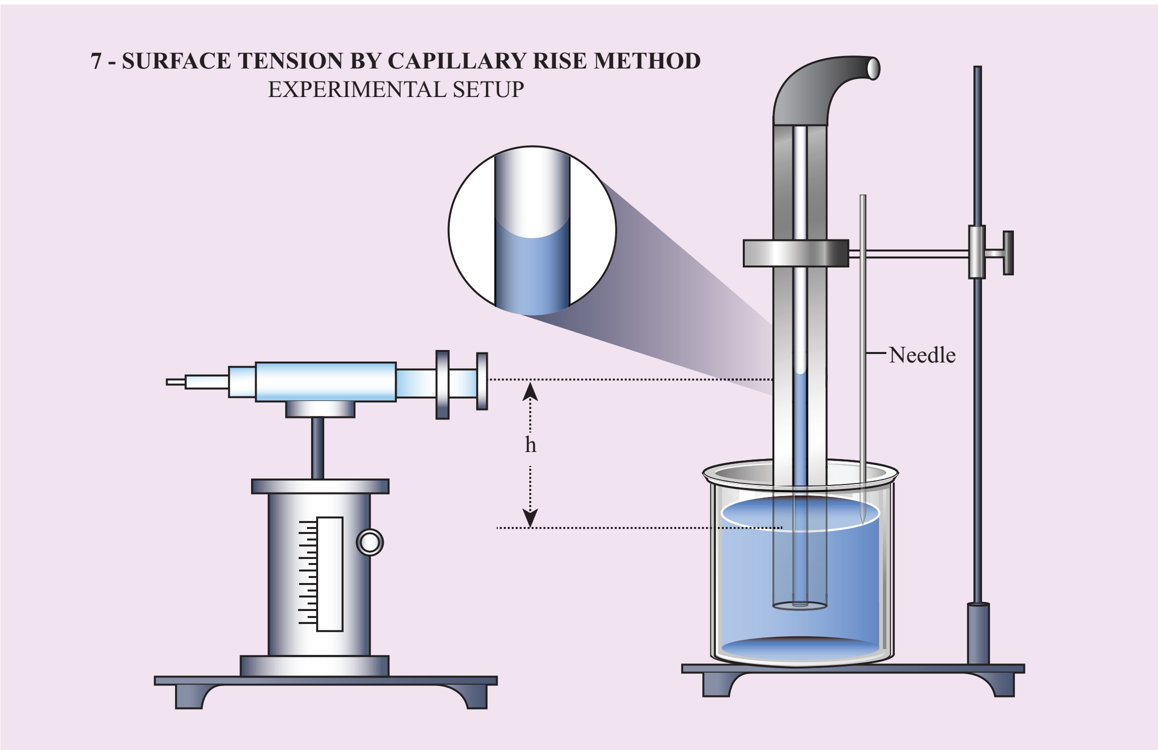
$h \rightarrow$ height of the liquid in the capillary tube (m)

$r \rightarrow$ radius of the capillary tube (m)

$\sigma \rightarrow$ Density of water ($kg\,m^{-3}$) ($\sigma = 1000\ kg\,m^{-3}$)

$g \rightarrow$ Acceleration due to gravity ($g = 9.8\ m\,s^{-2}$)

DIAGRAM

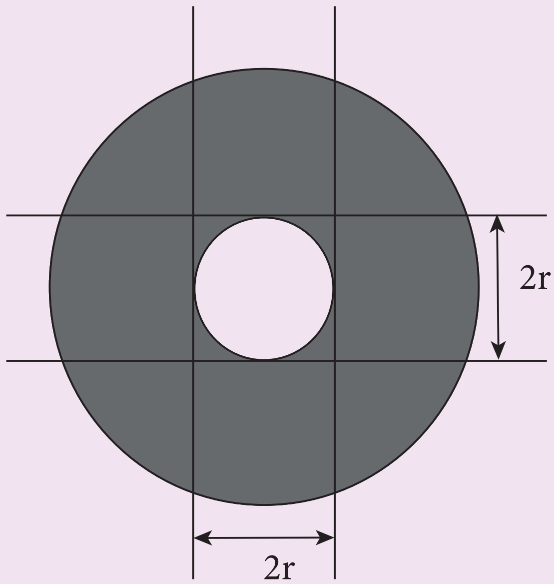


PROCEDURE

- A clean and dry capillary tube is taken and fixed in a stand
- A beaker containing water is placed on an adjustable platform and the capillary tube is dipped inside the beaker so that a little amount of water is raised inside.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- Fix a needle near the capillary tube so that the needle touches the water surface
- A Vernier microscope is focused at the water meniscus level and the corresponding reading is taken after making the cross wire coincidence.
- Vernier microscope is focused to the tip of the needle and again reading is taken and noted.
- The difference between the two readings of the vertical scale gives the height (h) of the liquid raised in the tube.
- Now to find the radius of the tube, lower the height of the support base and remove the beaker, carefully rotate the capillary tube so that the immersed lower end face towards you.
- Focus the tube using Vernier microscope to clearly see the inner walls of the tube.
- Let the vertical cross wire coincide with the left side inner walls of the tube. Note down the reading (L_1)
- Turn the microscope screws in horizontal direction to view the right side inner wall of the tube. Note the reading (R_1). Thus the radius of the tube can be calculated as $\frac{1}{2}(L_1 - R_1)$.
- Finally calculate the surface tension using the given formula.



RADIUS OF THE CAPILLARY TUBE

OBSERVATIONS

To measure height of the liquid (h)

Least count of the microscope = ________ $\times 0.001 cm$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

Trial No.	Microscope reading for the position of Lower meniscus of liquid			Microscope reading for the position of Lower tip of the needle			Height of the liquid h (cm)
	MSR	VSR	TR (cm)	MSR	VSR	TR (cm)	
						Mean h =	

Radius of the capillary tube

Tube	Microscope reading for the position of inner left wall of the tube L_1			Microscope reading for the position of inner right wall of the tube R_1			Radius of the capillary tube $r = \frac{1}{2}(L_1 - R_1)$ (cm)
	MSR	VSR	TR (cm)	MSR	VSR	TR (cm)	

CALCULATION

Mean rise of the liquid in the capillary tube h = __________ cm

__________ m

Diameter of the capillary tube 2r = __________ cm

Radius of the capillary tube r = __________ m

Density of the liquid $\sigma = 1000 \text{ kg m}^{-3}$

Acceleration due to gravity $g = 9.8 \text{ m s}^{-2}$

Surface tension $T = \frac{hr\sigma g}{2}$

$= \_\_\_\_\_\_\_\_\_\_ N\,m^{-1}$

RESULT

Surface tension of the given liquid by capillary rise method T = __________ $N\,m^{-1}$

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

8. NEWTON'S LAW OF COOLING USING CALORIMETER

AIM

To study the relationship between the temperature of a hot body and time by plotting a cooling curve.

APPARATUS REQUIRED

Copper calorimeter with stirrer, one holed rubber cork, thermometer, stop clock, heater / burner, water, clamp and stand

NEWTON'S LAW OF COOLING

Newton's law of cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature. (i.e., the temperature of its surroundings)

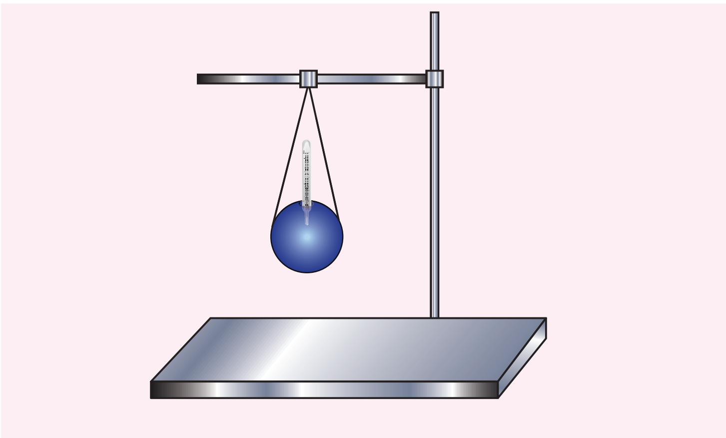
$$\frac{dT}{dt} \propto (T - T_0)$$

where $\frac{dT}{dt} \rightarrow$ Rate of change of temperature (°C)

$T \rightarrow$ Temperature of water (°C)

$T_0 \rightarrow$ Room Temperature (°C)

DIAGRAM



EXPERIMENTAL SETUP OF NEWTON'S LAW OF COOLING

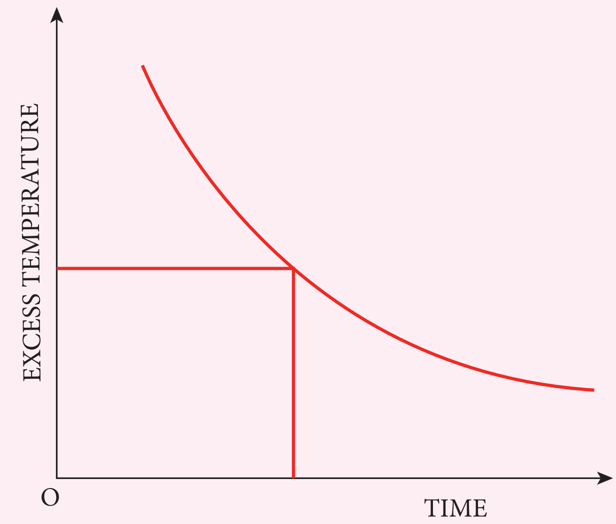
PROCEDURE

- Note the room temperature as (T_0) using the thermometer.
- Hot water about 90°C is poured into the calorimeter.
- Close the calorimeter with one holed rubber cork
- Insert the thermometer into calorimeter through the hole in rubber cork
- Start the stop clock and observe the time for every one degree fall of temperature from 80°C

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

- Take sufficient amount of reading, say closer to room temperature
- The observations are tabulated
- Draw a graph by taking time along the x axis and excess temperature along y axis.

MODEL GRAPH



RELATION BETWEEN TIME AND TEMPERATURE

ROOM TEMPERATURE (T_0) = ________°C

OBSERVATIONS

Measuring the change in temperature of water with time

Time (s)	Temperature of water (T) °C	Excess temperature ($T - T_0$) °C

RESULT

The cooling curve is plotted and thus Newton's law of cooling is verified.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

9. STUDY OF RELATION BETWEEN FREQUENCY AND LENGTH OF A GIVEN WIRE UNDER CONSTANT TENSION USING SONOMETER

AIM

To study the relation between frequency and length of a given wire under constant tension using a sonometer.

APPARATUS REQUIRED

Sonometer, six tuning forks of known frequencies, Metre scale, rubber pad, paper rider, hanger with half – kilogram masses, wooden bridges

FORMULA

The frequency n of the fundamental mode of vibration of a string

is given by $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ Hz

a) For a given m and fixed T.

$n \propto \frac{1}{l}$ (or) $nl =$ constant

where $n \rightarrow$ Frequency of the fundamental mode of vibration of the string (Hz)

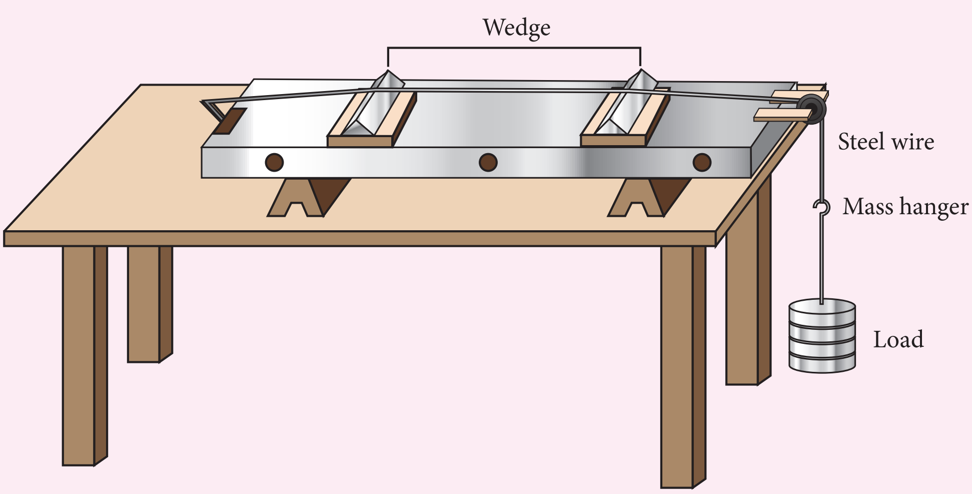
$m \rightarrow$ Mass per unit length of the string ($kg\ m^{-1}$)

$l \rightarrow$ Length of the string between the wedges (m)

$T \rightarrow$ Tension in the string (including the mass of the hanger) = Mg (N)

$M \rightarrow$ Mass suspended, including the mass of the hanger (Kg)

DIAGRAM



SONOMETER - STUDY OF RELATION BETWEEN FREQUENCY AND LENGTH OF A GIVEN WIRE UNDER CONSTANT TENSION USING SONOMETER

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

PROCEDURE

- Set up the sonometer on the table and clean the groove on the pulley to ensure minimum friction
- Stretch the wire by placing suitable mass in the hanger
- Set the tuning fork into vibrations by striking it against the rubber pad. Plug the sonometer wire and compare the two sounds.
- Adjust the vibrating length of the wire by sliding the bridge B till the sounds appear alike.
- For the final adjustment, place a small paper rider R in the middle of the wire AB.
- Sound the tuning fork and place its shank stem on the bridge A or on the sonometer box and slowly adjust the position of bridge B until the paper rider is agitated violently indicating resonance.
- The length of the wire between the wedges A and B is measured using meter scale. It is the resonant length. Now the frequency of vibration of the fundamental mode equals the frequency of the tuning fork.
- Repeat the above procedure for other tuning forks by keeping the same load in the hanger.

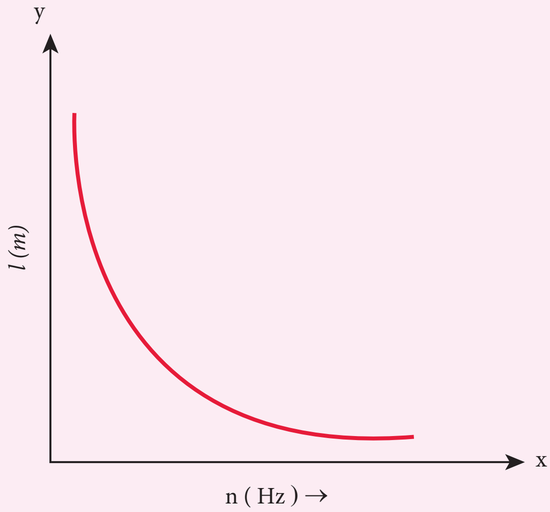
OBSERVATIONS

Tension (constant) on the wire (mass suspended from the hanger including its own mass)
T = ________ N

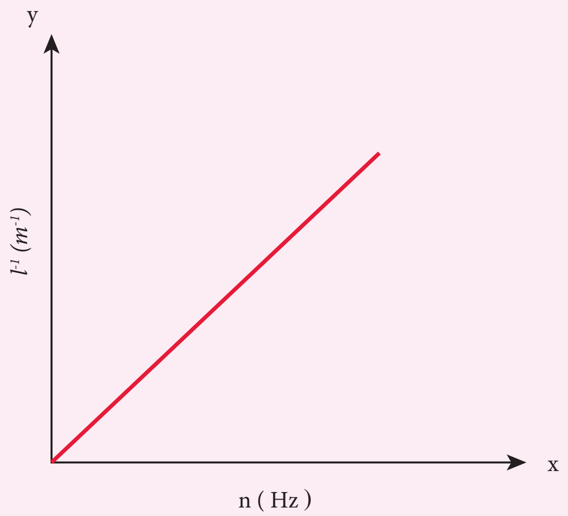
Variation of frequency with length		
Frequency of the tuning fork 'n ' (Hz)	Resonant length 'l'	nl
n_1 =		
n_2 =		
n_3 =		
n_4 =		
n_5 =		
n_6 =		

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

GRAPH:



Graph 1: Relation between frequency and length



Graph 2: Relation between frequency and inverse of length

Sonometer - study of relation between length of the given wire and tension for a consant frequency

CALCULATION

The product nl for all the tuning forks remain constant (last column in the table)

RESULT

- For a given tension, the resonant length of a given stretched string varies as reciprocal of the frequency (i.e., $n \propto \frac{1}{l}$)
- The product nl is a constant and found to be _______ (Hz m)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

10. STUDY OF RELATION BETWEEN LENGTH OF THE GIVEN WIRE AND TENSION FOR A CONSTANT FREQUENCY USING SONOMETER

AIM

To study the relationship between the length of a given wire and tension for constant frequency using a sonometer

APPARATUS REQUIRED

Sonometer, tuning fork of known frequency, meter scale, rubber pad, paper rider, hanger with half – kilogram masses, wooden bridges.

FORMULA

The frequency of the fundamental mode of vibration of a string is given by,

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

If n is a constant, for a given wire (m is constant)

$\frac{\sqrt{T}}{l}$ is constant.

where n $\rightarrow$ Frequency of the fundamental mode of vibration of a string (Hz)

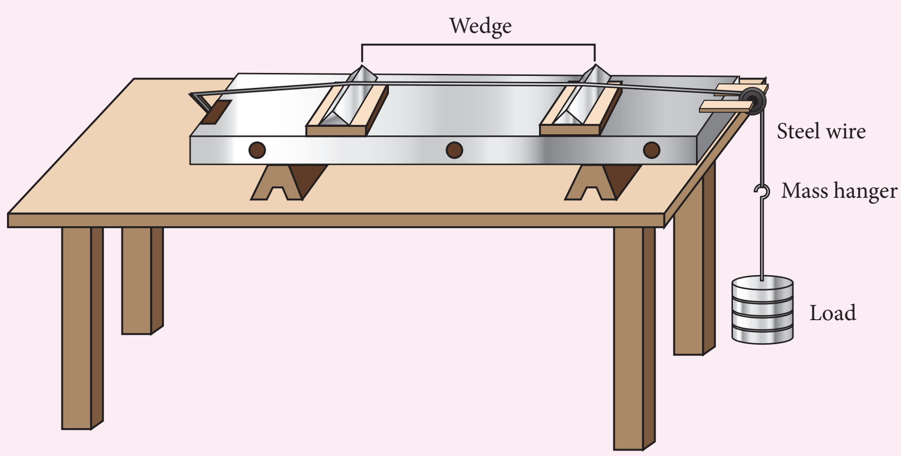
$m \rightarrow$ Mass per unit length of string ($kg\ m^{-1}$)

T $\rightarrow$ Tension in the string (including the weight of the hanger) = Mg (N)

$l \rightarrow$ Length of the string between the wedges (metre)

M $\rightarrow$ Mass suspended, including the mass of the hanger (kg)

DIAGRAM



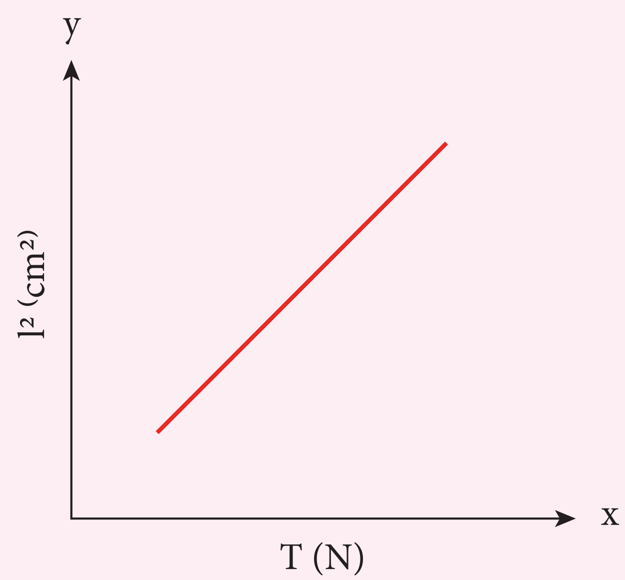
STUDY OF RELATION BETWEEN LENGTH OF THE GIVEN WIRE AND TENSION FOR A CONSTANT FREQUENCY USING SONOMETER

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

PROCEDURE

- Set up the sonometer on the table and clean the groove on the pulley to ensure that it has minimum friction.
- Set a tuning fork of known frequency into vibration by striking it against the rubber pad. Plug the sonometer wire and compare the sound due to the vibration of tuning fork and the plugged wire.
- Adjust the vibrating length of the wire by the adjusting the bridge B till the two sounds appear alike.
- Place a mass of 1 kg for initial reading in the load hanger.
- For final adjustment place a small paper rider R in the middle of the wire AB.
- Now, strike the tuning fork and place its shank stem on the bridge A and then slowly adjust the position of the bridge B till the paper rider is agitated violently (might eventually falls) indicating resonance.
- Measure the length of the wire between wedges at A and B which is the fundamental mode corresponding to the frequency of the tuning fork.
- Increase the load on the hanger in steps of 0.5 kg and each time find the resonating length as done before with the same tuning fork.
- Record the observations in the tabular column.

MODEL GRAPH



Relation between tension T (N) and length l^2 (cm²)

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

OBSERVATIONS

Frequency of the tuning fork = _________ Hz

Variation of resonant length with tension

Sl.No.	Mass M (kg)	Tension T=Mg (N)	$\sqrt{T}$	Vibrating length l (m)	$\frac{\sqrt{T}}{l}$

CALCULATION

Calculate the value $\frac{\sqrt{T}}{l}$ for the tension applied in each case.

RESULT

- The resonating length varies as square root of tension for a given frequency of vibration of a stretched string.
- $\frac{\sqrt{T}}{l}$ is found to be a constant.

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

SOME IMPORTANT CONSTANTS IN PHYSICS

Name	Symbols	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Acceleration due to gravity (sea level, at 45° latitude)	g	9.8 m s^{-2}
Planck constant	h	6.626×10^{-34} J s
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Avogadro number	N_A	6.023×10^{23} mol^{-1}
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Stefan – Boltmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Wien's constant	b	2.898×10^{-3} m K
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Standard atmospheric pressure	1 *atm*	1.013×10^{5} Pa

SOLVED EXAMPLES

BVQHFM

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

(THE GREEK ALPHABET)

(கிரேக்க எழுத்துகள்)

The Greek Alphabet	Upper Case	Lower Case
Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	€	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	ϕ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω

COMPETITIVE EXAM CORNER

BVZSHA

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

LOGARITHM TABLE

										Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0.000	0.004	0.009	0.013	0.017	0.021	0.025	0.029	0.033	0.037	4	8	11	17	21	25	29	33	37
11	0.041	0.045	0.049	0.053	0.057	0.061	0.064	0.068	0.072	0.076	4	8	11	15	19	23	26	30	34
12	0.079	0.083	0.086	0.090	0.093	0.097	0.100	0.104	0.107	0.111	3	7	10	14	17	21	24	28	31
13	0.114	0.117	0.121	0.124	0.127	0.130	0.134	0.137	0.140	0.143	3	6	10	13	16	19	23	26	29
14	0.146	0.149	0.152	0.155	0.158	0.161	0.164	0.167	0.170	0.173	3	6	9	12	15	18	21	24	27
15	0.176	0.179	0.182	0.185	0.188	0.190	0.193	0.196	0.199	0.201	3	6	8	11	14	17	20	22	25
16	0.204	0.207	0.210	0.212	0.215	0.217	0.220	0.223	0.225	0.228	3	5	8	11	13	16	18	21	24
17	0.230	0.233	0.236	0.238	0.241	0.243	0.246	0.248	0.250	0.253	2	5	7	10	12	15	17	20	22
18	0.255	0.258	0.260	0.262	0.265	0.267	0.270	0.272	0.274	0.276	2	5	7	9	12	14	16	19	21
19	0.279	0.281	0.283	0.286	0.288	0.290	0.292	0.294	0.297	0.299	2	4	7	9	11	13	16	18	20
20	0.301	0.303	0.305	0.307	0.310	0.312	0.314	0.316	0.318	0.320	2	4	6	8	11	13	15	17	19
21	0.322	0.324	0.326	0.328	0.330	0.332	0.334	0.336	0.338	0.340	2	4	6	8	10	12	14	16	18
22	0.342	0.344	0.346	0.348	0.350	0.352	0.354	0.356	0.358	0.360	2	4	6	8	10	12	14	15	17
23	0.362	0.364	0.365	0.367	0.369	0.371	0.373	0.375	0.377	0.378	2	4	6	7	9	11	13	15	17
24	0.380	0.382	0.384	0.386	0.387	0.389	0.391	0.393	0.394	0.396	2	4	5	7	9	11	12	14	16
25	0.398	0.400	0.401	0.403	0.405	0.407	0.408	0.410	0.412	0.413	2	3	5	7	9	10	12	14	15
26	0.415	0.417	0.418	0.420	0.422	0.423	0.425	0.427	0.428	0.430	2	3	5	7	8	10	11	13	15
27	0.431	0.433	0.435	0.436	0.438	0.439	0.441	0.442	0.444	0.446	2	3	5	6	8	9	11	13	14
28	0.447	0.449	0.450	0.452	0.453	0.455	0.456	0.458	0.459	0.461	2	3	5	6	8	9	11	12	14
29	0.462	0.464	0.465	0.467	0.468	0.470	0.471	0.473	0.474	0.476	1	3	4	6	7	9	10	12	13
30	0.477	0.479	0.480	0.481	0.483	0.484	0.486	0.487	0.489	0.490	1	3	4	6	7	9	10	11	13
31	0.491	0.493	0.494	0.496	0.497	0.498	0.500	0.501	0.502	0.504	1	3	4	6	7	8	10	11	12
32	0.505	0.507	0.508	0.509	0.511	0.512	0.513	0.515	0.516	0.517	1	3	4	5	7	8	9	11	12
33	0.519	0.520	0.521	0.522	0.524	0.525	0.526	0.528	0.529	0.530	1	3	4	5	6	8	9	10	12
34	0.531	0.533	0.534	0.535	0.537	0.538	0.539	0.540	0.542	0.543	1	3	4	5	6	8	9	10	11
35	0.544	0.545	0.547	0.548	0.549	0.550	0.551	0.553	0.554	0.555	1	2	4	5	6	7	9	10	11
36	0.556	0.558	0.559	0.560	0.561	0.562	0.563	0.565	0.566	0.567	1	2	3	5	6	7	8	10	11
37	0.568	0.569	0.571	0.572	0.573	0.574	0.575	0.576	0.577	0.579	1	2	3	5	6	7	8	9	10
38	0.580	0.581	0.582	0.583	0.584	0.585	0.587	0.588	0.589	0.590	1	2	3	5	6	7	8	9	10
39	0.591	0.592	0.593	0.594	0.595	0.597	0.598	0.599	0.600	0.601	1	2	3	4	5	7	8	9	10
40	0.602	0.603	0.604	0.605	0.606	0.607	0.609	0.610	0.611	0.612	1	2	3	4	5	6	8	9	10
41	0.613	0.614	0.615	0.616	0.617	0.618	0.619	0.620	0.621	0.622	1	2	3	4	5	6	7	8	9
42	0.623	0.624	0.625	0.626	0.627	0.628	0.629	0.630	0.631	0.632	1	2	3	4	5	6	7	8	9
43	0.633	0.634	0.635	0.636	0.637	0.638	0.639	0.640	0.641	0.642	1	2	3	4	5	6	7	8	9
44	0.643	0.644	0.645	0.646	0.647	0.648	0.649	0.650	0.651	0.652	1	2	3	4	5	6	7	8	9
45	0.653	0.654	0.655	0.656	0.657	0.658	0.659	0.660	0.661	0.662	1	2	3	4	5	6	7	8	9
46	0.663	0.664	0.665	0.666	0.667	0.667	0.668	0.669	0.670	0.671	1	2	3	4	5	6	7	7	8
47	0.672	0.673	0.674	0.675	0.676	0.677	0.678	0.679	0.679	0.680	1	2	3	4	5	5	6	7	8
48	0.681	0.682	0.683	0.684	0.685	0.686	0.687	0.688	0.688	0.689	1	2	3	4	4	5	6	7	8
49	0.690	0.691	0.692	0.693	0.694	0.695	0.695	0.696	0.697	0.698	1	2	3	4	4	5	6	7	8
50	0.699	0.700	0.701	0.702	0.702	0.703	0.704	0.705	0.706	0.707	1	2	3	4	4	5	6	7	8
51	0.708	0.708	0.709	0.710	0.711	0.712	0.713	0.713	0.714	0.715	1	2	3	4	4	5	6	7	8
52	0.716	0.717	0.718	0.719	0.719	0.720	0.721	0.722	0.723	0.723	1	2	2	4	4	5	6	7	7
53	0.724	0.725	0.726	0.727	0.728	0.728	0.729	0.730	0.731	0.732	1	2	2	4	4	5	6	6	7
54	0.732	0.733	0.734	0.735	0.736	0.736	0.737	0.738	0.739	0.740	1	2	2	4	4	5	6	6	7

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

LOGARITHM TABLE

											Mean Difference								
55	0.740	0.741	0.742	0.743	0.744	0.744	0.745	0.746	0.747	0.747	1	2	2	4	4	5	5	6	7
56	0.748	0.749	0.750	0.751	0.751	0.752	0.753	0.754	0.754	0.755	1	2	2	4	4	5	5	6	7
57	0.756	0.757	0.757	0.758	0.759	0.760	0.760	0.761	0.762	0.763	1	2	2	4	4	5	5	6	7
58	0.763	0.764	0.765	0.766	0.766	0.767	0.768	0.769	0.769	0.770	1	1	2	4	4	4	5	6	7
59	0.771	0.772	0.772	0.773	0.774	0.775	0.775	0.776	0.777	0.777	1	1	2	3	4	4	5	6	7
60	0.778	0.779	0.780	0.780	0.781	0.782	0.782	0.783	0.784	0.785	1	1	2	3	4	4	5	6	6
61	0.785	0.786	0.787	0.787	0.788	0.789	0.790	0.790	0.791	0.792	1	1	2	3	4	4	5	6	6
62	0.792	0.793	0.794	0.794	0.795	0.796	0.797	0.797	0.798	0.799	1	1	2	3	3	4	5	6	6
63	0.799	0.800	0.801	0.801	0.802	0.803	0.803	0.804	0.805	0.806	1	1	2	3	3	4	5	5	6
64	0.806	0.807	0.808	0.808	0.809	0.810	0.810	0.811	0.812	0.812	1	1	2	3	3	4	5	5	6
65	0.813	0.814	0.814	0.815	0.816	0.816	0.817	0.818	0.818	0.819	1	1	2	3	3	4	5	5	6
66	0.820	0.820	0.821	0.822	0.822	0.823	0.823	0.824	0.825	0.825	1	1	2	3	3	4	5	5	6
67	0.826	0.827	0.827	0.828	0.829	0.829	0.830	0.831	0.831	0.832	1	1	2	3	3	4	5	5	6
68	0.833	0.833	0.834	0.834	0.835	0.836	0.836	0.837	0.838	0.838	1	1	2	3	3	4	4	5	6
69	0.839	0.839	0.840	0.841	0.841	0.842	0.843	0.843	0.844	0.844	1	1	2	2	3	4	4	5	6
70	0.845	0.846	0.846	0.847	0.848	0.848	0.849	0.849	0.850	0.851	1	1	2	2	3	4	4	5	6
71	0.851	0.852	0.852	0.853	0.854	0.854	0.855	0.856	0.856	0.857	1	1	2	2	3	4	4	5	5
72	0.857	0.858	0.859	0.859	0.860	0.860	0.861	0.862	0.862	0.863	1	1	2	2	3	4	4	5	5
73	0.863	0.864	0.865	0.865	0.866	0.866	0.867	0.867	0.868	0.869	1	1	2	2	3	4	4	5	5
74	0.869	0.870	0.870	0.871	0.872	0.872	0.873	0.873	0.874	0.874	1	1	2	2	3	4	4	5	5
75	0.875	0.876	0.876	0.877	0.877	0.878	0.879	0.879	0.880	0.880	1	1	2	2	3	3	4	5	5
76	0.881	0.881	0.882	0.883	0.883	0.884	0.884	0.885	0.885	0.886	1	1	2	2	3	3	4	5	5
77	0.886	0.887	0.888	0.888	0.889	0.889	0.890	0.890	0.891	0.892	1	1	2	2	3	3	4	4	5
78	0.892	0.893	0.893	0.894	0.894	0.895	0.895	0.896	0.897	0.897	1	1	2	2	3	3	4	4	5
79	0.898	0.898	0.899	0.899	0.900	0.900	0.901	0.901	0.902	0.903	1	1	2	2	3	3	4	4	5
80	0.903	0.904	0.904	0.905	0.905	0.906	0.906	0.907	0.907	0.908	1	1	2	2	3	3	4	4	5
81	0.908	0.909	0.910	0.910	0.911	0.911	0.912	0.912	0.913	0.913	1	1	2	2	3	3	4	4	5
82	0.914	0.914	0.915	0.915	0.916	0.916	0.917	0.918	0.918	0.919	1	1	2	2	3	3	4	4	5
83	0.919	0.920	0.920	0.921	0.921	0.922	0.922	0.923	0.923	0.924	1	1	2	2	3	3	4	4	5
84	0.924	0.925	0.925	0.926	0.926	0.927	0.927	0.928	0.928	0.929	1	1	2	2	3	3	4	4	5
85	0.929	0.930	0.930	0.931	0.931	0.932	0.932	0.933	0.933	0.934	1	1	2	2	3	3	4	4	5
86	0.934	0.935	0.936	0.936	0.937	0.937	0.938	0.938	0.939	0.939	1	1	2	2	3	3	4	4	5
87	0.940	0.940	0.941	0.941	0.942	0.942	0.943	0.943	0.943	0.944	0	1	1	2	2	3	3	4	5
88	0.944	0.945	0.945	0.946	0.946	0.947	0.947	0.948	0.948	0.949	0	1	1	2	2	3	3	4	4
89	0.949	0.950	0.950	0.951	0.951	0.952	0.952	0.953	0.953	0.954	0	1	1	2	2	3	3	4	4
90	0.954	0.955	0.955	0.956	0.956	0.957	0.957	0.958	0.958	0.959	0	1	1	2	2	3	3	4	4
91	0.959	0.960	0.960	0.960	0.961	0.961	0.962	0.962	0.963	0.963	0	1	1	2	2	3	3	4	4
92	0.964	0.964	0.965	0.965	0.966	0.966	0.967	0.967	0.968	0.968	0	1	1	2	2	3	3	4	4
93	0.968	0.969	0.969	0.970	0.970	0.971	0.971	0.972	0.972	0.973	0	1	1	2	2	3	3	4	4
94	0.973	0.974	0.974	0.975	0.975	0.975	0.976	0.976	0.977	0.977	0	1	1	2	2	3	3	4	4
95	0.978	0.978	0.979	0.979	0.980	0.980	0.980	0.981	0.981	0.982	0	1	1	2	2	3	3	4	4
96	0.982	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986	0.986	0	1	1	2	2	3	3	4	4
97	0.987	0.987	0.988	0.988	0.989	0.989	0.989	0.990	0.990	0.991	0	1	1	2	2	3	3	4	4
98	0.991	0.992	0.992	0.993	0.993	0.993	0.994	0.994	0.995	0.995	0	1	1	2	2	3	3	4	4
99	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.999	0.999	1.000	0	1	1	2	2	3	3	3	4

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ANTILOG TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.00	1.000	1.002	1.005	1.007	1.009	1.012	1.014	1.016	1.019	1.021	0	0	1	1	1	1	2	2	2
0.01	1.023	1.026	1.028	1.030	1.033	1.035	1.038	1.040	1.042	1.045	0	0	1	1	1	1	2	2	2
0.02	1.047	1.050	1.052	1.054	1.057	1.059	1.062	1.064	1.067	1.069	0	0	1	1	1	1	2	2	2
0.03	1.072	1.074	1.076	1.079	1.081	1.084	1.086	1.089	1.091	1.094	0	0	1	1	1	1	2	2	2
0.04	1.096	1.099	1.102	1.104	1.107	1.109	1.112	1.114	1.117	1.119	0	1	1	1	1	2	2	2	2
0.05	1.122	1.125	1.127	1.130	1.132	1.135	1.138	1.140	1.143	1.146	0	1	1	1	1	2	2	2	2
0.06	1.148	1.151	1.153	1.156	1.159	1.161	1.164	1.167	1.169	1.172	0	1	1	1	1	2	2	2	2
0.07	1.175	1.178	1.180	1.183	1.186	1.189	1.191	1.194	1.197	1.199	0	1	1	1	1	2	2	2	2
0.08	1.202	1.205	1.208	1.211	1.213	1.216	1.219	1.222	1.225	1.227	0	1	1	1	1	2	2	2	3
0.09	1.230	1.233	1.236	1.239	1.242	1.245	1.247	1.250	1.253	1.256	0	1	1	1	1	2	2	2	3
0.10	1.259	1.262	1.265	1.268	1.271	1.274	1.276	1.279	1.282	1.285	0	1	1	1	1	2	2	2	3
0.11	1.288	1.291	1.294	1.297	1.300	1.303	1.306	1.309	1.312	1.315	0	1	1	1	2	2	2	2	3
0.12	1.318	1.321	1.324	1.327	1.330	1.334	1.337	1.340	1.343	1.346	0	1	1	1	2	2	2	2	3
0.13	1.349	1.352	1.355	1.358	1.361	1.365	1.368	1.371	1.374	1.377	0	1	1	1	2	2	2	3	3
0.14	1.380	1.384	1.387	1.390	1.393	1.396	1.400	1.403	1.406	1.409	0	1	1	1	2	2	2	3	3
0.15	1.413	1.416	1.419	1.422	1.426	1.429	1.432	1.435	1.439	1.442	0	1	1	1	2	2	2	3	3
0.16	1.445	1.449	1.452	1.455	1.459	1.462	1.466	1.469	1.472	1.476	0	1	1	1	2	2	2	3	3
0.17	1.479	1.483	1.486	1.489	1.493	1.496	1.500	1.503	1.507	1.510	0	1	1	1	2	2	2	3	3
0.18	1.514	1.517	1.521	1.524	1.528	1.531	1.535	1.538	1.542	1.545	0	1	1	1	2	2	2	3	3
0.19	1.549	1.552	1.556	1.560	1.563	1.567	1.570	1.574	1.578	1.581	0	1	1	1	2	2	3	3	3
0.20	1.585	1.589	1.592	1.596	1.600	1.603	1.607	1.611	1.614	1.618	0	1	1	1	2	2	3	3	3
0.21	1.622	1.626	1.629	1.633	1.637	1.641	1.644	1.648	1.652	1.656	0	1	1	2	2	2	3	3	3
0.22	1.660	1.663	1.667	1.671	1.675	1.679	1.683	1.687	1.690	1.694	0	1	1	2	2	2	3	3	3
0.23	1.698	1.702	1.706	1.710	1.714	1.718	1.722	1.726	1.730	1.734	0	1	1	2	2	2	3	3	4
0.24	1.738	1.742	1.746	1.750	1.754	1.758	1.762	1.766	1.770	1.774	0	1	1	2	2	2	3	3	4
0.25	1.778	1.782	1.786	1.791	1.795	1.799	1.803	1.807	1.811	1.816	0	1	1	2	2	2	3	3	4
0.26	1.820	1.824	1.828	1.832	1.837	1.841	1.845	1.849	1.854	1.858	0	1	1	2	2	3	3	3	4
0.27	1.862	1.866	1.871	1.875	1.879	1.884	1.888	1.892	1.897	1.901	0	1	1	2	2	3	3	3	4
0.28	1.905	1.910	1.914	1.919	1.923	1.928	1.932	1.936	1.941	1.945	0	1	1	2	2	3	3	4	4
0.29	1.950	1.954	1.959	1.963	1.968	1.972	1.977	1.982	1.986	1.991	0	1	1	2	2	3	3	4	4
0.30	1.995	2.000	2.004	2.009	2.014	2.018	2.023	2.028	2.032	2.037	0	1	1	2	2	3	3	4	4
0.31	2.042	2.046	2.051	2.056	2.061	2.065	2.070	2.075	2.080	2.084	0	1	1	2	2	3	3	4	4
0.32	2.089	2.094	2.099	2.104	2.109	2.113	2.118	2.123	2.128	2.133	0	1	1	2	2	3	3	4	4
0.33	2.138	2.143	2.148	2.153	2.158	2.163	2.168	2.173	2.178	2.183	0	1	1	2	2	3	3	4	4
0.34	2.188	2.193	2.198	2.203	2.208	2.213	2.218	2.223	2.228	2.234	1	1	2	2	3	3	4	4	5
0.35	2.239	2.244	2.249	2.254	2.259	2.265	2.270	2.275	2.280	2.286	1	1	2	2	3	3	4	4	5
0.36	2.291	2.296	2.301	2.307	2.312	2.317	2.323	2.328	2.333	2.339	1	1	2	2	3	3	4	4	5
0.37	2.344	2.350	2.355	2.360	2.366	2.371	2.377	2.382	2.388	2.393	1	1	2	2	3	3	4	4	5
0.38	2.399	2.404	2.410	2.415	2.421	2.427	2.432	2.438	2.443	2.449	1	1	2	2	3	3	4	4	5
0.39	2.455	2.460	2.466	2.472	2.477	2.483	2.489	2.495	2.500	2.506	1	1	2	2	3	3	4	5	5
0.40	2.512	2.518	2.523	2.529	2.535	2.541	2.547	2.553	2.559	2.564	1	1	2	2	3	4	4	5	5
0.41	2.570	2.576	2.582	2.588	2.594	2.600	2.606	2.612	2.618	2.624	1	1	2	2	3	4	4	5	5
0.42	2.630	2.636	2.642	2.649	2.655	2.661	2.667	2.673	2.679	2.685	1	1	2	2	3	4	4	5	6
0.43	2.692	2.698	2.704	2.710	2.716	2.723	2.729	2.735	2.742	2.748	1	1	2	3	3	4	4	5	6
0.44	2.754	2.761	2.767	2.773	2.780	2.786	2.793	2.799	2.805	2.812	1	1	2	3	3	4	4	5	6
0.45	2.818	2.825	2.831	2.838	2.844	2.851	2.858	2.864	2.871	2.877	1	1	2	3	3	4	5	5	6
0.46	2.884	2.891	2.897	2.904	2.911	2.917	2.924	2.931	2.938	2.944	1	1	2	3	3	4	5	5	6
0.47	2.951	2.958	2.965	2.972	2.979	2.985	2.992	2.999	3.006	3.013	1	1	2	3	3	4	5	5	6
0.48	3.020	3.027	3.034	3.041	3.048	3.055	3.062	3.069	3.076	3.083	1	1	2	3	4	4	5	6	6
0.49	3.090	3.097	3.105	3.112	3.119	3.126	3.133	3.141	3.148	3.155	1	1	2	3	4	4	5	6	6

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

ANTILOG TABLE

											Mean Difference								
0.50	3.162	3.170	3.177	3.184	3.192	3.199	3.206	3.214	3.221	3.228	1	1	2	3	4	4	5	6	7
0.51	3.236	3.243	3.251	3.258	3.266	3.273	3.281	3.289	3.296	3.304	1	2	2	3	4	5	5	6	7
0.52	3.311	3.319	3.327	3.334	3.342	3.350	3.357	3.365	3.373	3.381	1	2	2	3	4	5	5	6	7
0.53	3.388	3.396	3.404	3.412	3.420	3.428	3.436	3.443	3.451	3.459	1	2	2	3	4	5	6	6	7
0.54	3.467	3.475	3.483	3.491	3.499	3.508	3.516	3.524	3.532	3.540	1	2	2	3	4	5	6	6	7
0.55	3.548	3.556	3.565	3.573	3.581	3.589	3.597	3.606	3.614	3.622	1	2	2	3	4	5	6	7	7
0.56	3.631	3.639	3.648	3.656	3.664	3.673	3.681	3.690	3.698	3.707	1	2	2	3	4	5	6	7	8
0.57	3.715	3.724	3.733	3.741	3.750	3.758	3.767	3.776	3.784	3.793	1	2	3	3	4	5	6	7	8
0.58	3.802	3.811	3.819	3.828	3.837	3.846	3.855	3.864	3.873	3.882	1	2	3	4	4	5	6	7	8
0.59	3.890	3.899	3.908	3.917	3.926	3.936	3.945	3.954	3.963	3.972	1	2	3	4	5	5	6	7	8
0.60	3.981	3.990	3.999	4.009	4.018	4.027	4.036	4.046	4.055	4.064	1	2	3	4	5	6	6	7	8
0.61	4.074	4.083	4.093	4.102	4.111	4.121	4.130	4.140	4.150	4.159	1	2	3	4	5	6	7	8	9
0.62	4.169	4.178	4.188	4.198	4.207	4.217	4.227	4.236	4.246	4.256	1	2	3	4	5	6	7	8	9
0.63	4.266	4.276	4.285	4.295	4.305	4.315	4.325	4.335	4.345	4.355	1	2	3	4	5	6	7	8	9
0.64	4.365	4.375	4.385	4.395	4.406	4.416	4.426	4.436	4.446	4.457	1	2	3	4	5	6	7	8	9
0.65	4.467	4.477	4.487	4.498	4.508	4.519	4.529	4.539	4.550	4.560	1	2	3	4	5	6	7	8	9
0.66	4.571	4.581	4.592	4.603	4.613	4.624	4.634	4.645	4.656	4.667	1	2	3	4	5	6	7	9	10
0.67	4.677	4.688	4.699	4.710	4.721	4.732	4.742	4.753	4.764	4.775	1	2	3	4	5	7	7	9	10
0.68	4.786	4.797	4.808	4.819	4.831	4.842	4.853	4.864	4.875	4.887	1	2	3	4	5	7	8	9	10
0.69	4.898	4.909	4.920	4.932	4.943	4.955	4.966	4.977	4.989	5.000	1	2	3	4	5	7	8	9	10
0.70	5.012	5.023	5.035	5.047	5.058	5.070	5.082	5.093	5.105	5.117	1	2	3	4	5	7	8	9	11
0.71	5.129	5.140	5.152	5.164	5.176	5.188	5.200	5.212	5.224	5.236	1	2	4	5	6	7	8	10	11
0.72	5.248	5.260	5.272	5.284	5.297	5.309	5.321	5.333	5.346	5.358	1	2	4	5	6	7	9	10	11
0.73	5.370	5.383	5.395	5.408	5.420	5.433	5.445	5.458	5.470	5.483	1	3	4	5	6	8	9	10	11
0.74	5.495	5.508	5.521	5.534	5.546	5.559	5.572	5.585	5.598	5.610	1	3	4	5	6	8	9	10	12
0.75	5.623	5.636	5.649	5.662	5.675	5.689	5.702	5.715	5.728	5.741	1	3	4	5	7	8	9	10	12
0.76	5.754	5.768	5.781	5.794	5.808	5.821	5.834	5.848	5.861	5.875	1	3	4	5	7	8	9	11	12
0.77	5.888	5.902	5.916	5.929	5.943	5.957	5.970	5.984	5.998	6.012	1	3	4	5	7	8	10	11	12
0.78	6.026	6.039	6.053	6.067	6.081	6.095	6.109	6.124	6.138	6.152	1	3	4	6	7	8	10	11	13
0.79	6.166	6.180	6.194	6.209	6.223	6.237	6.252	6.266	6.281	6.295	1	3	4	6	7	9	10	11	13
0.80	6.310	6.324	6.339	6.353	6.368	6.383	6.397	6.412	6.427	6.442	1	3	4	6	7	9	10	12	13
0.81	6.457	6.471	6.486	6.501	6.516	6.531	6.546	6.561	6.577	6.592	2	3	5	6	8	9	11	12	14
0.82	6.607	6.622	6.637	6.653	6.668	6.683	6.699	6.714	6.730	6.745	2	3	5	6	8	9	11	12	14
0.83	6.761	6.776	6.792	6.808	6.823	6.839	6.855	6.871	6.887	6.902	2	3	5	6	8	9	11	13	14
0.84	6.918	6.934	6.950	6.966	6.982	6.998	7.015	7.031	7.047	7.063	2	3	5	6	8	10	11	13	15
0.85	7.079	7.096	7.112	7.129	7.145	7.161	7.178	7.194	7.211	7.228	2	3	5	7	8	10	12	13	15
0.86	7.244	7.261	7.278	7.295	7.311	7.328	7.345	7.362	7.379	7.396	2	3	5	7	8	10	12	13	15
0.87	7.413	7.430	7.447	7.464	7.482	7.499	7.516	7.534	7.551	7.568	2	3	5	7	9	10	12	14	16
0.88	7.586	7.603	7.621	7.638	7.656	7.674	7.691	7.709	7.727	7.745	2	3	5	7	9	10	12	14	16
0.89	7.762	7.780	7.798	7.816	7.834	7.852	7.870	7.889	7.907	7.925	2	4	5	7	9	11	12	14	16
0.90	7.943	7.962	7.980	7.998	8.017	8.035	8.054	8.072	8.091	8.110	2	4	6	7	9	11	13	15	17
0.91	8.128	8.147	8.166	8.185	8.204	8.222	8.241	8.260	8.279	8.299	2	4	6	8	9	11	13	15	17
0.92	8.318	8.337	8.356	8.375	8.395	8.414	8.433	8.453	8.472	8.492	2	4	6	8	10	12	14	15	17
0.93	8.511	8.531	8.551	8.570	8.590	8.610	8.630	8.650	8.670	8.690	2	4	6	8	10	12	14	16	18
0.94	8.710	8.730	8.750	8.770	8.790	8.810	8.831	8.851	8.872	8.892	2	4	6	8	10	12	14	16	18
0.95	8.913	8.933	8.954	8.974	8.995	9.016	9.036	9.057	9.078	9.099	2	4	6	8	10	12	15	17	19
0.96	9.120	9.141	9.162	9.183	9.204	9.226	9.247	9.268	9.290	9.311	2	4	6	8	11	13	15	17	19
0.97	9.333	9.354	9.376	9.397	9.419	9.441	9.462	9.484	9.506	9.528	2	4	7	9	11	13	15	17	20
0.98	9.550	9.572	9.594	9.616	9.638	9.661	9.683	9.705	9.727	9.750	2	4	7	9	11	13	16	18	20
0.99	9.772	9.795	9.817	9.840	9.863	9.886	9.908	9.931	9.954	9.977	2	5	7	9	11	14	16	18	20

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

GLOSSARY
கலைச்சொற்கள்

BH9NIX

1.	Altitude	-	குத்துயரம்
2.	Astronomy	-	வானியல்
3.	Angle of Contact	-	தொடுகோணம்
4.	Aerofoil lift	-	விமான இறக்கை உயர்த்தல்
5.	Adiabatic process	-	வெப்ப பரிமாற்றமில்லா நிகழ்வு
6.	Average (or) Mean speed	-	சராசரி வேகம்
7.	Angular Harmonic motion	-	கோணச்சீரிசை இயக்கம்
8.	Beats	-	விம்மல்கள்
9.	Buoyant force	-	மிதப்பு விசை
10.	Breaking of rupture point	-	முறிவுப்புள்ளி
11.	Compressive stress	-	அமுக்கத்தகைவு
12.	Calorimeter	-	வெப்ப அளவீட்டியல்
13.	Conduction	-	வெப்பக்கடத்தல்
14.	Change of state	-	நிலை மாற்றம்
15.	Capillary rise or fall	-	நுண்புழை ஏற்றம் அல்லது இறக்கம்
16.	Compress	-	அமுக்கம் / இறுக்கம்
17.	Compliance	-	இணக்கம்
18.	Coefficient of performance	-	செயல்திறன் குணகம்
19.	Degree of freedom	-	சுதந்திர இயக்கக்கூறு
20.	Damped oscillation	-	தடையுறு இயல்பு அதிர்வுகள்
21.	Elongate	-	நீட்சி
22.	Elastic limit	-	மீட்சி எல்லை
23.	Escape speed	-	விடுபடு வேகம்
24.	Epicycle	-	பெரு வட்டத்தில் அமையும் சிறு வட்டச் சுழற்சி
25.	Epoch	-	தொடக்கக்கட்டம்
26.	Equilibrium	-	சமநிலை
27.	Flexible constant	-	நெகிழ்வு தன்மை மாறிலி
28.	Free oscillations	-	தனி அலைவியக்கம்
29.	Force constant	-	விசை மாறிலி
30.	Forced Oscillation	-	திணிக்கப்பட்ட அலைவுகள்
31.	Frequency	-	அதிர்வெண்
32.	Geocentric model	-	புவிமையக் கோட்பாட்டு மாதிரி
33.	Gravitational field	-	ஈர்ப்பு புலச்செறிவு (அல்லது) ஈர்ப்புப்புலம்

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

34. Gravitational Potential - ஈர்ப்பழுத்தம்
35. Gravitational Potential energy - ஈர்ப்பழுத்த ஆற்றல்
36. Geo- Stationary satellite - புவி நிலைத் துணைக்கோள்
37. Hydrostatic paradox - நீர்ம நிலையியல் முரண்பாடு
38. Heat engine - வெப்ப இயந்திரம்
39. Hydraulic lift - நீரியல் தூக்கி
40. Harmonics - சீரிசை
41. Heliocentric model - சூரிய மையக்கோட்பாடு
42. Interference - குறுக்கீட்டு விளைவு
43. Isothermal process - வெப்பநிலை மாறா நிகழ்வு
44. Isobaric process - அழுத்தம் மாறா நிகழ்வு
45. Isochoric process - பருமன் மாறா நிகழ்வு
46. Lateral strain - பக்கவாட்டுத்திரிபு
47. Longitudinal strain - நீளவாட்டுத்திரிபு
48. Longitudinal stress - நீட்சித்தகைவு
49. Loudness - உரப்பு / ஒலி உரப்பு
50. Lunar Eclipse - சந்திர கிரகணம்
51. Latent heat - உள்ளுறை வெப்பம்
52. Latitude - குறுக்குக்கோடு / அட்சக்கோடு
53. Law of equipartition of energy - ஆற்றல் சமபங்கீட்டு விதி
54. Mean square speed - சராசரி இருமடிவேகம்
55. Most probable speed - மிகவும் சாத்தியமான வேகம்
56. Mean free path - சராசரி மோதலிடைத்தூரம்
57. Maintained Oscillation - நிலைநிறுத்தப்பட்ட அதிர்வுகள்
58. Mean position - நடுநிலை
59. Number density - எண் அடர்த்தி
60. Node - கணு
61. Natural oscillation - இயல்பு அதிர்வுகள்
62. Non-periodic motion - சீரலைவற்ற இயக்கம்
63. Oscillatory motion - அலைவியக்கம்
64. Orbital velocity - சுற்றியக்க திசைவேகம்
65. Overtone - மேற்சுரம்
66. Polar satellite - துருவ துணைக்கோள்
67. Phase - கட்டம்
68. Periodic motion - சீரலைவு இயக்கம்
69. Phase difference - கட்ட வேறுபாடு
70. Penumbra - பகுதி நிழல்
71. Retrograde motion - பின்நோக்கு இயக்கம்
72. Restoring force - மீள்விசை
73 . Restoring Torque - மீள்திருப்பு விசை
74. Random motion - ஒழுங்கற்ற இயக்கம் / சீரற்ற இயக்கம்
75. Root mean square speed (Vrms) - சராசரி இருமடிமூல வேகம்
76. Radiation - வெப்பக்கதிர்வீச்சு

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

77.	Resonance	-	ஒத்ததிர்வு
78.	Ripples	-	சிற்றலைகள்
79.	Standard (or) Normal temperature and pressure	-	படித்தர (அல்லது) இயல்பு வெப்பநிலை மற்றும் அழுத்தம்
80.	Stethoscope	-	இதயத்துடிப்பு அறியும் கருவி
81.	Sonometer	-	சுரமானி
82.	Superposition	-	மேற்பொருந்துதல்
83.	Spring constant / force constant	-	சுருள்மாறிலி
84.	Stiffness constant	-	முறுக்கு மாறிலி
85.	Simple pendulum	-	தனி ஊசல்
86.	Satellite	-	துணைக்கோள்
87.	Stationary waves	-	நிலை அலைகள்
88.	Shearing Stress	-	சறுக்குப்பெயர்ச்சித் தகைவு
89.	Surface tension	-	பரப்பு இழுவிசை
90.	Streamlined flow	-	வரிச்சீர் ஓட்டம்
91.	Specific heat capacity	-	தன்வெப்ப ஏற்புத்திறன்
92.	Speed distribution function	-	வேகப் பகிர்வுச் சார்பு
93.	Specific heat capacity at constant pressure	-	அழுத்தம் மாறா தன்வெப்ப ஏற்புத்திறன்
94.	Specific heat capacity at constant volume	-	பருமன் மாறா தன்வெப்ப ஏற்புத்திறன்
95.	Simple Hormonic motion	-	தனிச்சீரிசை இயக்கம்
96.	Tensile stress	-	இழுவிசைத்தகைவு
97.	Terminal velocity	-	முற்றுத் திசைவேகம்
98.	Time period	-	அலைவுக்காலம்
99.	Turbulent flow	-	சுழற்சி ஓட்டம்
100.	Tsunami	-	ஆழிப்பேரலை
101.	Thermal conductivity	-	வெப்பக்கடத்துத்திறன்
102.	Thermometer	-	வெப்பநிலைமானி
103.	Triple Point	-	முப்புள்ளி
104.	Universal gas constant	-	பொது வாயுமாறிலி
105.	Umbra	-	கருநிழல்
106.	Viscosity	-	பாகுநிலை
107.	Water striders	-	நீர்தாண்டிப்பூச்சிகள்
108.	Wavicle	-	அலைத்துகள்
109.	Zig – Zag path	-	குறுக்கு – நெடுக்கானப் பாதை

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

State Council of Educational Research and Training, Chennai-6

Higher Secondary Physics

List of Subject Experts in Textbook Writing

Domain Expert & Mentor

Prof. Dr. Rita John
Professor and Head,
Department of Theoretical Physics
University of Madras, Chennai.

Reviewers

Dr. V. N. Mani.
Principal Scientist F, Head (C-MET)
Department of Electronic &
Information Technology, Hyderabad,
Government of India.

Prof. Dr. P. Ravindran
Dept. of. Physics,
School of Basic and Applied Sciences,
Central University of Tamil Nadu, Thiruvarur.

Dr. Rajeev Shesha Joshi
Assistant Professor
School of Physical Sciences,
Central University of Karnataka.

Content Writers
Content developed by

Dr.K.Rajarajan
Associate Professor
Department of Physics
R.V. Government. Arts College
Chengalpattu.

Dr. S. S.Naina Mohammed
Assistant Professor
PG & Research Department of Physics,
Government Arts College,
Udumalpet, Tiruppur Dt.

Mr. C. Joseph Prabagar
Assistant Professor
PG & Research Department of Physics,
Loyola College, Chennai

Content modified by

Prof. Dr. Rita John
Prof. Dr. P. Ravindran
Dr. Rajeev Shesha Joshi

Tamil Translators

Dr. S. Sritharan
Head Master,
Government Hr. Sec. School,
Karaikurichipudur, Namakkal Dt.

Mr. A .Ganesh
Head Master,
Government Higher School,
Somangalam, Kanchipuram Dt.

E.Elangovan
Head master,
Government Hr. Sec. School,
Ramanayakenpettai,
Vellore Dt.

Mr.M.Palanikumar
PGT, MNMSP Senthi Kumara Nadar Higher Secondary School, Mallankinaru
Virudhunagar Dt.

Dr.K.Vasudevan
PGT, Govt. ADW. Hr. Sec. School
Kalangani, Namakkal Dt

Mr. T. Thamaraiselvan
PGT, Government Hr. Sec. School,
Aranthangi, Pudukottai Dt.

Content Readers

Dr.V.Sivamadhavi
Associate Professor
Department of Physics
Bharathi Women's College Chennai

Mr.M.Palanichamy
Head Master, Government Hr. Sec. School
Krishnarayapuram Karur Dt.

Dr.S.Ravikasi Venkatraman
Head Master, Government Girls. Hr. Sec. School
Desur, Tiruvannamalai Dt.

Mr.N.Thamaraikannan
PG Assistant, JGN Hr. Sec. School
Tambaram, Kanchipuram Dt.

Mr.T.Subbiah
PG Assistant, Government Girls Hr. Sec. School
Achirupakkam, Kanchipuram Dt.

Subject Coordinators

Mrs. P. Nantha
Senior Lecturer
SCERT, Chennai

Mrs. D. Shanmugasundari
Graduate Assistant
P.U.M. School Melathulukkankulam
Kariyapatti Union, Virudhunagar Dt.

Mr.G.Arulraja
Graduate Assistant
MKV Govt. Boys. Hr. Sec. School
Arani, Thiruvallur Dt.

ICT Coordinators

Mr.G. Bergin
PG Assistant (Physics)
Government Boys Higher Secondary School,
Sayalkudi
Ramanathapuram Dt.

Saranya.A
SGT, P.U.P.S,
Sellathapalayam,
Modakkurichi block,
Erode Dt.

Kamesh. R
SGT, P.U.P.S, Pinjanur,
Nallur Block, Cuddalore Dt.

QR CODE MANAGEMENT TEAM

R. Jaganathan
Pums Ganesapuram- Polur
Thiruvannamalai Dt.

N. Jagan
Gbhss Uthiramerur
Kancheepuram Dt.

J.F.Paul Edwin Roy
Pums Rakkipatti
Salem Dt.

Typist
Mrs.D.Kavitha

Art and Design Team

Layout
Asker Ali.M

Illustration
K. Sasikumar B.F.A
S. Duragadevi

Inhouse-QC
P. Arun Kamaraj
Jerald Wilson

Cover Design
Kathir Arumugam

Co-ordination
Ramesh Munisamy

This book has been printed on 80 G.S.M.
Elegant Maplitho paper.
Printed by offset at:

athiyamanteam.com | TNPSC Exam - Video Class+ Test Batch-| 8681859181

NOTE

NOTE

NOTE